MLE

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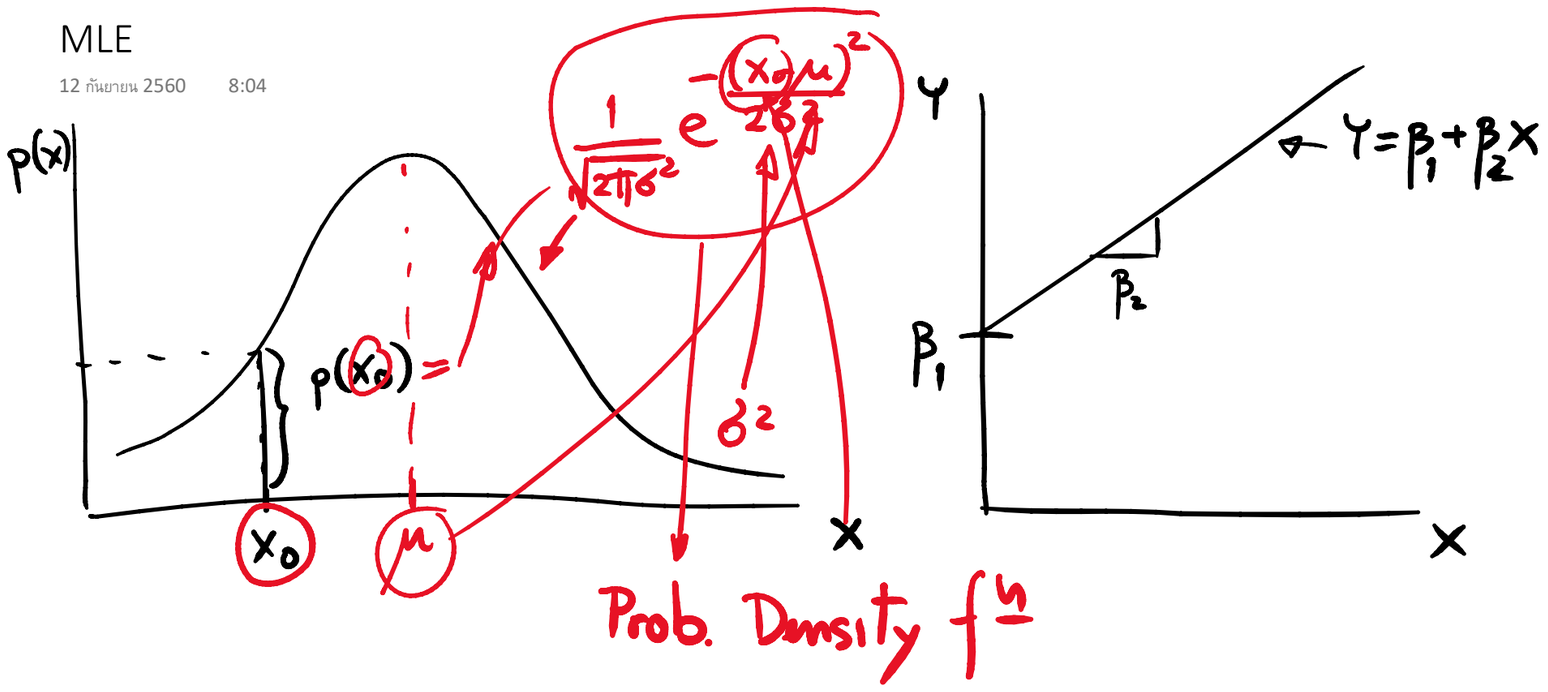


Prob. Density $f^{\underline{y}}$

**pdf** ⇒ X is unknown variable

$\mu, \sigma^2$ are given parameters.

Likelihood $f^{\underline{y}}$ ≡ pdf.

X is given variable

$\mu, \sigma^2$ are unknown parameters.

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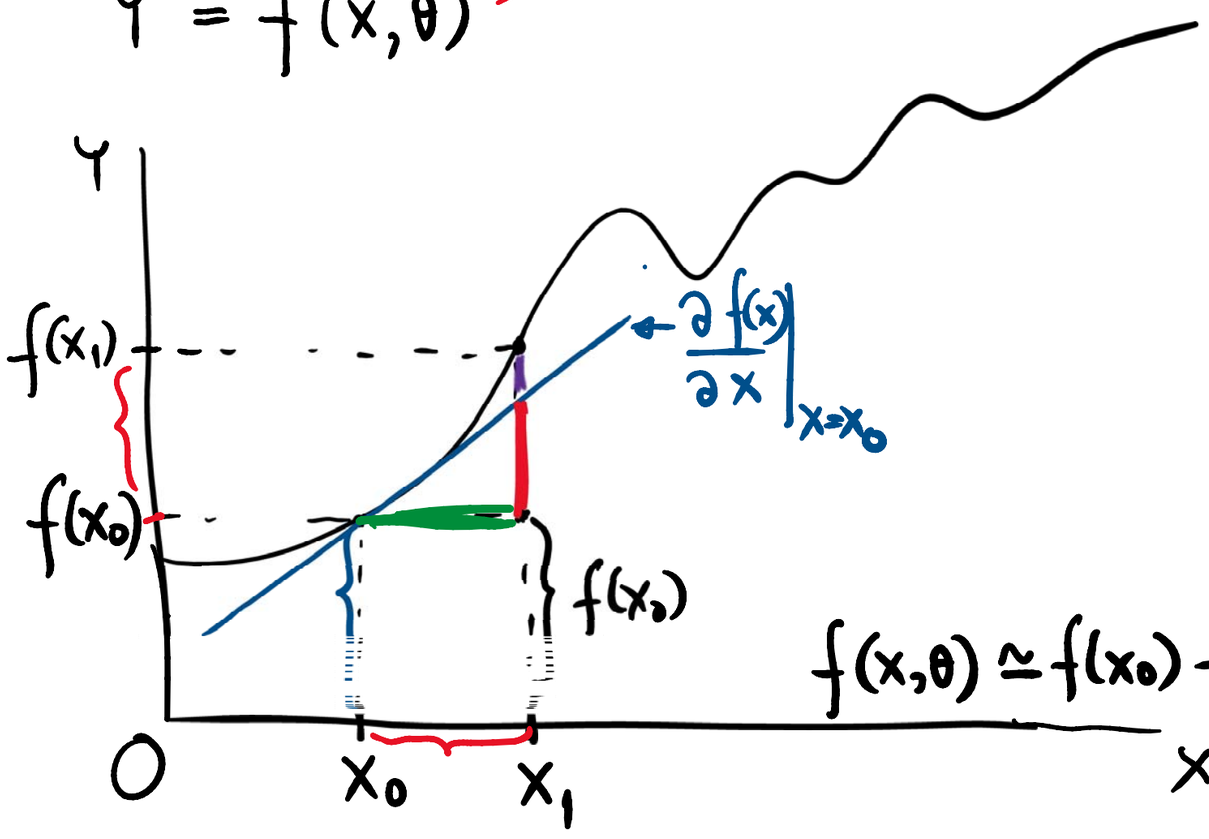
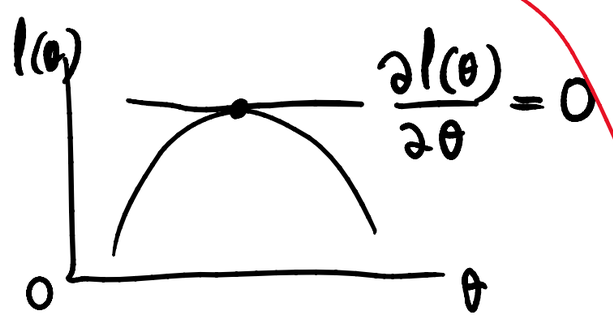
$Y = f(x,\theta)$ — Nonlinear.




$$f(x,\theta) \simeq f(x_0) + \frac{\partial f(x)}{\partial x}\Big|_{x=x_0} (x_1-x_0) + \Delta$$

$$y = f(x,\theta) \simeq f(x_0) + \frac{\partial f(x)}{\partial x}\Big|_{x=x_0} \cdot (x_1-x_0)$$

**Max** $l(\theta)$




$$\underset{k\times 1}{\theta} = \begin{bmatrix} \theta_1 \\ \theta_2 \\ \vdots \\ \theta_k \end{bmatrix}$$

**Gradient**

$$\underset{k\times 1}{G(\theta)} = \frac{\partial l(\theta)}{\partial \theta} = \begin{bmatrix} \frac{\partial l(\theta)}{\partial \theta_1} \\ \frac{\partial l(\theta)}{\partial \theta_2} \\ \vdots \\ \frac{\partial l(\theta)}{\partial \theta_k} \end{bmatrix} \simeq G(\theta_0) + \frac{\partial G(\theta)}{\partial \theta}\Big|_{\theta=\theta_0} \cdot (\theta-\theta_0) = 0$$

$$\Downarrow$$

$$G(\theta_0) + H(\theta_0)\cdot(\theta-\theta_0) = 0$$

$$\begin{bmatrix} \frac{\partial l(\theta)}{\partial \theta_k} \end{bmatrix}_{k \times 1}$$

## Hessian

$$\underset{k \times k}{H(\theta)} = \frac{\partial l(\theta)^2}{\partial \theta \partial \theta'} = \begin{bmatrix} \frac{\partial^2 l(\theta)}{\partial \theta_1 \partial \theta_1} & \frac{\partial^2 l(\theta)}{\partial \theta_1 \partial \theta_2} & \cdots & \frac{\partial^2 l(\theta)}{\partial \theta_1 \partial \theta_k} \\ \frac{\partial^2 l(\theta)}{\partial \theta_2 \partial \theta_1} & \frac{\partial^2 l(\theta)}{\partial \theta_2 \partial \theta_2} & \cdots & \frac{\partial^2 l(\theta)}{\partial \theta_2 \partial \theta_k} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial^2 l(\theta)}{\partial \theta_k \partial \theta_1} & \frac{\partial^2 l(\theta)}{\partial \theta_k \partial \theta_2} & \cdots & \frac{\partial^2 l(\theta)}{\partial \theta_k \partial \theta_k} \end{bmatrix}_{k \times k}$$

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$$G(\theta) \simeq \boxed{G(\theta_0) + H(\theta_0)(\theta - \theta_0) = 0}$$ FOC.

$$G(\theta_0) + H(\theta_0) \cdot (\theta - \theta_0) = 0$$

$$G(\theta_0) + H(\theta_0)\theta - H(\theta_0)\theta_0 = 0$$

$$H(\theta_0)^{-1} H(\theta_0)\,\theta = H(\theta_0)^{-1} H(\theta_0)\,\theta_0 - H(\theta_0)^{-1} G(\theta_0)$$

($H(\theta_0)$: $k \times k$, $\theta$: $k \times 1$; $H(\theta_0)^{-1} H(\theta_0)\,\theta_0$: $k \times k$, $k \times 1$; $H(\theta_0)^{-1} G(\theta_0)$: $k \times 1$)

$$\underset{k \times 1}{\theta} = \underset{k \times 1}{\theta_0} - \underset{k \times 1}{H(\theta_0)^{-1} \cdot G(\theta_0)}$$

($H(\theta_0)^{-1}$: $k \times k$, $G(\theta_0)$: $k \times 1$)

Initial Value

Convergence Value
Default $= 10^{-13}$
STATA

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MLE applies Nonlinear Optimization – Iterative Procedure

$$\hat{\theta}_{ML} = \theta_0 - \Delta$$

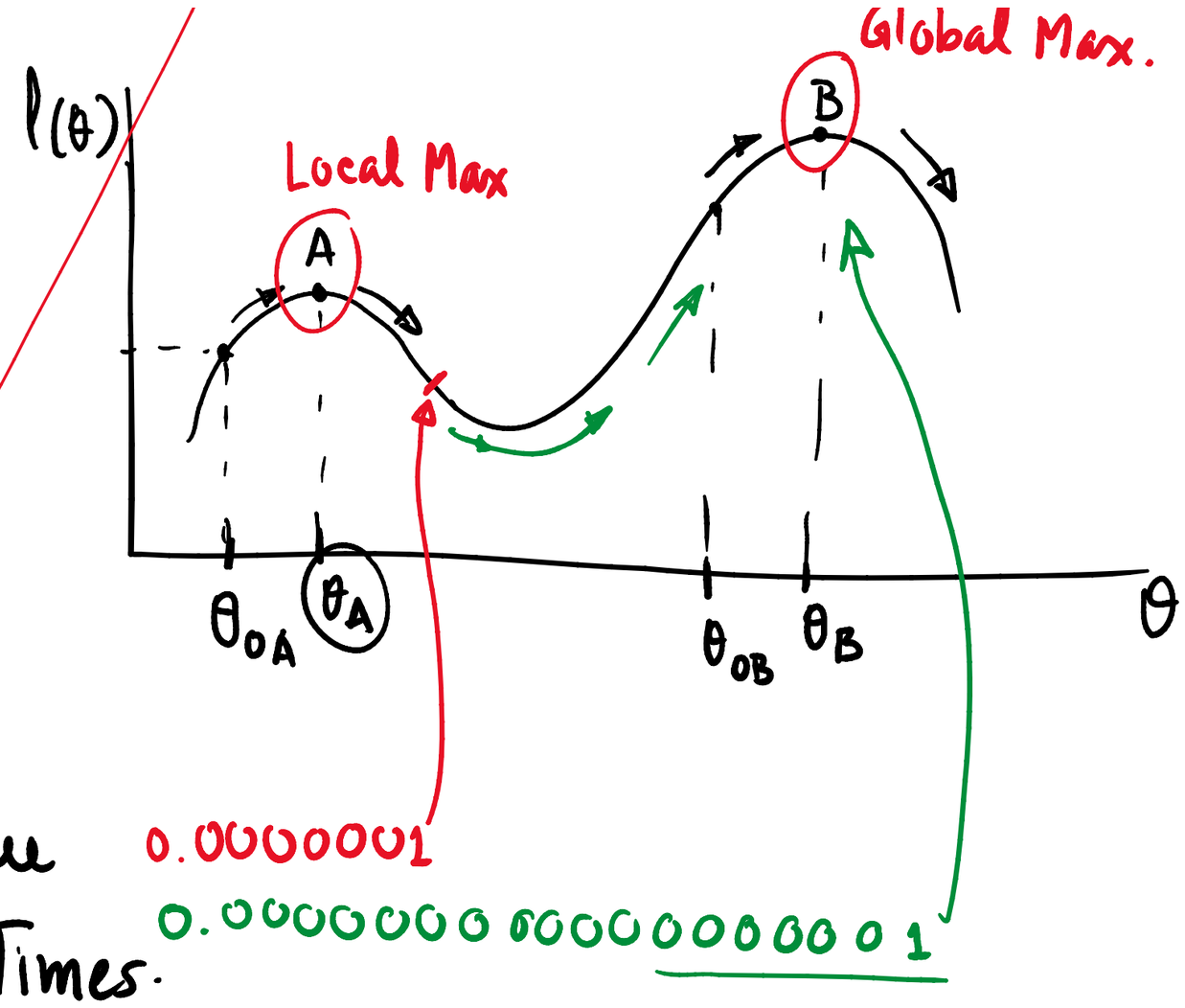
* Algorithm. – NR
– BHHH

ML Computation

Global Max.

## ML Computation

1. Initial Value
2. Algorithm
3. Convergence Value
4. #Max Iterative Times.




STATA = 10,000

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## Overall Test in OLS → Min RSS

① $Y_i = \beta_1 + \beta_2 X_{2i} + \beta_3 X_{3i} + \cdots + \beta_k X_{ki} + u_i$ — $RSS_{UR}$ - Unrestricted Model.

$H_0: \beta_2 = \beta_3 = \cdots = \beta_k = 0$ — k-1 restrictions.

$Y_i = \beta_1 + u_i$ — $RSS_R$ — Restricted Model.

$$F_{(k-1,n-k)} = \frac{RSS_R - RSS_{UR} \,/\, k-1}{RSS_{UR} \,/\, n-k}$$

($RSS_R - RSS_{UR} \sim \chi^2$, $RSS_{UR} \sim \chi^2$)

## Individual Test - t-test.

Ex. model ①. $H_0: \beta_2 = 0$ | $H_0: \beta_3 = 0$ | .. $H_0: \beta_k = 0$

$\hat{\beta} \sim t(\beta, s^2_{\hat{\beta}})$

$Y \sim N$ Nonstochastic = Fixed. $u \sim N(0, \sigma^2 I)$

t Parameter

Simple Regression

$\hat{\beta}_{OLS} = (X'X)^{-1}X'Y$ ~N

Parameter

Loss Degree of Freedom

$\hat{\beta}_2 = \frac{\sum(X_i - \bar{X})(Y_i - \bar{Y})}{\sum(X_i - \bar{X})^2}$

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## Overall Test - MLE — Max. likelihood

$$Y_i = \beta_1 + \beta_2 X_{2i} + \beta_3 X_{3i} + \cdots + \beta_k X_{ki} + u_i \quad \text{— Unrestricted.}$$

($\beta_2 = 0$, $\beta_3 = 0$, $\beta_k = 0$; $L_{UR}$)

$$H_0: \beta_2 = \beta_3 = \cdots = \beta_k = 0$$

$$Y_i = \beta_1 + u_i \quad (L_R) \quad \text{— Restricted Model.}$$

Likelihood Ratio

$$\log\left(\frac{L_{UR}}{L_R}\right)^2 = 2(\log L_{UR} - \log L_R) \sim \chi^2_{(k-1)}$$

($\log L_{UR} \sim \chi^2$, $\log L_R \sim \chi^2$)

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## OLS

1. Sign & Meaning.
2. Overall Test - F-test
3. $R^2$ - Model can explain Y by ?? %
   - Make comparison among models.
4. Individual Test
   t-test

## MLE

1. Sign & Meaning
2. Overall Test - $\chi^2$-test
   - LR
   - Wald
   - LM
3. No $R^2$ ⊗

   log L - log-likelihood Value.
4. Individual Test
   Z-test