BACHELOR OF ECONOMICS
THAMMASAT UNIVERSITY

EE325 Introductory Econometrics, Semester 1/2019 (Section 046402)

Due Date: Thursday 27th February 2020 by 09.30 via Assignment Submission in Moodle.

Instruction: Do all questions with your own handwriting and your own attempt.

Use 4 decimal places for numerical answers

1. In Table 1. $X_i$is total econometrics exam point (total points are 100) and Y_i is GPA of each BE student.

77.625

Table 1

.566

$(x_i-\bar{x})^2$	$x_i-\bar{x}$	$(y_i-\hat{y})^2)$	Student	Y_i	X_i	x_iY_i	x_i^2	$\hat{Y} = .572 + .034x_i$	$\hat{u}_i = Y_i - \hat{Y}$
213.891	-14.625	.007	1	2.8	63	176.4	3969	2.714	.086
31.641	-5.625	.144	2	3.4	72	244.8	5184	3.02	.38
0.141	.375	.05	3	3.0	78	234	6084	3.224	-.224
11.391	3.375	.03	4	3.5	81	283.5	6561	3.326	.174
87.891	9.375	.005	5	3.6	87	313.2	7569	3.53	.07
6.891	-2.625	.015	6	3.0	75	225	5625	3.122	-.122
6.891	-2.625	.178	7	2.7	75	202.5	5625	3.122	-.422
153.141	12.375	.005	8	3.7	90	333	8100	3.632	.068
511.828	0	.434	Σ	25.7	621	2012.4	48,717	25.686	0.01

1.1 Now consider the two-variable model $Y_i = \beta_1 + \beta_2 X_i + u_i,\ u_i \sim NIID(0, \sigma^2)$

Use OLS to find the estimator of $\beta_1 and \beta_2$. Interpret the regression.

1.2 Find $\hat{Y}_i \quad and\ \hat{u}_i$ and show that $\sum_{i=1}^{n} \hat{u}_i \approx 0$

1.3 Find $var(\hat{u}_i), var(\hat{\beta}_1), and\ var(\hat{\beta}_2)$

$$var(\hat{u}_i) = \frac{\Sigma(y_i-\hat{y})^2}{n-k} = \frac{.434}{6} = .071 = \sigma^2$$

1.1

$$\beta_2 = \frac{8(2012.4) - (621)(25.7)}{8(48717) - 621}$$

$$= \frac{139.5}{4095}$$

$$= .34$$

$$\beta_1 = \frac{25.7}{8} - (.34)\left(\frac{621}{8}\right)$$

$$= 3.2125 - 2.639$$

$$\beta_1 = .5735$$

$$\hat{y} = .5735 + .34x_i$$

interpret β_1 (intercept) when $x=0$, y will equal to .5735 on average

interpret β_2 (slope) when x increase by 1 unit, y will increase by .34 on average

1.2

$\hat{y} = .572 + .034x_i$	$\hat{u}_i = y_i - \hat{y}$
2.714	.086
3.02	.38
3.224	-.224
3.326	.174
3.53	.07
3.122	-.122
3.122	-.422
3.632	.068
25.686	0.01

$$\text{var}(\hat{u}_i) = \frac{\Sigma(y_i - \hat{y})^2}{n-k} = \frac{.434}{6} = .071 = \sigma^2$$

1.3

$$\text{var}(\hat{u}_i) = \frac{\Sigma(y_i - \hat{y})^2}{n-k}$$

$$= \frac{.434}{6}$$

$$= .071 \text{ aka } \sigma^2$$

$$\text{var}(\hat{\beta}_2) = \frac{\sigma^2}{\Sigma(x_i - \bar{x})^2}$$

$$= \frac{.072}{511.678}$$

$$= .0001$$

$$\text{var}(\hat{\beta}_1) = \hat{\sigma}^2\left(\frac{1}{8} + \frac{\bar{x}^2}{\Sigma x^2}\right)$$

$$= .072\left(\frac{1}{8} + \frac{6025.641}{511.878}\right)$$

$$= .072\left(\frac{1}{8} + 11.772\right)$$

$$= .072(11.897)$$

$$= .857$$

$\bar{x} = 20$

2. Data is listed in the table

$(x_i-\bar{x})^2$	$x_i-\bar{x}$	$(y_i-\hat{y})^2$	X_i	Y_i	X_iY_i	X^2	$\hat{Y} = -8.809 + .895x_i$	$\hat{U}_i = Y_i - \hat{Y}$
100	-10	.021	10	0	0	100	.145	-.145
64	-8	.004	12	2	24	144	1.936	.064
36	-6	1.621	14	5	70	196	3.727	1.272
16	-4	.232	16	6	96	256	5.518	.482
4	-2	.095	18	7	126	324	7.309	-.309
4	2	.801	22	10	220	484	10.891	-.891
16	4	7.193	24	10	240	576	12.682	-2.682
36	6	.278	26	15	390	676	14.473	.527
64	8	.07	28	16	448	784	16.264	-.264
100	10	3.783	30	20	600	900	18.055	1.945
440		14.098	200	91	2214	4440	91	.000015

2.1 From the simple regression model $Y_i = \beta_1 + \beta_2 X_i + u_i, u_i \sim NIID(0, \sigma^2)$

Find estimators of β_1 and β_2 from the OLS method and interpret the meaning.

2.2 Find the value of $\hat{Y}_i$ and $\hat{u}_i$. Show that $\sum \hat{u}_i \approx 0$

2.3 Plot graph and draw regression line. Does the line pass $(\bar{X}, \bar{Y})$?

2.4 If $X_i = 18$, what is the predicted Y? $\hat{Y} = \hat{\beta}_1 + \hat{\beta}_2 X_i (18)$

2.5 Find $var(\hat{u}_i)$, $var(\hat{\beta}_1)$, $var(\hat{\beta}_2)$

3. Consider the below regression function: consider the two-variable model

$\boldsymbol{Y_i = \beta_1 + \beta_2 X_i + u_i, u_i \sim NIID(0, \sigma^2)}$

Find an OLS estimator of β_1. Then, provide a proof that this is an unbiased estimator. Please state the assumption(s) of CLRM when used (pages 66-75 in Gujarati).

2.1

$$\hat{\beta}_1 = \bar{y} - \hat{\beta}_2 \bar{x}$$

$$\hat{\beta}_2 = \frac{n \sum x_i y_i - \sum x_i \sum y_i}{n \sum x_i^2 - (\sum x_i)^2}$$

$$\hat{\beta}_2 = \frac{(10)\,2214 - (200)(91)}{10\,(4440) - (200)^2}$$

$$\hat{\beta}_2 = \frac{22140 - 18{,}200}{44400 - 40{,}000}$$

$$\hat{\beta}_2 = \frac{3940}{4{,}400}$$

$$\hat{\beta}_2 = .895$$

$$\hat{\beta}_1 = \frac{91}{10} - (.895)\frac{200}{10}$$

$$\hat{\beta}_1 = 9.1 - 17.909$$

$$\hat{\beta}_1 = -8.809$$

$$\hat{y} = -8.809 + .895 x_i$$

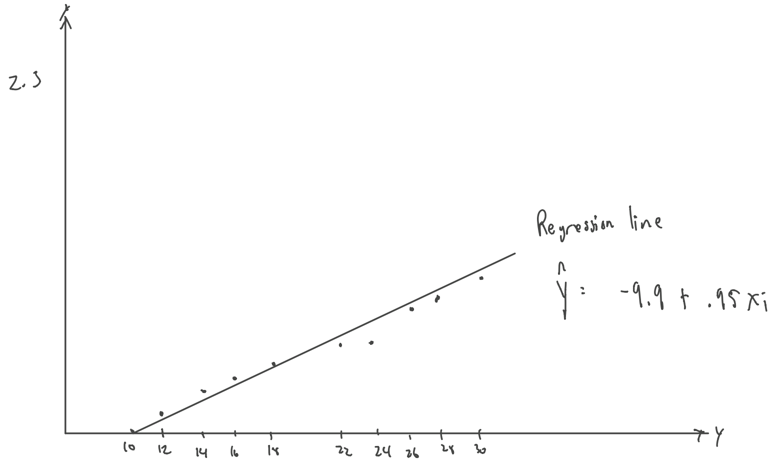
interpret β_1 (intercept), when $x=0$, y will be equaled to -8.809 on average

interpret β_2 (slope), when x increase by 1 unit, y will increase by $.895$ by average

2.2

$\hat{y} = -8.809 + .895x_i$	$\hat{u}_i = y_i - \hat{y}$
.145	-.145
1.936	.064
3.727	1.272
5.518	.482
7.309	-.309
10.891	-.891
12.682	-2.682
14.473	.527
16.264	-.264
18.055	1.945
91	.000015

2.3



2.4 $\hat{y} = \hat{\beta}_1 + \hat{\beta}_2 (18)$

$\hat{y} = -8.809 + .895 (18)$

$\hat{y} = 7.309$

$$\frac{\hat{\sigma}^2}{\Sigma(X_i - \bar{X})^2}$$

2.5 $var(\hat{u}_i) = \frac{\Sigma(y_i - \hat{y})^2}{8}$

$var(\hat{u}_i) = \frac{14.098}{8}$

$var(\hat{u}_i) = 1.762$ //

aka $\hat{\sigma}^2$ //

$var(\hat{\beta}_2) = \frac{\hat{\sigma}^2}{\Sigma(X_i - \bar{X})^2}$

$var(\hat{\beta}_2) = \frac{1.762}{440}$

$= .004$

$var(\hat{\beta}_1) = \hat{\sigma}^2 \left(\frac{1}{10} + \frac{\bar{X}^2}{\Sigma(X_i - \bar{X})^2} \right)$

$var(\hat{\beta}_1) = 1.762 \left(\frac{1}{10} + \frac{400}{440} \right)$

$= 1.762 (1.009)$

$= 1.778$ //

$var(\beta_2) = \frac{\sigma^2}{\Sigma(x_i - \bar{x})^2}$

3

D-I-Y VERIFY THAT $E(\hat{\beta}_1) = \beta_1$

From $\hat{\beta}_1 = \bar{y} - \hat{\beta}_2 \bar{X}$

$= \bar{y} - \sum_{i=1}^{n} k_i y_i \cdot \bar{X}$

$= \frac{\sum_{i=1}^{n} y_i}{n} - \bar{X} \sum_{i=1}^{n} k_i y_i$

$= \sum_{i=1}^{n} \left(\frac{1}{n} - \bar{X} \cdot k_i \right) y_i$

$= \sum_{i=1}^{n} \left(\frac{1}{n} - \bar{x} k_i \right)(\beta_1 + \beta_2 X_i + u_i)$

continue $\Sigma(\frac{1}{n} - \bar{x}k)\beta_1 + (\frac{1}{n} - \bar{x}k)\beta_2 x_i + (\frac{1}{n} - \bar{x}k) u_i)$

$E(\hat{\beta}_1) = E(\frac{1}{n}) - E(\cancel{\bar{x}k}\, \beta_1) - E(k(\beta_2 x_i)) + E(u_i)$

$E(\hat{\beta}_1) = \beta_1$

So, $\hat{\beta}_1$ is an unbiased estimator.