

1. firm 1 : $\pi_1 = TR_1 - TC_1$

$$= (a - bQ_1 - bQ_2 - bQ_3)Q_1 - c_1$$

$$\frac{d\pi_1}{dQ_1} = a - bQ_2 - bQ_3 - c - 2bQ_1$$

$$Q_1 = \frac{a - bQ_2 - bQ_3 - c}{2b}$$

from $\pi_1 = \pi_2 = \pi_3 = \pi_{AN}$

$$Q_2 = \frac{a - bQ_1 - bQ_3 - c}{2b}$$

$$Q_3 = \frac{a - bQ_1 - bQ_2 - c}{2b}$$

Sub Q_2 into Q_1 : $Q_1 = \frac{a - b\left(\frac{a - bQ_1 - bQ_3 - c}{2b}\right) - bQ_3 - c}{2b}$

$$Q_1 = \frac{a - bQ_3}{3b}$$

Sub Q_1 into Q_2 : $Q_2 = \frac{a - b\left(\frac{a - bQ_3}{3b}\right) - bQ_3 - c}{2b}$

$$Q_2 = \frac{a - bQ_3}{3b}$$

Sub Q_1 and Q_2 into Q_3

$$Q_3 = \frac{a - b\left(\frac{a - bQ_3}{3b}\right) - b\left(\frac{a - bQ_3}{3b}\right) - c}{2b}$$

$$Q_3 = \frac{a + 2bQ_3}{6b}$$

$$Q_3 = \frac{a}{4b}$$

∴ $Q_1 = Q_2 = Q_3$

$$P = a - bQ = a - bQ = a - 3b\left(\frac{a}{4b}\right) = \frac{a}{4}$$

∴ $\pi_1 = \pi_2 = \pi_3 = \pi_{AN}$

$$\pi = (P - C)Q = \left(\frac{a}{4} - c\right) = \frac{a}{4b} = \frac{a^2}{16b} - c$$

Done

$$2. \quad \text{Assume } Q_1 + Q_2 + Q_3 + \dots + Q_n = A$$

$$P = a - b(Q_1 + Q_2 + Q_3 + \dots + Q_n)$$

$$\pi_1 = a - b(Q_1 + Q_2 + Q_3 + \dots + Q_n)Q_1 - C_1$$

$$\frac{d\pi_1}{dQ_1} = a - 2bQ_1 + bQ_2 + bQ_3 + \dots + bQ_n = 0$$

$$Q_1 = \frac{a}{2b} - \frac{1}{2}(Q_2 + Q_3 + \dots + Q_n)$$

$$Q_n = \frac{a}{2b} - \frac{1}{2}(Q_1 + Q_2 + \dots + Q_{n-1})$$

$$Q_1 = \frac{1}{2}Q_1 = \frac{a}{2b} - \frac{A}{2}$$

$$Q_1 = \frac{a}{b} - A = Q_2 = Q_3 = \dots = Q_n$$

$$\text{As } A = Q_1 + Q_2 + Q_3 + \dots + Q_n$$

$$A = n \left(\frac{a}{b} - A \right)$$

$$A = \frac{na}{b} - nA$$

$$A + nA = \frac{na}{b}$$

$$A(1+n) = \frac{na}{b}$$

$$A = \frac{na}{(1+n)b}$$

$$P = a - bA = a - b \left(\frac{na}{(n+1)b} \right) = a - \frac{na}{n+1} = a - a \left(\frac{n}{n+1} \right)$$

$$= \frac{a(n+1) - an}{(n+1)} = \frac{a}{(n+1)}$$

$$\pi = P \cdot Q - C = \frac{a}{n+1} \cdot \frac{a}{(n+1)b} - C$$

$$= \frac{a^2}{(n+1)^2 b} - C$$

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3. If $n \rightarrow \infty$

$q_i = \frac{a}{(n+1)b} \rightarrow 0$, each firm will sell a nearly 0 unit.

$A = \frac{na}{(n+1)b} \rightarrow 0$, Q of firms combine will nearly ∞ units.

$P = \frac{a}{(n+1)b} \rightarrow 0$, when supply increase, P will decrease nearly 0.

$\pi = \frac{a^2}{(n+1)^2 b} - c$, each firm will lose profit.

If $n = 2$

$q_i = \frac{a}{(n+1)b} = \frac{a}{2b}$, monopoly will sell less.

$A = \frac{na}{(n+1)b} = \frac{a}{2b}$, firm will become monopolist.

$P = \frac{a}{(n+1)b} = \frac{a}{2b}$, monopoly will set higher price.

$\pi = \frac{a^2}{(n+1)^2 b} - c = \frac{a^2}{4b} - c$, monopolist will gain higher profit.

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