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EE325 Introductory Econometrics, Semester 1/2019 (Section 046402)

Due Date: Thursday 27th February 2020 by 09.30 via Assignment Submission in Moodle.

Instruction: Do all questions with your own handwriting and your own attempt.

Use 4 decimal places for numerical answers

1. In Table 1. X_i is total econometrics exam point (total points are 100) and Y_i is GPA of each BE student.

Table 1

$\bar{Y} = 3.2125$ $\quad \bar{X} = 77.625$

Student	Y_i	X_i	$x_i - \bar{x}$	$y_i - \bar{y}$	$(x_i-\bar{x})(y_i-\bar{y})$	$\Sigma(x_i-\bar{x})^2$
1	2.8	63	= -14.625	-0.4125	6.0328	= 511.875
2	3.4	72	= -5.625	0.1875	-1.0547	
3	3.0	78	= 0.375	-0.2125	-0.0797	
4	3.5	81	= 3.375	0.2875	0.9703	
5	3.6	87	= 9.375	0.3875	3.6328	
6	3.0	75	= -2.625	-0.2125	0.5578	
7	2.7	75	= -2.625	-0.5125	1.3453	
8	3.7	90	= 12.375	0.4875	6.0328	

$\Sigma = 17.4347$

1.1 Now consider the two-variable model $Y_i = \beta_1 + \beta_2 X_i + u_i,\ u_i \sim NIID(0, \sigma^2)$

Use OLS to find the estimator of $\beta_1 and \beta_2$. Interpret the regression.

1.2 Find $\hat{Y}_i \quad and\ \hat{u}_i$ and show that $\sum_{i=1}^{n} \hat{u}_i \approx 0$

1.3 Find $var(\hat{u}_i), var(\hat{\beta}_1), and\ var(\hat{\beta}_2)$

OLS: $\hat{\beta}_2 = \dfrac{\Sigma(Y_i-\bar{Y})(X_i-\bar{X})}{\Sigma(X_i-\bar{X})^2} = \dfrac{17.4347}{511.875} = 0.0341$

$\beta_1 = \bar{Y} - \hat{\beta}_2 \bar{X} = 3.2125 - (0.0341)(77.625)$

$= 0.3655$

$Y_i = \beta_1 + \beta_2 x_i$; $Y_i = 0.5655 + 0.0341 x_i + u_i$

1.2) $\hat{Y}_i$, $\hat{U}_i$

$\hat{Y}_i = 0.5655 + 0.0347 x_i$

$\hat{Y}_1 = 2.7138$

$\hat{Y}_2 = 3.0207$

$\hat{Y}_3 = 3.2253$

$\hat{Y}_4 = 3.3276$

$\hat{Y}_5 = 3.5322$

$\hat{Y}_6 = 3.1230$

$\hat{Y}_7 = 3.1230$

$\hat{Y}_8 = 3.6345$

$\hat{u}_i = Y_i - \hat{Y}_i$ $\quad \sum \hat{U}_i^2 = 0.4347$ $\quad 3.2125$

$\hat{u}_1 = 0.0862$

$\hat{u}_2 = 0.3793$

$\hat{U}_3 = -0.2253$

$\hat{U}_4 = 0.1724$

$\hat{U}_5 = 0.0678$

$\hat{U}_6 = -0.123$

$\hat{U}_7 = -0.423$

$\hat{U}_8 = 0.0655$

0.4347.

$$\sum_{i=1}^{8} \hat{U}_i = -0.0001 \approx 0$$

1.3) $$var(u) = \frac{\sum (Y_i - \hat{Y})^2}{n-2} = \frac{\sum \hat{u}^{2}}{n-2} = \frac{0.4347}{8-2} = 0.0725$$

SSR

$\sum x_i^2 = 48717$

$$var(\hat{\beta}_1) = \frac{\sum (x_i^2)}{n \sum x_i^2} \sigma^2 \; ; \; x_i = X_i - \bar{X} \; ; \; \sum x_i^2 = 511.875$$

$$= \frac{48717}{511.875(8)} (0.0725) = 0.8625$$

$$Var(\hat{\beta}_2) = \frac{\sigma^2}{\sum x_i^2} = \frac{0.0725}{511.875} = 0.0001.$$

2. Data is listed in the table

$\bar{X} = 20, \bar{Y} = 9.1$

	X_i	Y_i	$x_i - \bar{x}$	$y_i - \bar{y}$	$(x_i - \bar{x})(y_i - \bar{y})$	$\Sigma(x_i - \bar{x})(y_i - \bar{y})$
1	10	0	= -10	-9.1	91	= 394
2	12	2	= -8	-7.1	56.8	
3	14	5	-6	-4.1	24.6	
4	16	6	-4	-3.1	12.4	
5	18	7	-2	-2.1	4.2	
6	22	10	2	0.9	1.8	
7	24	10	4	0.9	3.6	
8	26	15	6	5.9	35.4	
9	28	16	8	6.9	55.2	
10	30	20	10	10.9	109	

2.1 From the simple regression model $Y_i = \beta_1 + \beta_2 X_i + u_i, u_i \sim NIID(0, \sigma^2)$

Find estimators of β_1 and β_2 from the OLS method and interpret the meaning.

2.2 Find the value of $\hat{Y}_i$ and $\hat{u}_i$. Show that $\sum \hat{u}_i \approx 0$

2.3 Plot graph and draw regression line. Does the line pass $(\bar{X}, \bar{Y})$?

2.4 If $X_i = 18$, what is the predicted Y?

2.5 Find $var(\hat{u}_i)$, $var(\hat{\beta}_1)$, $var(\hat{\beta}_2)$

3. Consider the below regression function: consider the two-variable model

$$\boldsymbol{Y_i = \beta_1 + \beta_2 X_i + u_i,\ u_i \sim NIID(0, \sigma^2)}$$

Find an OLS estimator of β_1. Then, provide a proof that this is an unbiased estimator. Please state the assumption(s) of CLRM when used (pages 66-75 in Gujarati).

OLS: $\hat{\beta}_2 = \dfrac{\Sigma(Y_i - \bar{Y})(X_i - \bar{X})}{\Sigma(X_i - \bar{X})^2} = \dfrac{394}{440} = 0.8955$

$\beta_1 = \bar{Y} - \hat{\beta}_2 \bar{X} = 9.1 - (0.8955)(20)$

$= -8.81$

$\hat{Y}_i = \hat{\beta}_1 + \hat{\beta}_2 X_i \; ; \; \hat{Y}_i = -8.81 + 0.8955 X_i$

"Practice makes Perfect."

2.2) $\hat{Y}_1 = 0.145$ $\qquad \hat{Y}_6 = 10.891$

$\hat{Y}_2 = 1.936$ $\qquad \hat{Y}_7 = 12.682$

$$\hat{Y}_3 = 3.727 \qquad \hat{Y}_8 = 14.473$$

$$\hat{Y}_4 = 5.518 \qquad \hat{Y}_9 = 16.264$$

$$\hat{Y}_5 = 7.309 \qquad \hat{Y}_{10} = 18.055$$

$$\hat{u}_i = Y_i - \hat{Y}_i$$

$$\hat{U}_1 = -0.145 \qquad \hat{U}_6 = -0.891$$

$$\hat{U}_2 = 0.064 \qquad \hat{U}_7 = -2.682$$

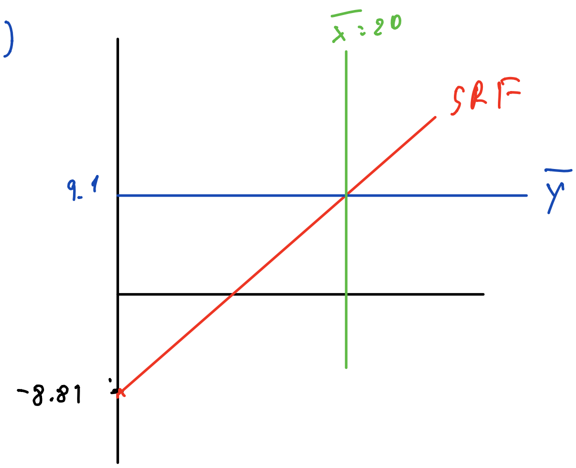
$$\hat{U}_3 = 1.273 \qquad \hat{U}_8 = 0.527$$

$$\hat{U}_4 = 0.482 \qquad \hat{U}_9 = -0.264$$

$$\hat{U}_5 = -0.309 \qquad \hat{U}_{10} = 1.945$$

$$\sum U_i = 0 \;//$$

2.3)



$$\hat{Y}_i = -8.81 + 0.8955x_i$$

$$\bar{Y} = 9.1, \quad \bar{x} = 20$$

$$\hat{Y}_i = -8.81 + 0.8955(20) = 9.1$$

$\therefore$ it pass the SRF line

2.4) $\hat{Y}_i = -8.81 + 0.8955 X_i$ If $X_i = 18$

$= -8.81 + 0.8955(18)$

$= 7.309$

2.5) Find $var(\hat{u}_i)$, $var(\hat{\beta}_1)$, $var(\hat{\beta}_2)$

$$var(\hat{u}_i) = \frac{\sum (Y_i - \hat{Y})^2}{n-2} = \frac{14.0909}{8} = 1.7614$$

$$var(\hat{\beta}_1) = \frac{\sum X_i^2}{n \sum (X_i - \bar{X})^2} = \frac{4,440}{10(440)} = 1.0091$$

$$var(\hat{\beta}_2) = \frac{\sigma^2}{\sum x_i^2} = \frac{1.7614}{4,440} = 0.0004$$

3) OLS $\hat{Y} = \hat{\beta}_1 + \hat{\beta}_2 \hat{x}_i + \hat{u}_i$

$\min \sum \hat{u}_i^2 \quad \hat{u}_i = \hat{y}_i - \hat{\beta}_i - \hat{\beta}_2 \hat{x}_i$

$\sum \hat{u}_i^2 = \sum (\hat{y}_i - \hat{\beta}_1 - \hat{\beta}_2 \hat{x}_i)^2$

$-2 \sum (\hat{y}_i - \hat{\beta}_1 - \hat{\beta}_2 \hat{x}_i) = 0$

$\sum \hat{y}_i - \sum \hat{\beta}_1 - \beta_2 \sum \hat{x}_i = 0$

$\sum \hat{\beta}_1 = \sum \hat{y}_i - \beta_2 \sum \hat{x}_i$

$n \hat{\beta}_1 = \sum \hat{y}_i - \beta_2 \sum \hat{x}_i$

$\hat{\beta}_1 = \bar{y}_i - \beta_2 \bar{x}$

unbiased when $E(\hat{\beta}_1) = \beta_1$

$\hat{\beta}_1 = \bar{y} - \beta_2 \bar{x}$

$\hat{\beta}_1 = \beta_1 + \beta_2 \bar{x} - \beta_2 \bar{x}$

$\hat{\beta}_1 = \beta_1$

$E(\hat{\beta}_1) = \beta_1$