Properties of Stock Options

Chapter 9

Outline

- Factors affecting option prices
- Upper and lower bound for option prices
- Put-Call parity
- Early exercise: calls and puts on non-dividend paying stocks
- The effect of dividends

Notation

- c : European call option price
- p : European put option price
- S_0 : Stock price today
- K : Strike price
- T : Life of option
- σ: Volatility of stock price
- C : American Call option price
- P : American Put option price
- S_T :Stock price at option maturity
- D : Present value of dividends during option's life
- r : Risk-free rate for maturity T with cont comp

Effect of Variables on Option Pricing

Variable	c	p	C	P
S_0	+	−	+	−
K	−	+	−	+
T	?	?	+	+
σ	+	+	+	+
r	+	−	+	−
D	−	+	−	+

American vs European Options

An American option is worth at least as much as the corresponding European option

$$C \geq c$$

$$P \geq p$$

Upper and lower bonds

- Upper bound on a call option $S_0 \geq c$
- Upper bound on a put option $K \geq p$
- Lower Bound for European Call Option Prices; No Dividends $c \geq \max(S_0 - Ke^{-rT}, 0)$
- Lower Bound for European Put Option Prices; No Dividends

  $p \geq \max(Ke^{-rT} - S_0, 0)$

Lower bond for call option (no dividend)

- Consider two portfolios
  - Portfolio A: European call on a stock + PV of the strike price in cash
  - Portfolio B: The stock
- A is worth MAX(S_T , K) at the maturity of the options
  - If stock price > strike price then the option is exercised and A is worth S_T
  - If the stock price < strike price then the option expires worthless and we have K
- Since value of A >= value of B at maturity,
- $c + Ke^{-rT} \geq S_0 \rightarrow c \geq \max(S_0 - Ke^{-rT}, 0)$

Calls: An Arbitrage Opportunity?

- Suppose $c = 3$, $S_0 = 20$, $T = 1$, $r = 10\%$, $K = 18$, $D = 0$
- $S_0 - Ke^{-rT} = 3.71 > c$
- There is an arbitrage opportunity
- Today: buy option for $3, short the stock to get $20, invest $16.28 ($18e^{-0.1}$) for one year, you get 0.71
- At expiration: if $S_T > K$: exercise option to buy stock for $18, use stock to close out short position, receive 18 from investment, total=0
- If $K > S_T$: buy stock for S_T ; use stock to close out short position; receive $18 from investment; net gain 18- S_T

Lower bond for put option (no dividend)

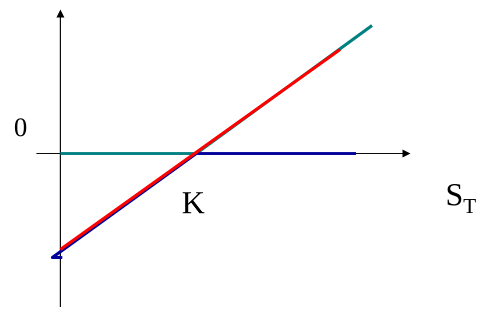
- Consider two portfolios
  - Portfolio A: European put option + one share
  - Portfolio B: Cash Ke^{-rT}
- A is worth MAX(S_T , K) at the maturity of the options
  - If stock price > strike price then the option expires and we get S_T
  - If the stock price < strike price then the option is exercised portfolio A is worth K
- At maturity portfolio B is worth K
- Since value of A >= value of B at maturity,
- $p + S_0 \geq Ke^{-rT} \rightarrow p \geq \max(Ke^{-rT} - S_0, 0)$ since $p \geq 0$

Puts: An Arbitrage Opportunity?

- Suppose p = 1, S_0 = 37, T = 0.5, r = 10%, K = 40, D = 0
- $Ke^{-rT} - S_0 = 2.01 > p$
- There an arbitrage opportunity
- Today: Borrow $38 for 6 months, buy 1 option for $1, buy one stock for $37, get $1
- At expiration: if S_T > 40: sell the stock for S_T ; use $40 to repay loan; net gain = S_T - 40
- If S_T < 40: exercise the option to sell stock for $40, use $40 to repay loan, total =0

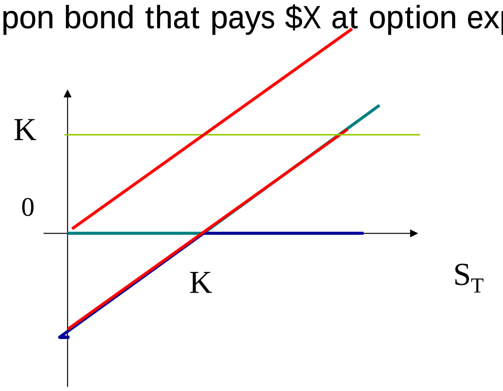
Put-call parity (no dividends)

Suppose you buy a call and write (sell) a put with the same exercise price and expiration date



Put-call parity (no dividends)

Suppose in addition you buy today a risk-free zero coupon bond that pays $X at option expiry



Put-call parity

- Portfolio A: Buy one call, write (sell) one put with the same exercise price and expiration date, buy a risk-free bond (no coupon) that pays $K at expiration
- What is the payoff at T of this portfolio?
- It is S_T
- Therefore we have $C_T - P_T + K = S_T$

Put-call parity

- Pay off of each portfolio at maturity

  Portfolio A: $C_T - P_T + K$

  Portfolio B: Buy the stock
- Law of one price: Two investments strategies that have the same payoff in the future in all states of the world must have the same price today

$$C_T - P_T + K = S_T$$

- Put-call Parity Theorem (no dividends)

$$C_t - P_t + PV(K) = S_t \longrightarrow C_t = P_t + S_t - Ke^{-rT}$$

Arbitrage Opportunities

- Suppose that c = 3, S_0= 31, T = 0.25, r =10%, K = 30, D = 0
- Is there an arbitrage opportunity when p = 2.25 ?
- $P_t = C_t - S_t + Ke^{-rT} = 3 + 30e^{-0.1*0.25} - 31 = 1.26$
- Yes!

Arbitrage Opportunities

- P > C-S+PV(X)
- Sell the LHS and buy the RHS
- Investment now S_T>30 S_T<30

Investment	now	$S_T>30$	$S_T<30$
Sell put	2.25	0	$-(30-S_T)$
Buy call	-3	S_T-30	0
short stock	31	$-S_T$	$-S_T$
Lend money PV(K)	-29.26	30	30
Total	.99	0	0

Concept check

- Suppose that c = 3, S_0= 31, T = 0.25, r =10%, K = 30, D = 0
- Is there an arbitrage opportunity when p = 1?
- If so, what is your arbitrage trading strategy?

G) Yes. Buy put, sell call, buy stock, borrow money

Y) Yes. Buy put, buy call, short stock, lend money

R) No

Arbitrage Opportunities

- P < C-S+PV(X)
- Sell the RHS and buy the LHS
- Investment now S_T>30 S_T<30

Investment	now	S_T>30	S_T<30
Buy put	-1	0	(30-S_T)
Short call	3	-(S_T-30)	0
Buy stock	-31	S_T	S_T
Borrow money PV(K)	29.26	-30	-30
Total	.26	0	0

Early Exercise of American options

- Usually there is some chance that an American option will be exercised early
- An exception is an American call on a non-dividend paying stock. This should never be exercised early.
- Reasons:
  - No income is sacrificed
  - We delay paying the strike price
  - Holding the call provides insurance against stock price falling below strike price

American call option (no dividend)

- Formal argument
- For a European call $c_t \geq S_t - Ke^{-rT}$
- For an American call $C_t \geq c_t \geq S_t - Ke^{-rT}$
- Because interest rates r>0, we have

$$C_t > S_t - K$$

- The value of holding the call option is more valuable than the intrinsic value of exercising immediately

An Extreme Situation

- For an American call option:

S_0 = 100; T = 0.25; K = 60; D = 0

Should you exercise immediately?

- What should you do if
  - You want to hold the stock for the next 3 months?
  - You do not feel that the stock is worth holding for the next 3 months?

Should Puts Be Exercised Early ?

Are there any advantages to exercising an American put when

S_0 = 60; T = 0.25; r=10%

K = 100; D = 0

The Impact of Dividends on Lower Bounds to Option Prices

$$c \geq S_0 - D - Ke^{-rT}$$

$$p \geq D + Ke^{-rT} - S_0$$

Extensions of Put-Call Parity

- American options; $D = 0$

$S_0 - K < C - P < S_0 - Ke^{-rT}$

Equation 9.4 p. 216

- European options; $D > 0$

$c + D + Ke^{-rT} = p + S_0$

Equation 9.7 p. 220

- American options; $D > 0$

$S_0 - D - K < C - P < S_0 - Ke^{-rT}$

Equation 9.8 p. 220

Summary

- Factors affecting option prices
- Upper and lower bound for option prices
- Put-Call parity
- Early exercise: calls and puts on non-dividend paying stocks
- The effect of dividends