Quiz 1: Date: April 19, 2022 from 11.00-12.30

Question 1 (10 Points) Score....................

Consider the one-period model of consumption and portfolio choice. Let an individual in this economy has the utility function as follow:

$$U(C) = ln(C)$$

Also, let $\frac{C_1}{C_0}$ is distributed as log-normal with mean equals μ_c and its variance is σ_c.
Please read and answer the following questions carefully and completely.

Score....................

Question 1.1 (10 marks) Calculate the risk free rate R_f in terms of the individual's consumption, C_0 and C_1. Then,explain the relationship between the level of consumption and the risk free rate in this economy.

$m_{01} = \frac{\delta U'(C_1)}{U'(C_0)} = \delta \left(\frac{C_0}{C_1}\right) = \delta e^{\ln\left(\frac{C_0}{C_1}\right)}$

$\frac{1}{R_f} = E(m_{01})$

$\frac{1}{R_f} = \delta E\left[e^{\ln\left(\frac{C_0}{C_1}\right)}\right]$

$\frac{1}{R_f} = \delta E\left[e^{\mu_c + \frac{1}{2}(r-1)\sigma_c^2}\right]$

$\ln(R_f) = -\ln(\delta) + \mu_c - \frac{1}{2}(r-1)\sigma_c^2$

Score....................

Question 1.2 (10 marks) Calculate the elasticity of intertemporal substitution in this setting. If in the next year, the interest rate is falling, Will the individual's consumption level increase or decrease? Why? To support your answer, use the concepts of income effect and substitution effect.

$$U'(c_0) = R_f \delta E(U'(c_1))$$

$$\frac{1}{c_0} = R_f \delta E\left[\frac{1}{c_1}\right]$$

$$\frac{1}{R_f} = \delta E\left[\frac{c_0}{c_1}\right]$$

$$R_f = \frac{1}{\delta}\left(\frac{c_1}{c_0}\right)$$

$$\frac{\partial R_f}{\partial\left(\frac{c_1}{c_0}\right)} = \frac{1}{\delta}$$

$$\frac{\partial c_1/c_0 / \frac{c_1}{c_0}}{\partial R_f / R_f} = \frac{c_1/\cancel{c_0}}{\cancel{R_f}} \cdot \frac{\cancel{R_f}}{\frac{c_1}{\cancel{c_0}}}$$

$$\varepsilon = 1$$

$\therefore$ Decrease

Score.....................

Question 1.3 (10 marks) Solve for the pricing kernel P_i of any risky asset i in this economy. Then explain the meaning of this pricing kernel.

$$E[m_{01} R_i] = 1$$

$$1 = E[m_{01}]E[R_i] + Cov(m_{01}, R_i)$$

$$\frac{1}{E(m_{01})} = E[R_i] + \frac{Cov(m_{01}, R_i)}{E[m_{01}]}$$

$$R_f = E(R_i) + \frac{Cov(m_{01}, R_i)}{E(m_{01})}$$

$$E(R_i) = R_f - \frac{Cov(m_{01}, R_i)}{E(m_{01})}$$

$$= R_f - \left[\frac{Cov(U'(C_1), R_i)}{E(U'(C_{01}))}\right]$$

Score.....................

Question 1.4 (10 marks) Calculate Hansen-Jaganathan Bound and explain the meaning.