

EE325 Section 1 HW 2 Due Thursday February 20<sup>th</sup> (23:00 hr.),2020

Use 4 decimal places for numerical answers

1. In Table 1.a.  $X_i$  is total microeconomics exam point (total points are 100) and  $Y_i$  is GPA of each student.

Table 1.a

Student	$Y_i$	$X_i$
1	2.8	63
2	3.4	72
3	3	78
4	3.5	81
5	3.6	87
6	3.0	75
7	2.7	75
8	3.7	90

$$\bar{x} = \frac{63+72+78+81+87+75+75+90}{8}$$

$$\bar{x} = 77.625$$

$$\bar{y} = \frac{2.8+3.4+3+3.5+3.6+3.0+2.7+3.7}{8}$$

$$\bar{y} = 3.2125$$

1.1 Now consider the two-variable  $Y_i = \beta_0 + \beta_1 X_i + u_i$ ,  $u_i \sim NIID(0, \sigma^2)$ . Use OLS to find the estimator of  $\beta_0$  and  $\beta_1$ . (Note: *NIID* = Normally, Identically, and Independently Distributed).

$$\hat{\beta}_1 = \frac{\sum_i (x_i - \bar{x})(y_i - \bar{y})}{\sum_i (x_i - \bar{x})^2}$$

$$= \frac{(63-77.625)(2.8-3.2125) + (72-77.625)(3.4-3.2125) + (78-77.625)(3-3.2125) + (81-77.625)(3.5-3.2125) + (87-77.625)(3.6-3.2125) + (75-77.625)(3-3.2125) + (75-77.625)(2.7-3.2125) + (90-77.625)(3.7-3.2125)}{(63-77.625)^2 + (72-77.625)^2 + (78-77.625)^2 + (81-77.625)^2 + (87-77.625)^2 + (75-77.625)^2 + (75-77.625)^2 + (90-77.625)^2}$$

$$= \frac{17.4374}{511.875}$$

$$= 0.0341$$

$$\hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x}$$

$$= 3.2125 - (0.0341)(77.625)$$

$$= 0.5655$$

1.2 For each observation  $i$ , find  $\hat{Y}_i$  and  $\hat{u}_i$ . Show that  $\sum_{i=0}^N \hat{u}_i = 0$ .

$y$	$x$	$\hat{y}$	$\hat{u}$
2.8	63	2.7143	0.0857
3.4	72	3.0209	0.3791
3	78	3.2253	-0.2253
3.5	81	3.3275	0.1725
3.6	87	3.5319	0.0651
3	75	3.1231	-0.01231
2.7	75	3.1231	-0.4231
3.7	90	3.6341	0.0639

calculated by  $\hat{y}_i = \hat{\beta}_0 + \hat{\beta}_1 x_i$   
 $\hat{u}_i = y_i - \hat{y}_i$

$$\sum_{i=0}^N \hat{u}_i = 0$$

$$\hat{u}_i = y_i - \bar{y}$$

$$= (2.8-3.2125) + (3.4-3.2125) + (3-3.2125) + (3.5-3.2125) + (3.6-3.2125) + (3-3.2125) + (2.7-3.2125) + (3.7-3.2125)$$

$$= 0$$

1.3 Find  $var(\hat{u}_i)$ ,  $var(\hat{\beta}_0)$ ,  $var(\hat{\beta}_1)$

2. Data is listed in the table

$X_i$	$Y_i$
10	0
12	2
14	5
16	6
18	7
22	10
24	10
26	15
28	16
30	20

$$\bar{x} = \frac{10+12+14+16+18+22+24+26+28+30}{10} = 20$$

$$\bar{y} = \frac{0+2+5+6+7+10+10+15+16+20}{10} = 9.1$$

2.1 From the simple regression model  $Y_i = \beta_0 + \beta_1 X_i + u_i$ ,  $u_i \sim NIID(0, \sigma^2)$ . Find estimators of  $\beta_0$  and  $\beta_1$  from the OLS method and interpret the meaning.

$$\hat{\beta}_1 = \frac{394}{440} = 0.8955$$

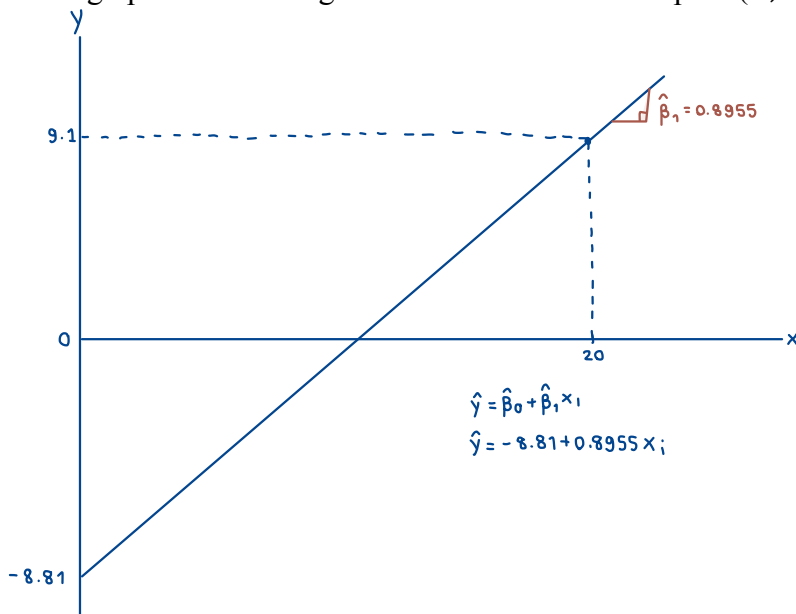
$$\hat{\beta}_0 = 9.1 - (0.8955)(20) = -8.81$$

2.2 Find the value of  $\hat{Y}_i$  and  $\hat{u}_i$ . Show that  $\sum_{i=0}^N \hat{u}_i = 0$ .

$x_i$	$y_i$	$\hat{y}_i$	$\hat{u}_i$
10	0	0.145	-0.145
12	2	1.936	0.064
14	5	3.727	1.273
16	6	5.518	0.482
18	7	7.309	-0.309
22	10	10.891	-0.891
24	10	12.682	-2.682
26	15	14.473	0.527
28	16	16.264	-0.264
30	20	18.055	1.945
			<u>1.945</u>

$\sum_{i=0}^N \hat{u}_i = 0$

2.3 Plot graph and draw regression line. Does the line pass  $(\bar{X}, \bar{Y})$ ?



2.4 If  $X_i = 16$ , what is the predicted Y? estimated  $y = \hat{y}$

$$x_i = 16, \hat{y} = 5.518$$

2.5 Find  $var(\hat{u}_i)$ ,  $var(\hat{\beta}_0)$ ,  $var(\hat{\beta}_1)$

3. Consider the below regression function:

$$Y_i = \beta_1 X_i + u_i,$$

Where  $u_i \sim NIID(0, \sigma^2)$ . Find an OLS estimator of  $\beta_1$ . Then, provide a proof that this is an unbiased estimator. Please state the SLR assumption(s) when used.

$$0 = \sum_i (y_i - \hat{\beta}_0 - \hat{\beta}_1 x_i)$$

$$= \sum_i y_i - n \hat{\beta}_0 - \hat{\beta}_1 \sum_i x_i$$

$$\hat{\beta}_0 = \frac{\sum_i y_i}{n} - \hat{\beta}_1 \frac{\sum_i x_i}{n}$$

$$\hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x}$$

But in this case  $\hat{\beta}_0 = 0$

$$0 = \sum_i x_i (y_i - (0) - \hat{\beta}_1 x_i)$$

$$= \sum_i x_i y_i - \sum_i \hat{\beta}_1 x_i^2$$

$$\hat{\beta}_1 = \frac{\sum_i x_i y_i}{\sum_i x_i^2}$$