Introductory to mathematical economics

EE320

Fall 2015

Kittichai Saelee

Agendas

- Logistics about this course
- What is this course all about?
- First start with Structure of economy analysis and element of mathematical economics model

Logistics

- About the instructor:
  - Kittichai Saelee (Golf)
    - (no, it's not "Chalee" as shown on the website!)
  - Have been on my sabbatical leave for doing Ph.D. in economics at UW-Madison.
  - Area of interest: macroeconomics, monetary economics, and structural estimations.
  - Email: kittichai_lee[at]econ.tu.ac.th
  - Office: rm#516. (Tue/Thu: 1.30-2.30 pm, or by appointment)

Logistics

- We meet twice a week in rm# 201.
  - I don’t check for your class attendance. But, I encourage everyone to attend my class.
  - Teaching will be lecture-based.

Logistics

- Grading is based on class's distribution.
  - Homework (10%)
    - Homework would be the same for both sections.
    - Posted on Moodle.
    - Tentatively 6 times. The lowest one will be dropped out.
    - Grading method will be decided later.
  - Quiz (10%)
    - Different across sections.
  - Mid-term (30%)
  - Final (50%)

Logistics

- Materials:
  - Chiang and Wainwright (henceforth CW)
  - This is very a classic one. Any editions fine! Well-written. Easy to read. Lot of examples, applications, and exercises.
  - Lecture note will be uploaded on BE moodle. Any announcements will be made via Moodle as well.
  - Thanks to Aj. Panit, I've prepared my materials based on teaching notes previously used by him. So, his kind courtesy in sharing his material is gratefully noted here.

Logistics

- To do well in this course,
  - Study hard "*regularly*".
  - I suggest that you form up your study group.
  - Help each other.
  - Assignment is really helpful.
- Don't piggy-back your friend. Free-riding doesn't help in the long-term. It only works to get you 10% of homework, but the rest of 90% is on your own.

Agendas

- Logistics about this course
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What is this course all about?

- This course is mainly about the application of mathematics for economics analysis.
  - That is, we show students how to represent and analyze all the existing economic theories that you've learned in ee211/ee212 in mathematical form.

Why using math in economic analysis

- Sound difficult, so why bother using mathematics in economic analysis.
  - Limitations of the other two approaches in addressing the analysis when it requires understanding the relationship of variables in multidimensional array.
  - Mathematical model is a ground for taking the model to test with data. Mathematical model can be adapted into statistical model, which we can validate any existing theories.

Why using math in economic analysis

- In term of economic theory, we are not going to introduce any new concepts. All these should be already seen in 212/213.
- In term of mathematics, we will focus on three important tools:
  - Method of solving system of equations (linear and non-linear)
  - Univariate and multivariate calculus
  - Optimization theory

What is this course all about?

- Method of solving system of equations
  - Most economic concepts can be formulated as system of equations, which equilibrium outcome of the model can be obtained by solving for the solution of such a system of equations.

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n &= b_1 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n &= b_2 \\ \vdots \qquad \vdots \qquad & \qquad \vdots \quad \vdots \\ a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n &= b_m. \end{aligned}$$

  - Several economic problems can be analyzed under linear setting. However, introducing non-linearity might gain some additional insights.

What is this course all about?

- Univariate calculus
  - Single variable: y $= f(x)$,
    $\frac{dy}{dx} = f'(x)$, $\quad higher\ order\ f''(x), \ldots . f^n(x)$
  - Applying the method to derive extreme points

- Multivariate calculus
  - Multiple variable: y $= f(x_1, x_2, \ldots, x_n)$
  - Various concepts of finding the derivative for multivariable case: partial derivative, total differential, etc.
  - Applying the method to derive extreme points for multivariate case.

What is this course all about?

- Optimization theory is the central toolkit for mathematical economic model.
- Mathematically, this field combines together three different strands of mathematics.
  - Linear algebra: solving system of equation
  - Real analysis: characteristic and property of functions
  - Calculus: derivative
- Method has been extensively used in engineering field for searching for an optimized action plan.
- Economist have started using this tools after his famous dissertation of Paul Samuelson. The concept of optimization problem matches perfectly well with the idea of economic decision making, in which agent choose for actions that best generates the highest return/benefits, given the constraints facing.

Agendas

- Logistics about this course
- What is this course all about?
- A first start with structure of economy analysis and element of mathematical economics model

Structure of economic analysis

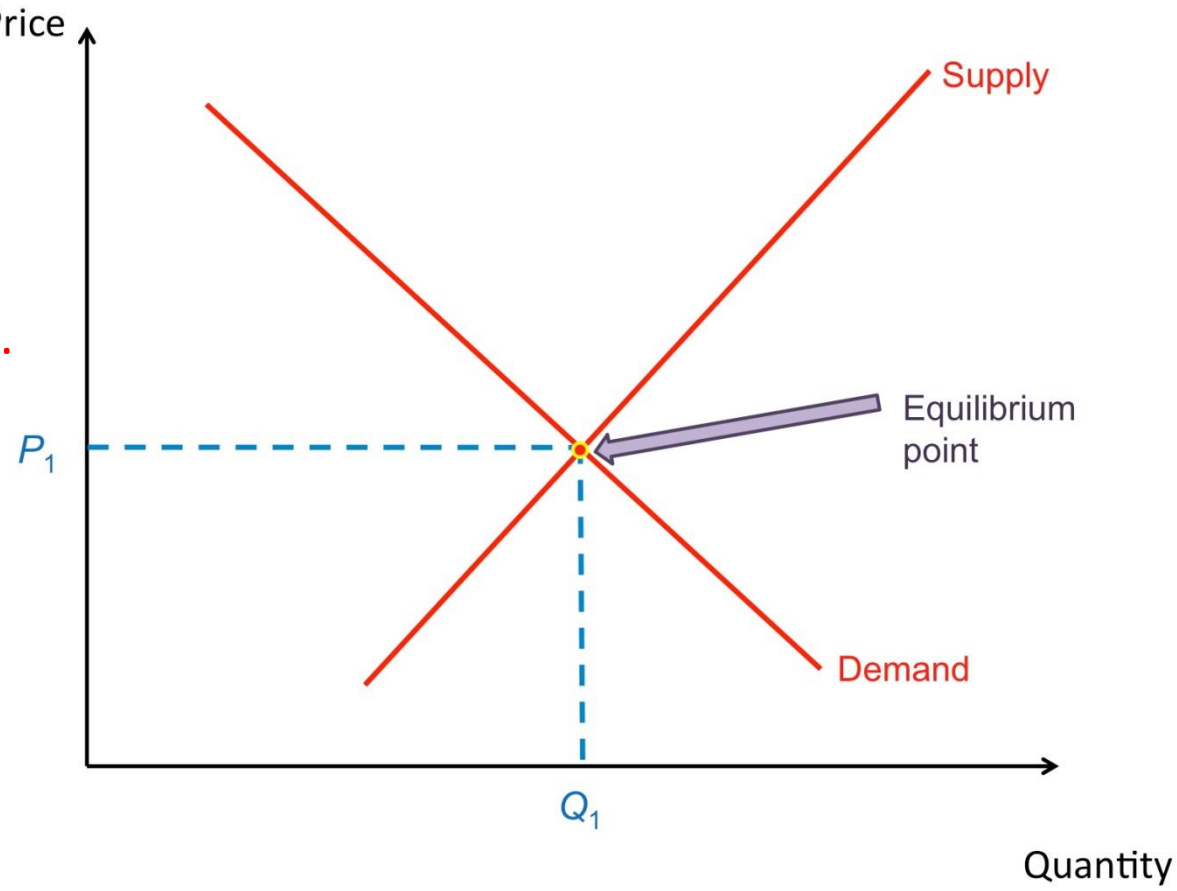
- How do we study economics?
  - Model-based analysis
- Model is an abstract form of the problem that we are interested in.
  - The problem that we are interested in aims at getting us some insights, helping us understand something about how the world/economy works.
  - Typically, all the results in a certain model hinges on the assumptions used in the analysis.

Structure of economic analysis

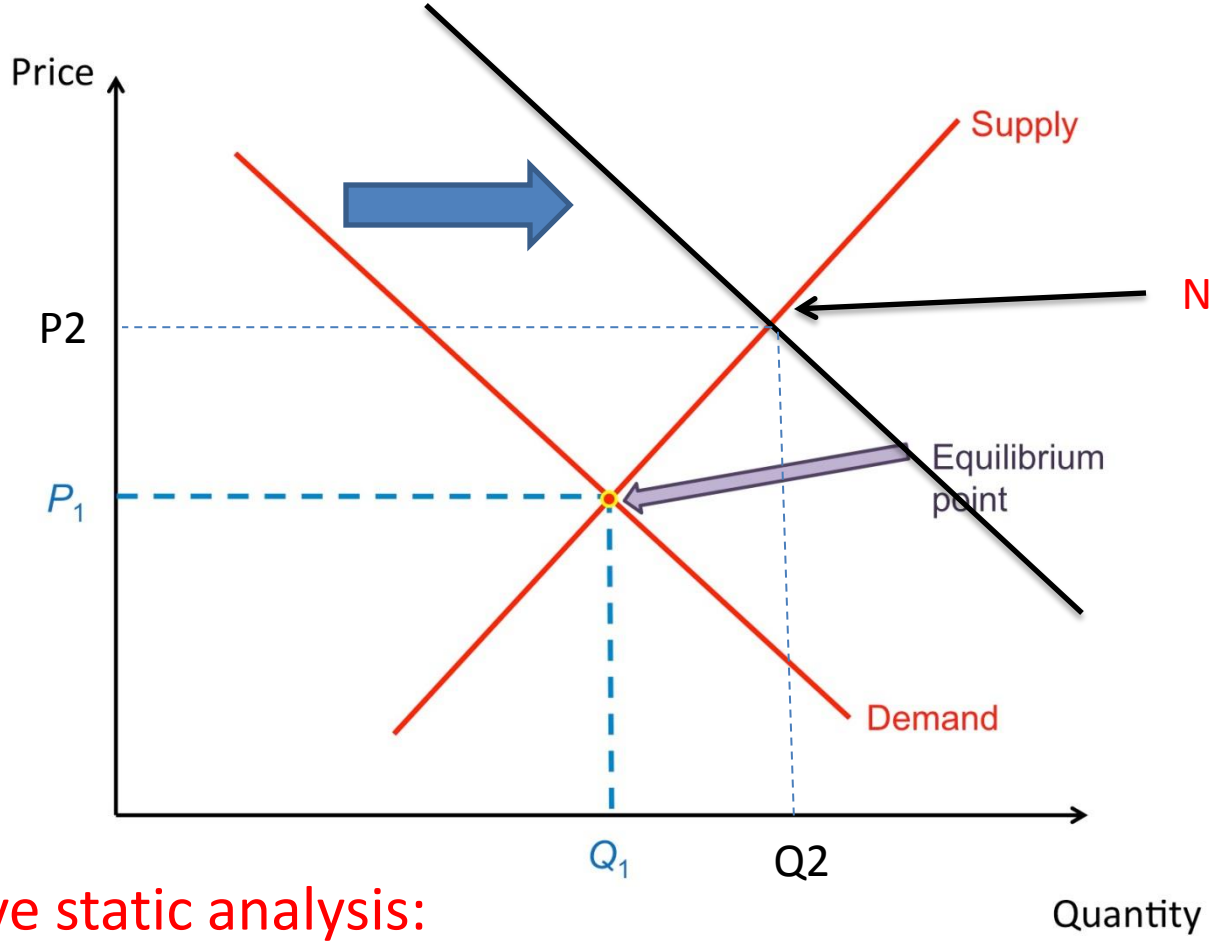
- Micro: Price determination model
  - Question: at what price and quantity of output, market will trade?
  - So we need to characterize behavior of two parties in trading arrangements: buyers and sellers
  - Buyers = demand and Sellers = supply
  - Conventionally, behavioral assumptions are
    - Demand: quantity demanded is negatively related to price
    - Supply: quantity supplied is positively related to price.
  - Note as usual that an underlying assumption for this behavioral relationship is that Ceteris Paribus.

Structure of economic analysis

Static analysis:
- Draw demand/supply onto p-q diagram
- Equilibrium occurs when price clears the market. That is, demand matches with supply, aka no excess demand.



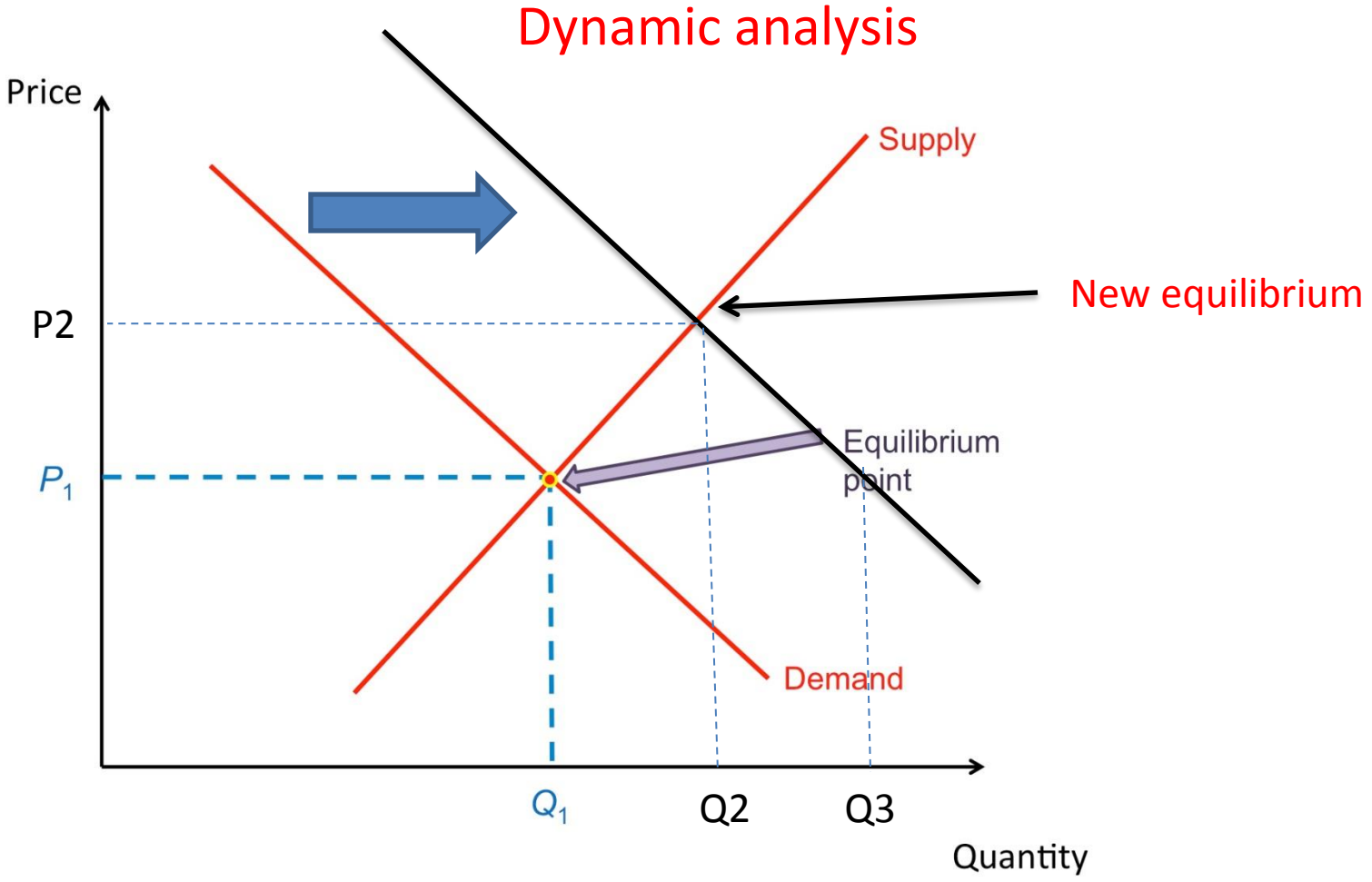
Structure of economic analysis



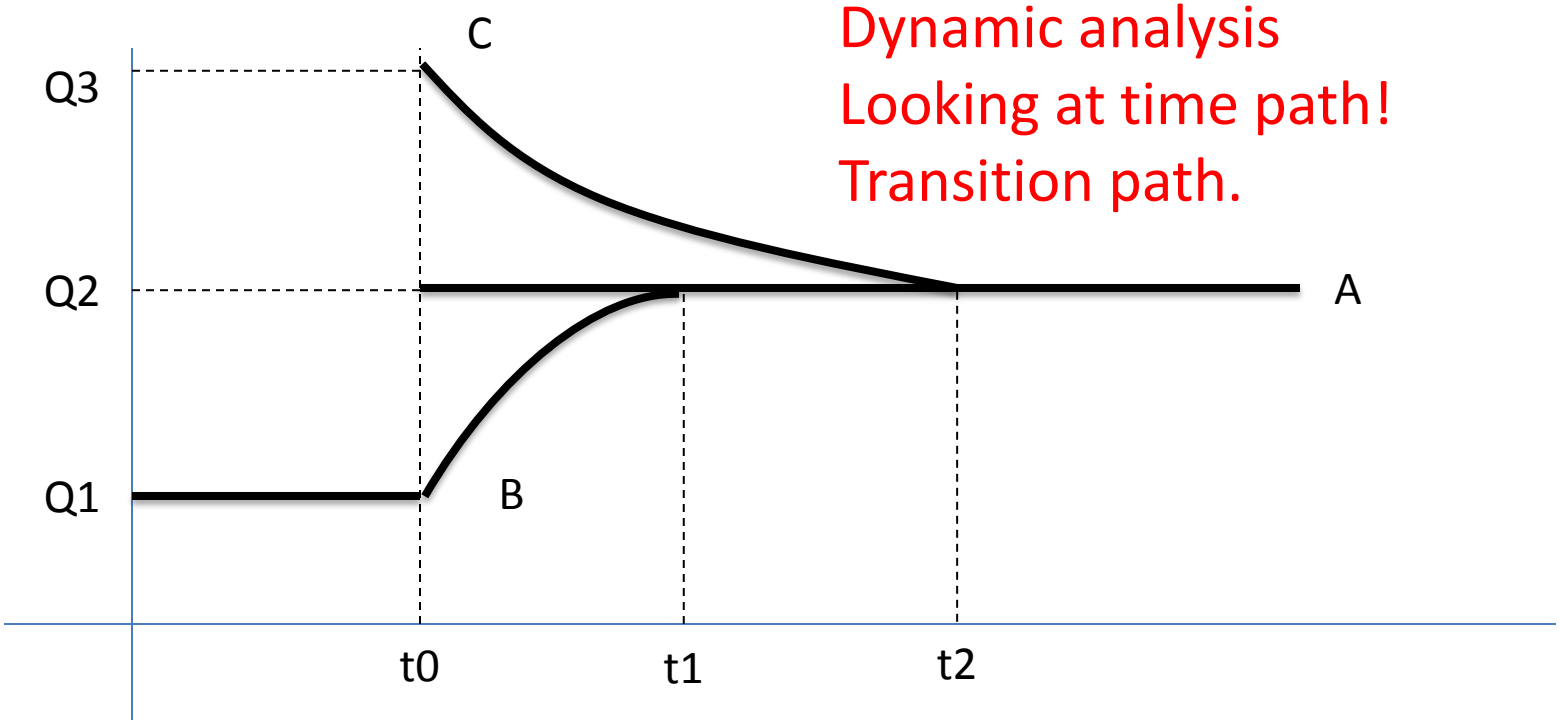
Comparative static analysis:

For example, what if income increase?

Structure of economic analysis



Structure of economic analysis



Dynamic analysis
Looking at time path!
Transition path.

Mathematical economic model

- Reformulating those relationships, which are illustrated by graphs/curves, into mathematical concepts.
- Elements of mathematical models.
  - So, mathematical economic model would have equations that describe economic relationship of economic variables.
  - Plus, set of conditions that tell us about how equilibrium outcome can occur.

Mathematical economic model

- Types of equations
  - Definitional equation
    - $\pi(Q) \equiv R(Q) - C(Q)$
  - Behavioral equation
    - $C = a + b * Y$ $\longrightarrow$ a, b *are* $parameters$!
  - Equilibrium conditions
    - $Q^d = Q^s$
    - $Y = C + I + G$
    - $\frac{M^S}{P} = L^d$

Mathematical economic model

- Type of variables in the model
  - Exogenous variables = treated as "given" in the model. Their values are given as known thing, and *will not be solved* within the model.
  - Endogenous variable = set of variables that will be solved for.
- Typically, the solution will depend on (i) exogenous variable and (ii) parameters.

Mathematical economic model

- Example: **National income model**
  - Known as Keynesian cross diagram model
  - Emphasizing at the role of government sector in stabilizing economy.
  - Adjusting output (GDP) toward desirable level.
  - Idea first appeared in economics field after his treatise work by John Meynad Keynes, and subsequently reinterpreted and put into formal analysis by Sir John Hicks.

Mathematical economic model

- Example: **National income model**
  - Simple two-equation model
    - Equilibrium condition

$$Y = C + I + G$$

    - Consumption function

$$C = \mathrm{a} + \mathrm{bY}$$

Mathematical economic model

- Static analysis:
  - characterizing the equilibrium solution of the model by solving for a solution of system of equations

$$Y^* = f(I, G, a, b)$$
$$C^* = g(I, G, a, b)$$

- Mathematically, this is call a reduced-form solution.
- From the example,

$$Y^* = \frac{a+I+G}{1-b},\ C^* = \frac{a+b(I+G)}{1-b}$$

Mathematical economic model

- Comparative Static analysis:
- From the example, if a = 1, I = 1, G = 1 and b = 0.5, this yields us,

$$Y^* = \frac{a+I+G}{1-b} = 6,\ C^* = \frac{a+b(I+G)}{1-b} = 4$$

- What if G is now 2, how big is the change in Y* and C*?
  - New Y* = 8.
  - A dollar increase in G changes in Y* = 2.
  - Multiplier of government expenditure is 2.
- Later on, we will apply *derivative method* to calculate the multiplier and figure out the impact of changes in exogenous variable/parameters on endogenous variable.
- Good thing is that we don't need to keep substituting numeric value of variables and parameters, but still we can figure out the size of multiplier.

Mathematical economic model

- Dynamic analysis:
  - Transition path for the change from Y* = 6 to Y* = 8.
  - Fortunately or unfortunately, we won't discuss this issue in this course.
  - You would need to know a more sophisticated technic called differential/difference equations.
  - Welcome to learn this on your own or from EE421.

Done for today

- Next time, we will cover topic on static equilibrium analysis, in more details.. Meaning is lot of examples and applications.
- We will start from basic mathematical concept for relation, function and system of equations.
- Then, we proceed to a variety of economic model that applies those mathematical tools developed/discussed.
- Read chapter 3 and 4 of CW