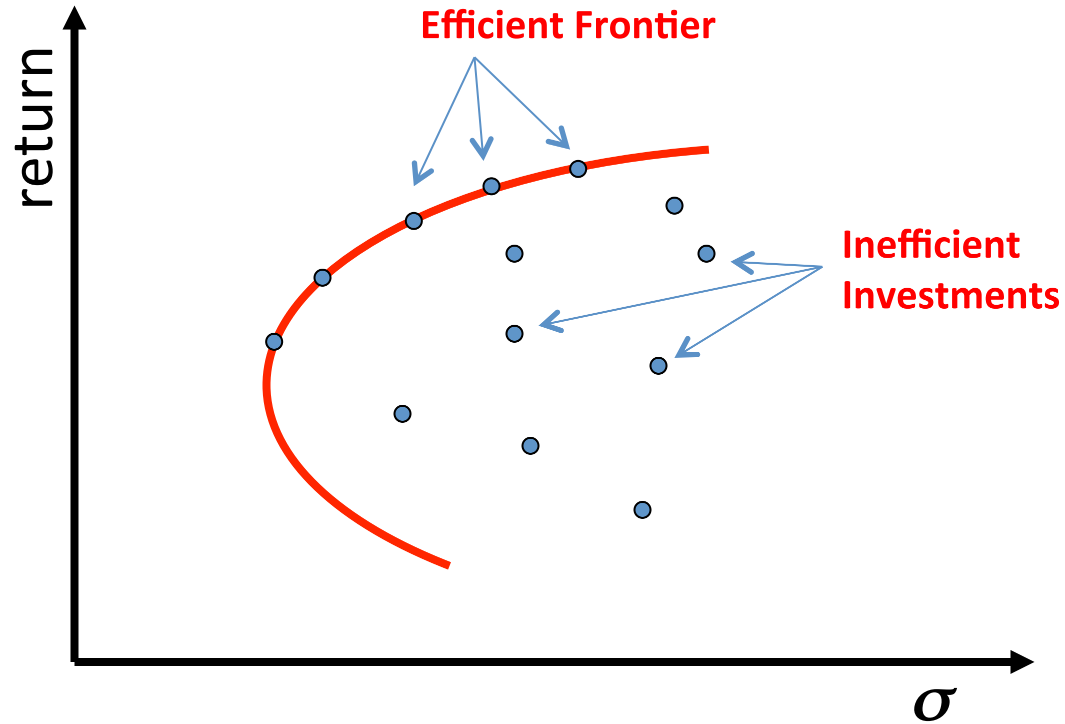
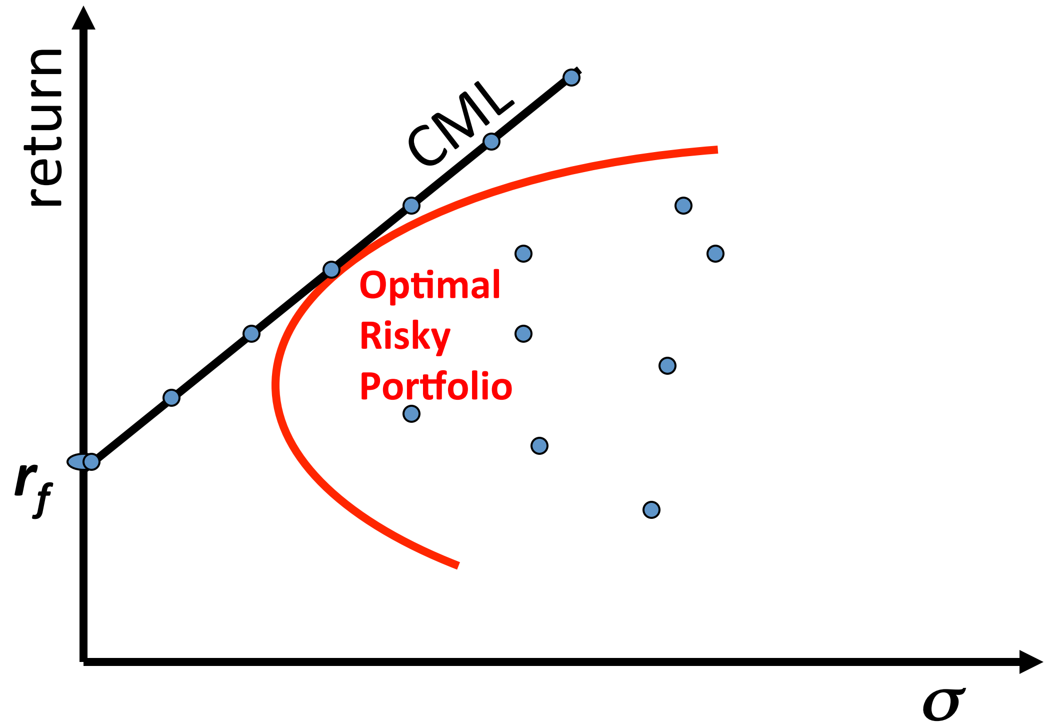
The Capital Asset Pricing Model

Efficient Investments



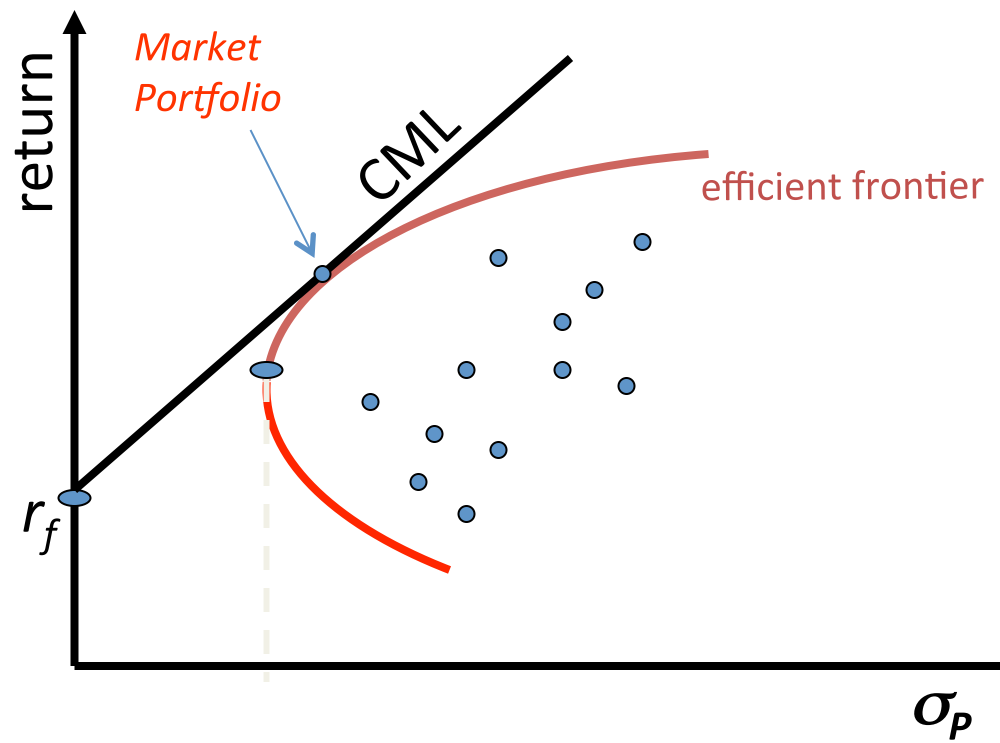
As we discussed, since investors want to minimize risk and maximize return, they will not invest in portfolios that are dominated on either dimension. This leads to an *Efficient Frontier.*

Market Equilibrium



If all investors have the same expectations about risk and return, all investors will have the same CAL because they all have the same optimal risky portfolio given the risk-free rate. This is the *Capital Market Line (CML).*

Market Equilibrium



With the capital market line identified, all investors choose a point along the line: some combination of the risk-free asset and the common optimal risky portfolio. In equilibrium this portfolio **has to be** the market portfolio *M*.

Market Equilibrium

- If all investors have the same set of expectations, they will choose the same optimal risky portfolio. All investors will always hold this portfolio. In market equilibrium, demand will equal supply. This optimal risky portfolio will then be the market portfolio, the value-weighted portfolio of all risky assets.
- Then with borrowing or lending, the investor selects a point along the CML, the Capital Market Line.

Intuition

- If everyone in the economy holds an efficient portfolio, how should securities be priced so that demand equals supply?
- If, for given expected returns, variances, and covariances, no investor wants to hold Xerox, what will happen?
  – Price (and expected return) of Xerox needs to adjust
- Equilibrium
  – Every investor is happy with her portfolio
  – Supply of assets is equal to demand for assets

Intuition

- Every investor solves the mean-variance problem and holds a combination of risk-free asset and portfolio of risky assets (tangency portfolio)
- With homogeneous expectations, tangency portfolio is the same for all investors
- In equilibrium the sum of all investors' desired portfolios must equal the supply of assets
- Aggregate supply of asset is the market portfolio
- Market portfolio has to be the tangency portfolio

The Market Portfolio

- Everybody will want to invest in Portfolio M and borrow or lend to be somewhere on the CML
  - Therefore this portfolio must include all risky assets (or else some assets would have no demand)

- Because the market is in equilibrium, all assets are included in this portfolio in proportion to their market value

The Market Portfolio

- Portfolio of all risky assets that exist in the economy
- Portfolio weights

$$w_{Mi} = \frac{N_i \times P_i}{\sum_j N_j \times P_j}$$

where N_i is the number of shares and P_i the market price of company i

- Should really include all assets
  - Real estate
  - Human capital

Market Indexes

- Report the value of a particular portfolio of securities.
- Examples:
  - S&P 500
    - A value-weighted portfolio of the 500 largest U.S. stocks
  - Wilshire 5000
    - A value-weighted index of all U.S. stocks listed on the major stock exchanges
  - Dow Jones Industrial Average (DJIA)
    - A price weighted portfolio of 30 large industrial stocks

Investing in a Market Index

- Index funds – mutual funds that invest in the S&P 500, the Wilshire 5000, or some other index.
- Exchange-traded funds (ETFs) – trade directly on an exchange but represent ownership in a portfolio of stocks.
  - Example: SPDRS (Standard and Poor's Depository Receipts) represent ownership in the S&P 500

Relationship between Risk and Return

- In the mid 1960 three economists Sharpe, Lintner and Treynor showed that in a competitive market the expected risk premium on a security varies in direct proportion to its beta (its measure of systematic risk)
- The relevant measure of risk is the covariance of the asset or portfolio with the market (beta)

CAPM

- Expected Return on the Market:

$$E[r_M] = \bar{r}_M = r_f + \text{Market Risk Premium}$$

- Expected return on an individual security:

$$E[r_i] = \bar{r}_i = r_f + \beta_i \times \underbrace{(\bar{r}_M - r_f)}_{\text{Market Risk Premium}}$$

$$where \quad \beta_i = \frac{\sigma_{iM}}{\sigma_M^2}$$

Expected Return on an Individual Security

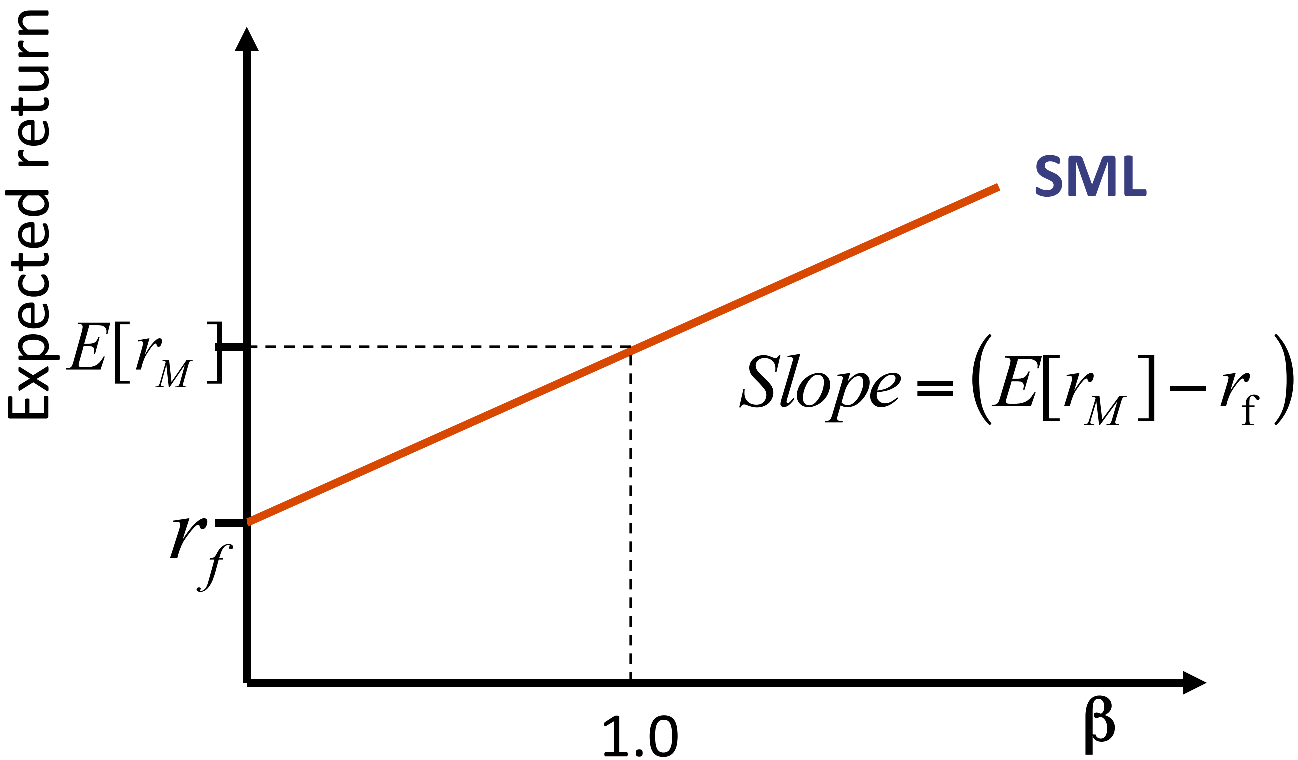
- This formula is called the Capital Asset Pricing Model (CAPM)

$$\bar{r}_i = r_f + \beta_i \times (\bar{r}_M - r_f)$$

Expected return on a security	=	Risk-free rate	+	Beta of the security	×	Market risk premium

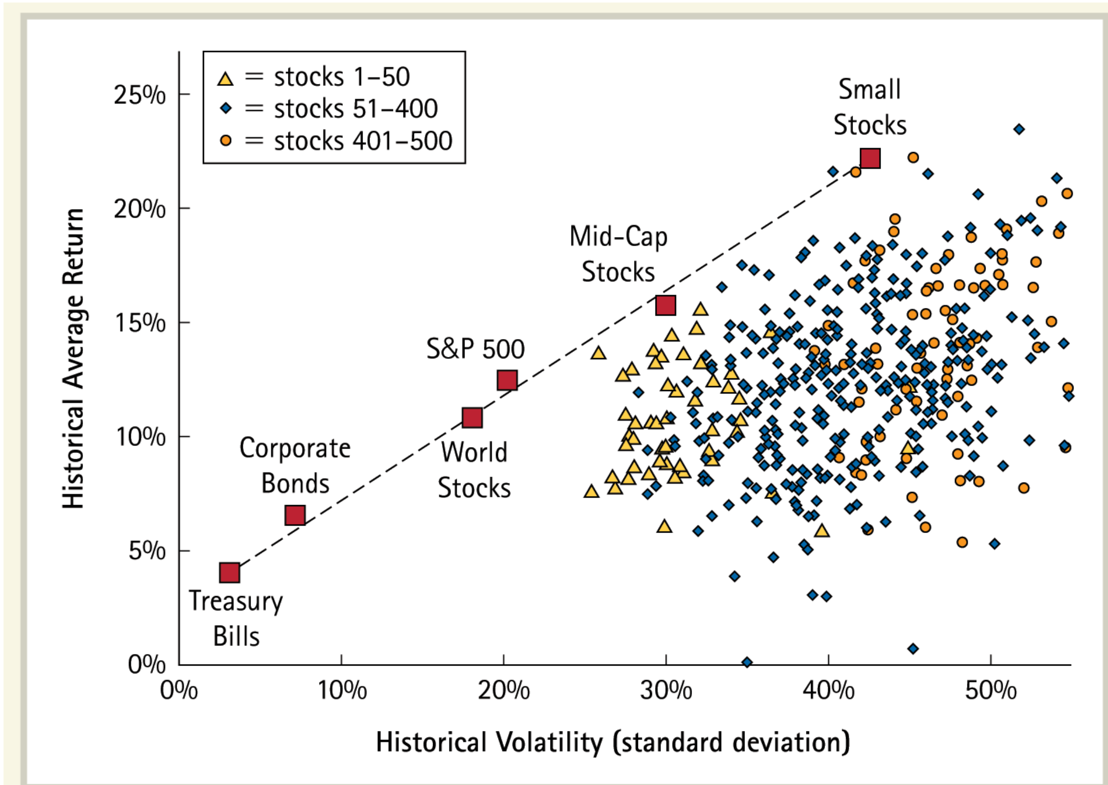
- If $\beta_i = 0$, then the expected return is r_f.
- If $\beta_i = 1$, then $\bar{r}_i = \bar{r}_M$

Relationship Between Risk & Expected Return (Security Market Line)



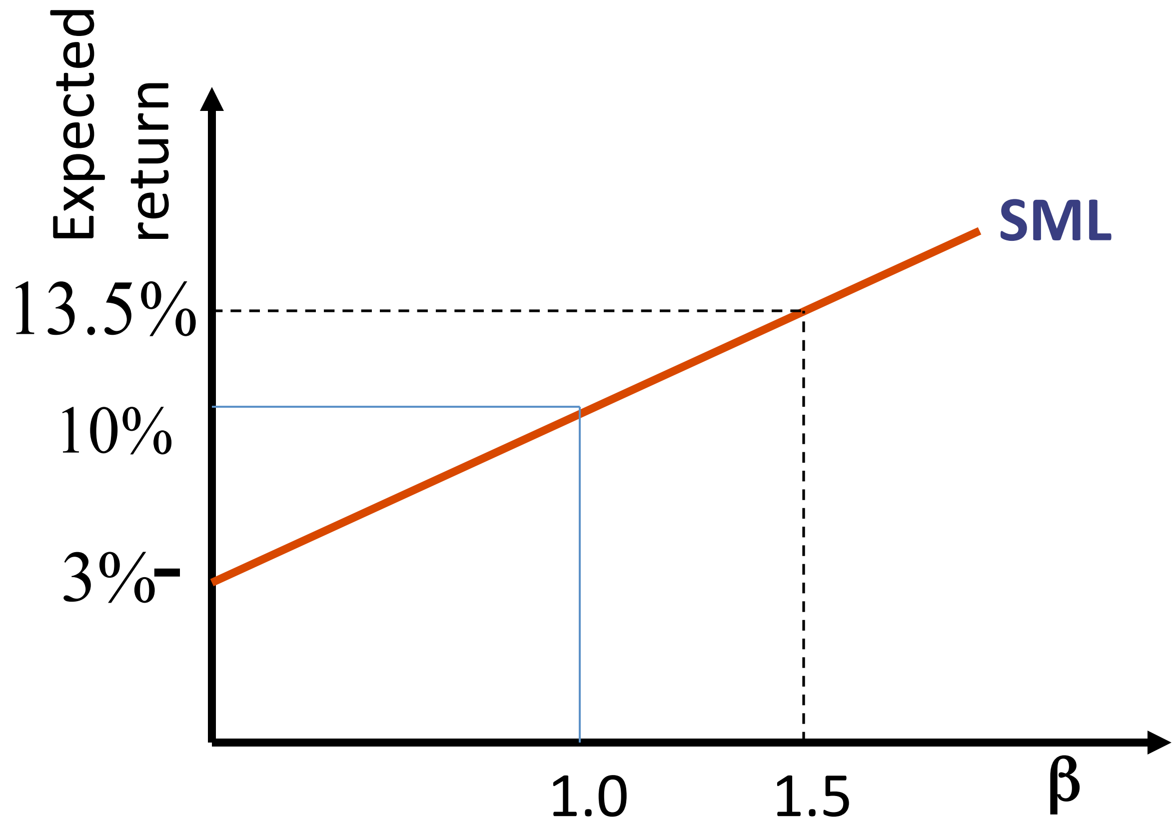
$$E[r_i] = r_f + \beta_i (E[r_M] - r_f)$$

Historical Volatility and Return for 500 Individual Stocks, by Size, Updated Quarterly, 1926–2004



Unlike the case for large portfolios, there is no precise relationship between volatility and average return for individual stocks. Individual stocks have higher volatility and lower average returns than the relationship shown for large portfolios.

Relationship Between Risk & Expected Return



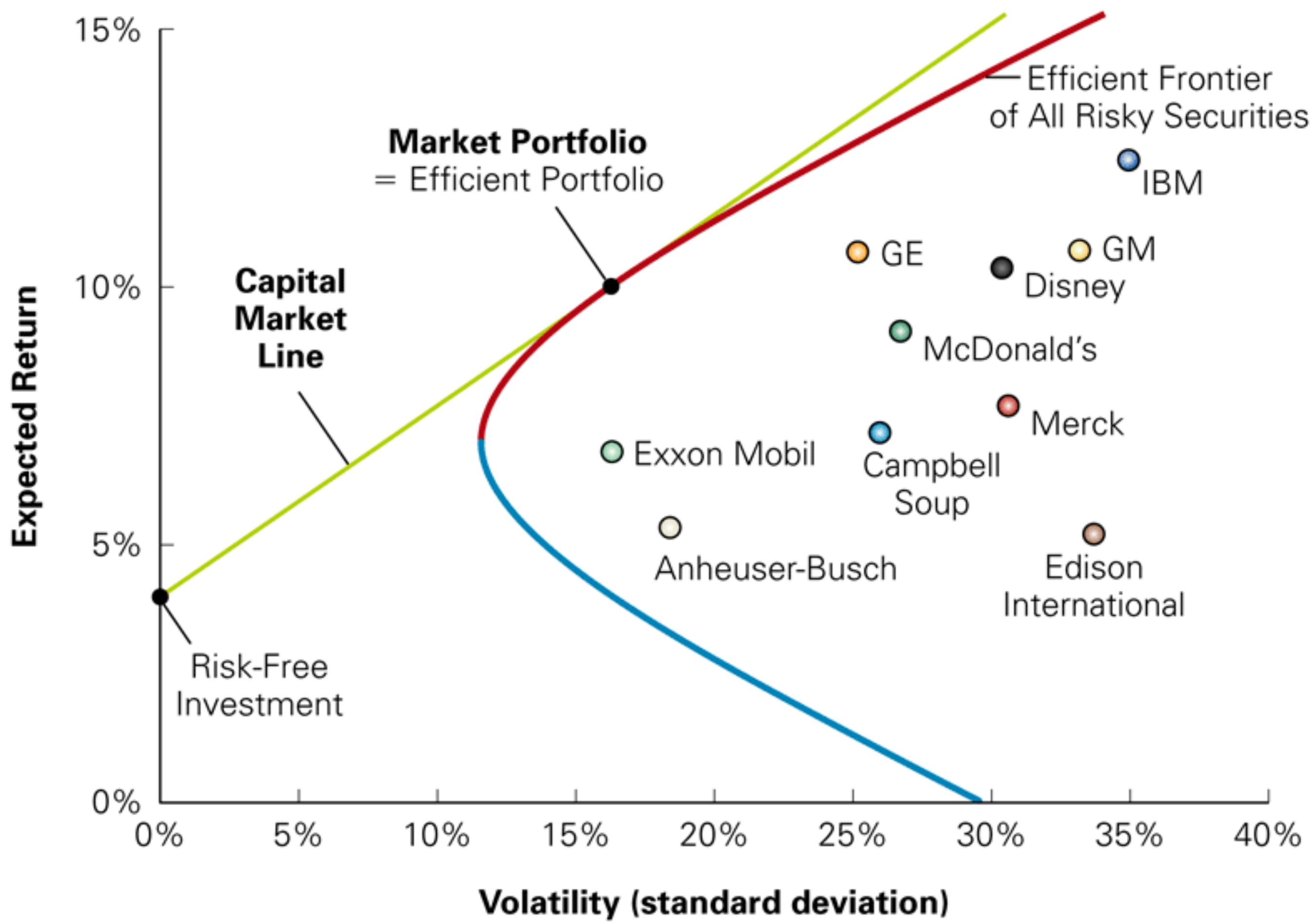
$$\beta_i = 1.5 \qquad r_f = 3\% \qquad \bar{r}_M = 10\%$$

$$\bar{r}_i = 3\% + 1.5 \times (10\% - 3\%) = 13.5\%$$

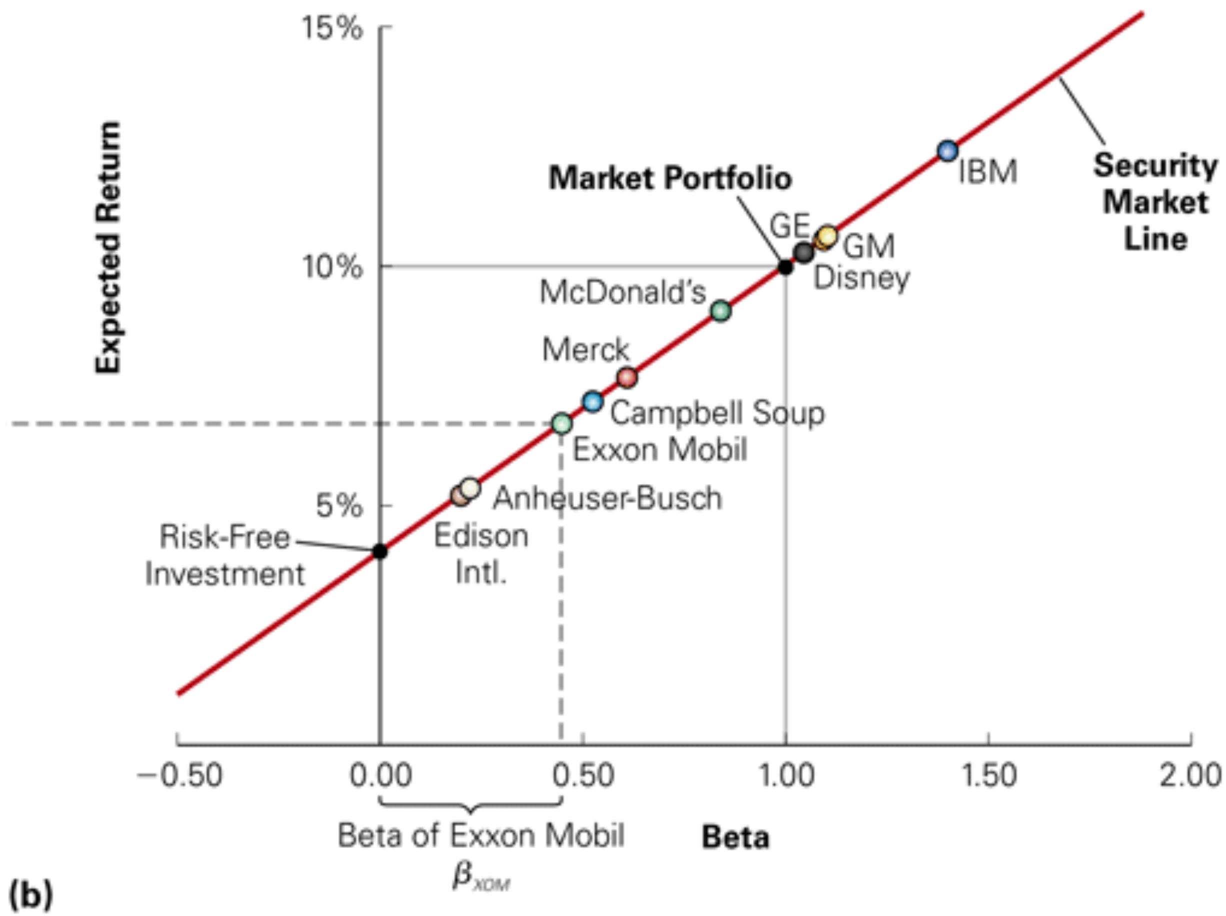
CAPM and SML

- The capital asset pricing model (CAPM): A plot of this relation is called the Security Market Line (SML)
- The SML gives the relation between expected return and risk (beta) for individual assets and portfolios
- Just as the CML gives the relation between expected return and risk (s.d.) for efficient portfolios

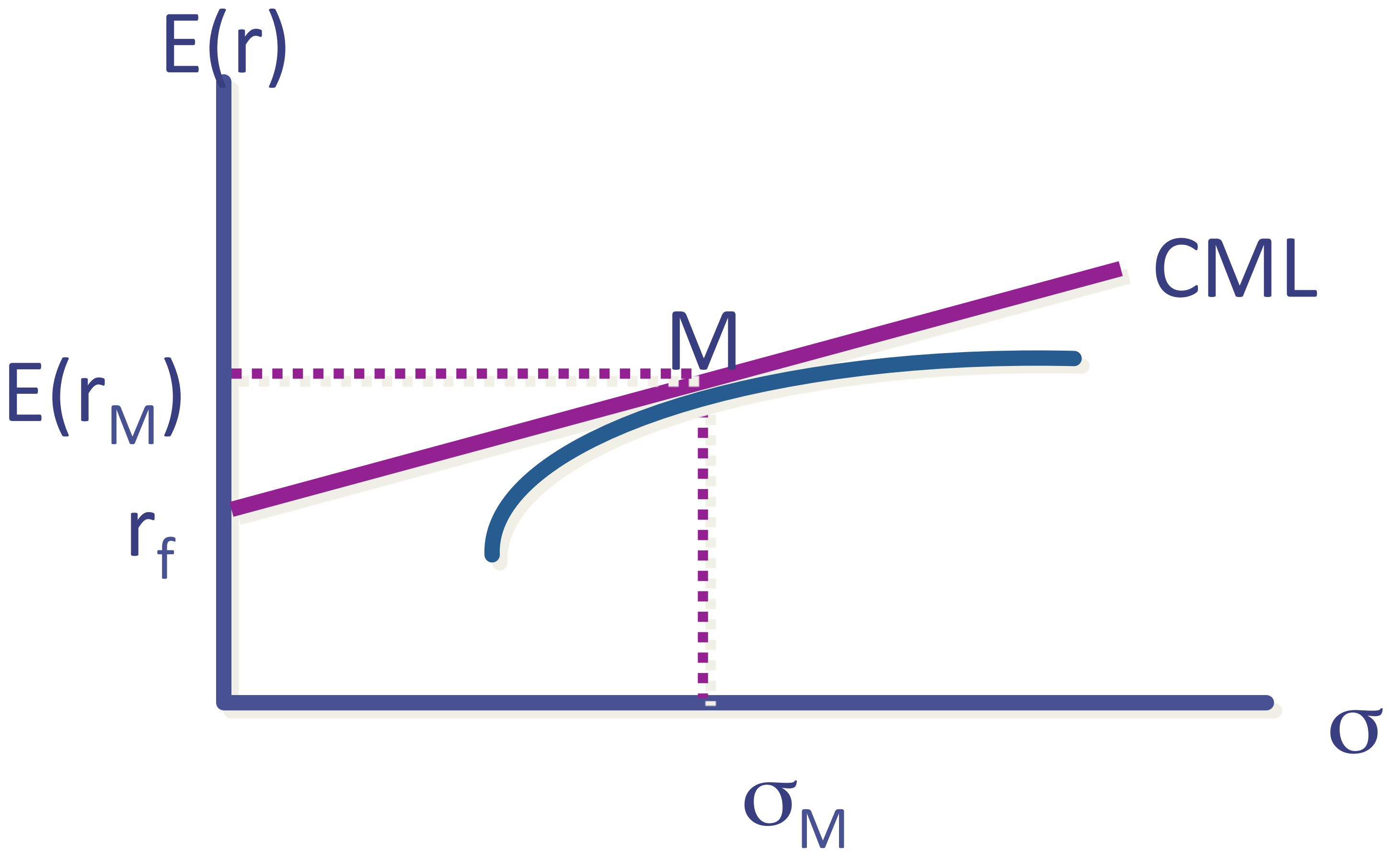
The Capital Market Line



Security Market Line



Capital Market Line



Capital Market Line

- The CML is a straight line with intercept r_f and slope equal to the Sharpe Ratio of the market portfolio

$$Slope = \frac{E[r_M] - r_f}{\sigma_M}$$

- The equation for the CML

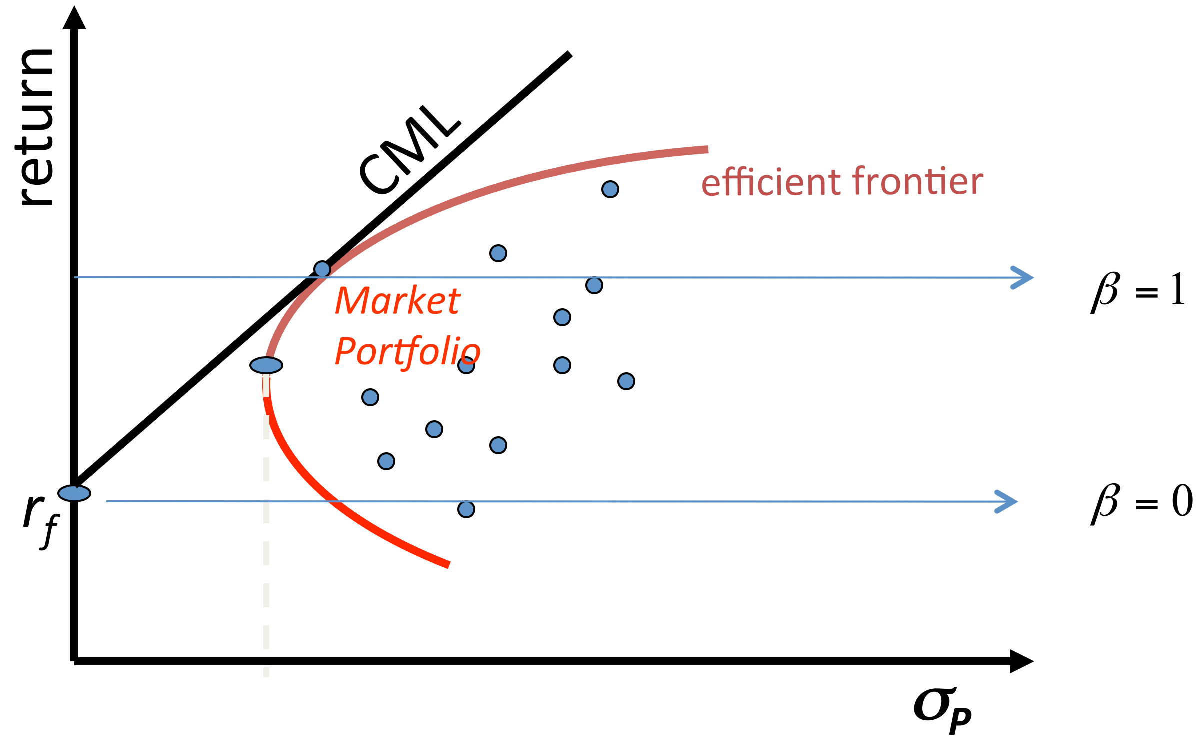
$$E[r_E] = r_f + \frac{E[r_M] - r_f}{\sigma_M}\sigma_E$$

- The subscript E emphasizes that the CML is only for efficient portfolios

CML vs. SML

- The CML plots the relation between expected returns and standard deviation
- The SML plots the relation between expected returns and β
- All portfolios, whether efficient or not, must lie on the SML but only efficient portfolios are on the CML
  - Investments with the same mean return can have different standard deviations, but must have the same β. In other words,the only relevant measure of risk for pricing securities is β (a measure of covariance with market portfolio)

Expected Return and Beta



CAPM assumptions

- Frictionless markets
  - No trading costs
  - No taxes
  - All assets can be traded
- Unlimited borrowing and lending
  - No restrictions on short sales
- Lending and borrowing rates are the same
- Investors care only about means and variances
- All investors are fully rational and have the same information (**homogeneous expectations**)
- These are strong assumptions. We would not expect it to hold exactly. Empirical tests.

Portfolio Betas

$$\bar{r}_i = r_f + \beta_i\left(\bar{r}_M - r_f\right)$$

$$\sum_i w_i \bar{r}_i = \sum_i w_i r_f + \sum_i w_i \beta_i \left(\bar{r}_M - r_f\right) = r_f \sum_i w_i + \left(\bar{r}_M - r_f\right)\sum_i w_i \beta_i$$

$$\bar{r}_P = r_f + \beta_P\left(\bar{r}_M - r_f\right)$$

Beta of a portfolio is portfolio-weighted average of individual assets

$$\beta_P = \sum_i w_i \beta_i$$

– Beta of a company is the value-weighted average of the betas of its subsidiaries

- CAPM (SML) holds also for all portfolios

Example

- Suppose the stock of the 3M Company (MMM) has a beta of 0.69 and the beta of Hewlett-Packard Co. (HPQ) stock is 1.77.
- Assume the risk-free interest rate is 5% and the expected return of the market portfolio is 12%.
- **What is the expected return of a portfolio of 40% of 3M stock and 60% Hewlett-Packard stock, according to the CAPM?**

Example

- **Solution**

$$\beta_P = \sum_i x_i \beta_i = (.40)(0.69) + (.60)(1.77) = 1.338$$

$$E[R_i] = r_f + \beta_i^{Mkt}(E[R_{Portfolio}] - r_f)$$

$$E[R_i] = 5\% + 1.338(12\% - 5\%) = 14.37\%$$

Example

- Consider a portfolio made up of asset A and a risk-free asset. For asset A, $E(R_A)$ = 20% and β_A = 1.6. The risk-free rate R_f = 8%. Note that for a risk-free asset, β = 0 by definition.

✧ We can calculate some different possible portfolio expected returns and betas by varying the percentages invested in these two assets.

✧ Note that when the investor borrows at the risk-free rate and invests the proceeds in asset A, the investment in asset A will exceed 100%.

Estimating β with regression

- In practice $$\hat{\beta}_i = \frac{\sigma_{iM}}{\sigma_M^2}$$

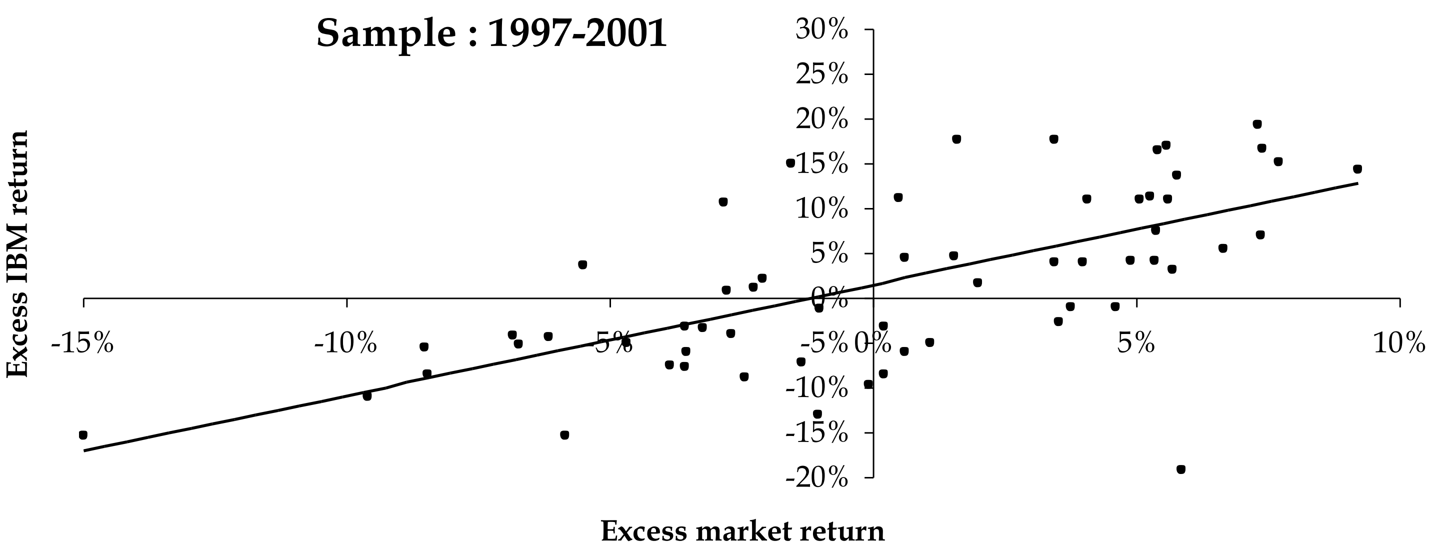
usually estimated using the regression in excess returns

$$r_{i,t} - r_{f,t} = \alpha_i + \beta_i \left(r_{M,t} - r_{f,t} \right) + \varepsilon_{i,t}$$

- Estimation issues
  - Betas may change over time
  - Don't use data from too long ago
  - Five years of weekly or monthly data is reasonable
  - Use Data Analysis / Regression or Linest in Excel

IBM example

$$r_{IBM,t} - r_{f,t} = 0.015 + 1.237\left(r_{M,t} - r_{f,t}\right) + \varepsilon_{IBM,t}$$



IBM example

- Assume riskfree rate of 3% and equity premium of 6%
- Expected return on IBM *assuming CAPM is true is* r=3%+1.237*6%= 10.42%
- In valuation applications, 10.42% would be the discount rate in the present value formula
  - E.g. If you expect IBM's price to be $50 in one year and expect it to pay a dividend of $2, then the current fair price should be $(50+2)/(1+.1042)=$47.1