1.) $E_0[m_{02} f_2] = E_0[m_{02}(S_2 - F_{02})] = E[m_{02} S_2] - E[m_{02} F_{02}]$

$$= S_0 - D_0 - \frac{1}{R_f^2} F_{02}$$

$$= 0$$

since we know that $F_{02} = R_f^2 (S_0 - D_0)$

2.) $u(C_t, t) = -\delta^t e^{-aC_t}$

$u_c(C_t, t) = a\delta^t e^{-aC_t}$

$$P_0 = E_0\left[\sum_{t=1}^{\infty} \frac{u_c(C_t, t)}{u_c(C_0, 0)} d_t\right]$$

$$= E_0\left[\sum_{t=1}^{\infty} \delta^t e^{-a(d_t - d_0)} d_t\right]$$

3.) $$\frac{P_t}{d_t} = E_t\left[\sum_{t=1}^{\infty} \delta^j \left(\frac{C_{t+j}}{C_t}\right)^{\gamma-1} \left(\frac{d_{t+j}}{d_t}\right)\right]$$

$$= E_t\left[\sum_{t=1}^{\infty} \delta^j e^{(\gamma-1)\ln(C_{t+j}/C_t) + \ln(d_{t+j}/d_t)}\right]$$

$$\frac{P_t}{d_t} = E_t\left[\sum_{j=1}^{\infty} \delta^j e^{(\gamma-1)(j\mu_c + \sigma_c \sum_{i=1}^{j} \eta_{t+j}) + j\mu_d + \sigma_d \sum_{i=1}^{j} \varepsilon_{t+j}}\right]$$

$$= E_t\left[\sum_{j=1}^{\infty} \delta^j e^{j[(\gamma-1)\mu_c + \mu_d] + \sum_{i=1}^{j} [(\gamma-1)\sigma_c \eta_{t+j} + \sigma_d \varepsilon_{t+j}]}\right]$$

$$= \sum_{j=1}^{\infty} e^{j[\ln\delta - (1-\gamma)\mu_c + \mu_d + \frac{1}{2}((1-\gamma)^2\sigma_c^2 + \sigma_d^2) - (1-\gamma)\sigma_c\sigma_d\rho]}$$

$$= \frac{1}{1 - \delta e^{-(1-\gamma)\mu_c + \mu_d + \frac{1}{2}((1-\gamma)^2\sigma_c^2 + \sigma_d^2) - (1-\gamma)\sigma_c\sigma_d\rho}} - 1$$

So, $P_t = d_t \dfrac{\delta e^a}{1 - \delta e^a}$; $a \equiv \mu_d - (1-\gamma)\mu_c + \frac{1}{2}[(1-\gamma)^2\sigma_c^2 + \sigma_d^2] - (1-\gamma)\rho\sigma_c\sigma_d$

4.) a.) $E_t[b_{t+1}] = \dfrac{R_f}{q_t} b_t q_t + E_t[e_{t+1}] q_t + (1-q_t) E_t[Z_{t+1}] = R_f b_t$

b.) From $E_t[b_{t+1}] = R_f b_t$

$$\lim_{i\to\infty} E_t[b_{t+i}] = \begin{cases} +\infty & \text{if } b_t > 0 \\ -\infty & \text{if } b_t < 0 \end{cases}$$

Oil is a limited, so b_t should > 0.

Rational speculative buble do not exist, since p_t can not exceed p_{solar}, b_t can not exceed $p_{solar} - p_t^*$.

4. c) A rational bubble can't be existed, since the bond price must be $p_t = d_t$ at maturity and be 0 after that. Only rational price is $p_t = p_t^*$

5. because $p_t = f_t + b_t$ w/ $b_t \neq 0$ could not happen,

$p_t = f_t = d_t$ at date T