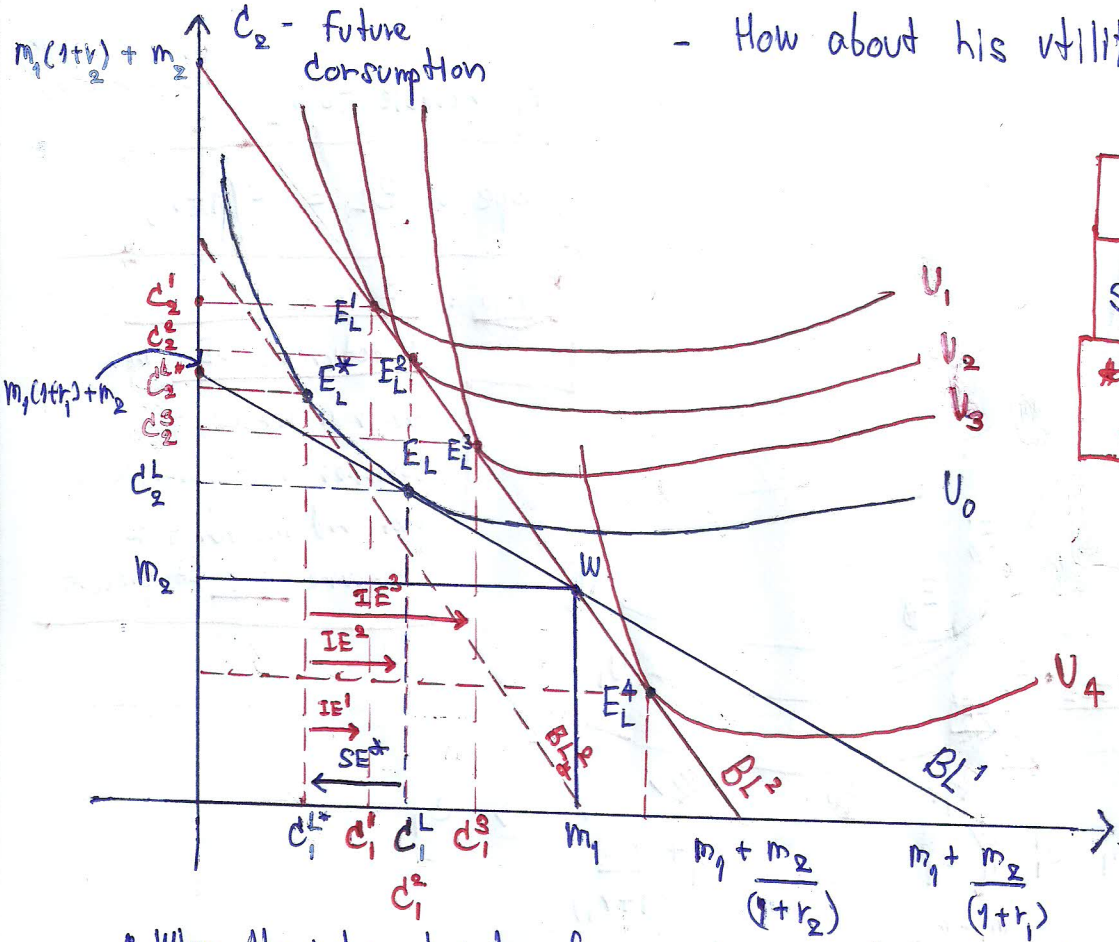
Solution Guide For HW. 3.

Q.1 - Show the outcome when interest rate rises and consumer is a lender.
- Does he remain a lender?
- How about his utility?



r_1 increase to r_2

slope of BL is $-(1+r)$

* Note: IE is opposite to SE because when the r increases, the lender is richer and may want to consume ~~less~~/more at present.

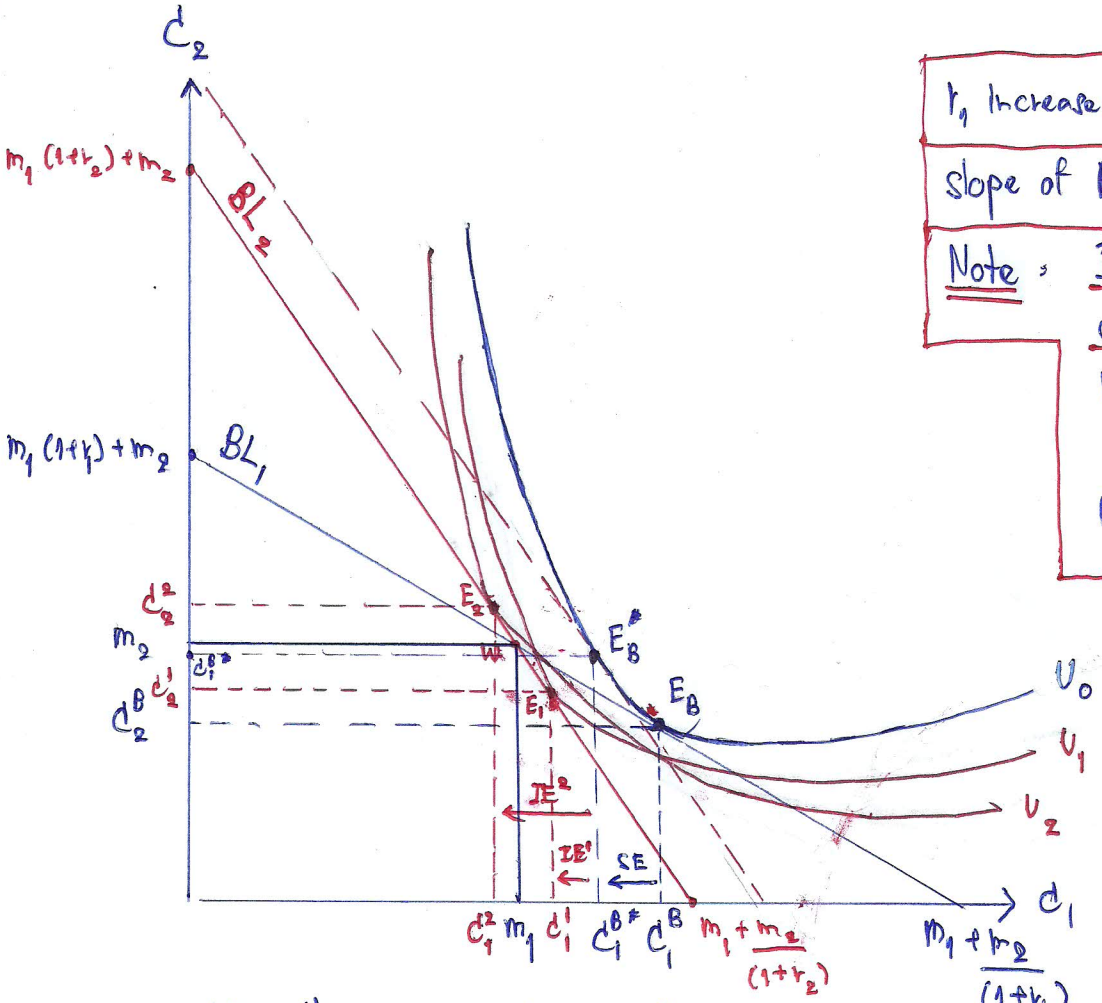
- When the interest rate from r_1 to r_2, the budget line will rotate clockwise around the endowment point (W) from BL^1 to BL^2. Using BL f^n:

$$C_2 = (1+r)m_1 + m_2 - (1+r)C_1$$ - future value form

If r increase the budget line will be steeper. (BL^1 to BL^2)

- For the lender/saver, the original consumption is at (C_1^L, C_2^L) where $C_1^L < m_1$ and $C_2^L > m_2$. And after the rise of interest rate, the new consumption could be E_L^1, E_L^2 or E_L^3 depends on the SE and IE.
- To separate SE and IE, we have to shift the new budget BL^2 tangent to the old IC (U_0) to see only SE (BL^2_*). We can see that, follow our logic, when r increases, the consumer will give away present consumption (C_1^L to C_1^{L*}) and increase future consumption (C_2^L to C_2^{L*}) (past E_L^*)
- And if we look at the IE, it will determine how much more or less the present consumption would be. If IE < SE, E_L^1 will be archieved with lower ($C_1^L > C_1^1$). If IE = SE, E_L^2 will be archieved with the same ($C_1^L = C_1^2$). If IE > SE, E_L^3 will be achelved with more ($C_1^L < C_1^3$)
- But no matter which E_L^1, E_L^2 or E_L^3 is archieved the lender will have higher utility (U_1, U_2, U_3) depends on the effects.
- The lender will still be lender because E_L^4 (corresponding to U_4) is not satisfied. The lender can archieved higher utility (U_1, U_2 or U_3)

Q. 2 : Show the outcome when interest rate rises and a consumer is a borrow
- Does he remain a borrower ?
- What happened to his utility ?



r_1 increases to r_2

slope of BL = $-(1+r)$

Note : IE is in the same direction as SE.

When r increases, the borrower is comparatively poor and will want to consume less at present

- When the interest rate rise from r_1 to r_2, the budget line rotate clockwise around W (follow Q.1) from BL^1 to BL^2.
- For borrower, the original consumption is at (C_1^B, C_2^B) where $C_1^B > m_1$ and $C_2^B < m_2$. And after the rise of interest rate, the new consumption could be E_1 or E_2.
- We could see that, when the interest rise, the SE will induce the consumer to give away present consumption / save more (C_1^B, C_1^{B*}). And with IE, the borrower will be comparatively poorer and will choose to save even more and increase their future consumption.
- So if IE is high enough (IE^2), the borrower may turn to be lender instead. But both cases, the borrower utility will be lower. We can see that they may archieve either U_1 or U_2 which is lower than U_0.

- As a sum, the utility of borrower in case of increasing interest rate will be lower. and the borrower may turn to be lender in the IE is high (they feel too poor to consume more than they get at period 1)