Assignment 4

1. Let a_1, a_2, a_3, and a_4 be some constant real numbers such that
$$a_1 < a_2 < a_3 < a_4.$$
Determine the solution set for each of the following inequalities in terms of a_1, a_2, a_3, and a_4.
   (a) $(x-a_1)(x-a_2)(x-a_3)(x-a_4) \geq 0$
   (b) $(x-a_1)^2(x-a_2)(x-a_3)^2(x-a_4)^3 \geq 0$

2. Let a and b be real numbers with $|b| < |a| < 1$ and $a > 0$.
   (a) Show that $a-1 \ < \ 0 \ < \ b+a \ < \ 2a \ < \ 1+a.$
   (b) Show that
$$\frac{2a^2-2a}{b+a} > \frac{a^2-1}{2a}.$$

3. Find the solution set for each of following inequalities.
   (a)
$$\frac{1}{x} < 8$$
   (b)
$$\frac{9-x}{x-2} \geq \frac{2-x}{x-9}$$
   (c)
$$\frac{|2x+5|-2}{3|x|-1} \leq 1$$
   (d)
$$\frac{|x^2-x+1|}{|3x+1|} > 1$$
   (e)
$$\frac{|x^2-2|+2}{x^2-3|x|+2} < 0$$

4. A math class counts the midterm exam scores as 1/3 of the grade and the final exam scores as 2/3 of the grade. Paul scored 48% on the midterm.

   To get an A, Paul must have the total score between 90% and 100% inclusive;
   to get a B, Paul must have the total score between 80% and 89% inclusive;
   to get a C, Paul must have the total score between 70% and 79% inclusive.

   Suppose that all scores with decimal digits get rounded up to the closest higher integers.

   (a) What range of scores (in %) on the final exam would make Paul get a C?

   (b) What is the highest grade that Paul could get for this class?

5. (Optional) The weekly demand (the number bought by consumers) for the a product is given by the formula

$$d = 9000 - 60p$$

where p is the price each in dollars.

(a)What is the demand when the price is 30 each?

(b) In what price range will the demand be above 6000?

6. (Optional) A store's revenue $R(x)$ (in Baht) on the sale of x cupcakes is determined by the formula $R(x) = 50x - x^2$. The cost $C(x)$(in Baht) for producing x cupcakes is given by the formula $C(x) = 2x + 400$. For what values of x is the store's profit positive? Note: profit = revenue - cost.

7. (Optional) Find the solution set for each of the following inequities.

(a) $|x-1| < 7$
(b) $3 < |x+2| < 4$
(c) $|2x+3| \leq 2$
(d) $|3x-1| \geq 1$
(e) $\frac{|x+2|}{2} > |x|$
(f) $|x+1| - |2x-1| < 0$
(g) $|2-x| - x \leq 0$
(h) $|x+1| > 3 - |x|$
(i) $x^2 - 2 \geq \frac{1}{2}|x-1|$
(j) $|x+3| \geq 2 - x$
(k) $|2x^2 + x - 1| \geq 2$
(l) $\frac{|x|}{2} + 3 > x^2$
(m) $|x| > \frac{2}{|x+1|}$
(n) $\frac{x^2}{x+1} < |x|$
(o) $\frac{4}{|x|} \leq \frac{1}{|x|}$
(p) $|x^2+1| < |x+1|$
(q) $(x^2+1)(|x+2| - |x|) \geq 0$
(r) $\frac{|1-x|}{x^3+2x^2+5x+4} \leq 0$