Answer Key Homework #2

CHAPTER 7: OPTIMAL RISKY PORTFOLIOS

4. The parameters of the opportunity set are:

$$E(r_S) = 20\%,\ \sigma_S = 30\%,\ E(r_B) = 12\%,\ \sigma_S = 30\%,\ \rho = 0.10$$

From the standard deviations and the correlation coefficient we generate the covariance matrix [note that $Cov(r_S, r_B) = \ \rho \times \sigma_S \times \sigma_B$]:

	Bonds	Stocks
Bonds	225	45
Stocks	45	900

The minimum-variance portfolio is computed as follows:

$$W_{Min}(S) = \frac{\sigma_B^2 - Cov(r_S, r_B)}{\sigma_S^2 - \sigma_B^2 - 2Cov(r_S, r_B)} = \frac{225 - 45}{900 + 225 - (2 \times 45)} = 0.1739$$

$$W_{Min}(B) = 1 - 0.1739 = 0.8261$$

The minimum variance portfolio mean and standard deviation are:

$$E(r_{Min}) = (0.1739 \times 0.20) + (0.8261 \times 0.12) = 0.1339 = 13.39\%$$

$$\sigma_{Min} = [w_S^2\sigma_S^2 + w_B^2\sigma_B^2 + 2w_S w_B Cov(r_S, r_B)]^{\frac{1}{2}}$$

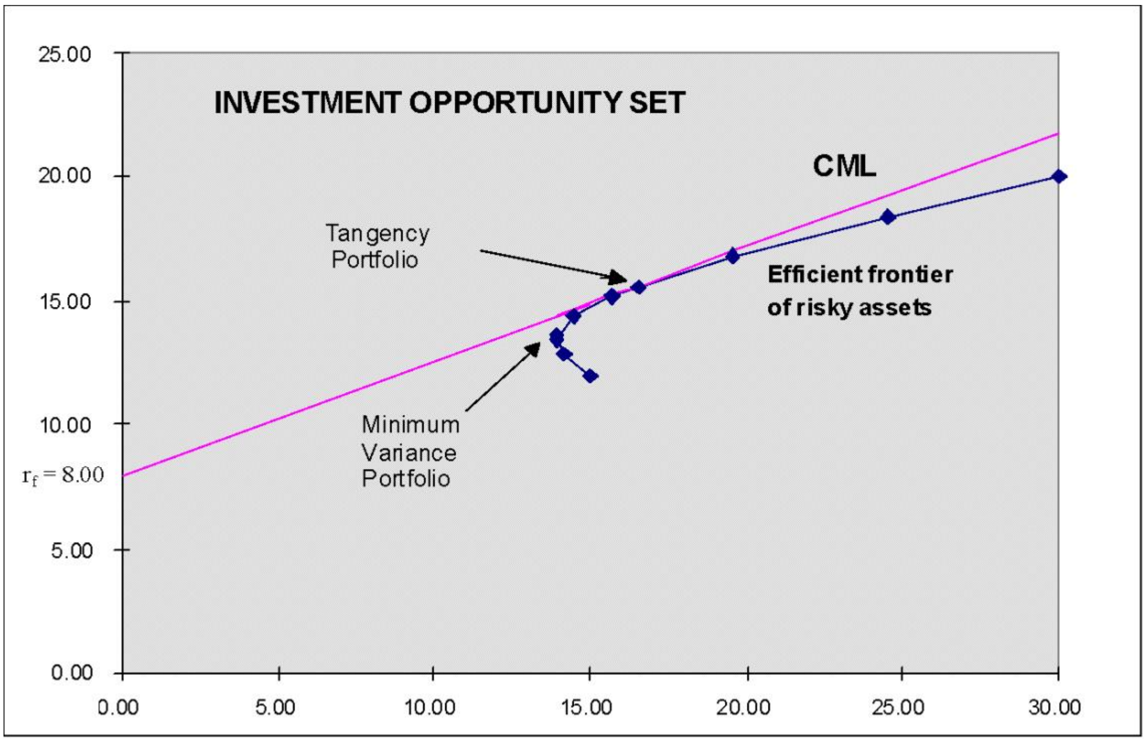
$$= [(0.1739^2 \times 900) + (0.8261^2 \times 225) + (2 \times 0.1739 \times 0.8261 \times 45)]^{\frac{1}{2}}$$

$$= 13.92\%$$

5.

Proportion in stock fund	Proportion in bond fund	Expected return	Standard Deviation	
0.00%	100.00%	12.00%	15.00%	
17.39%	82.61%	13.39%	13.92%	minimum variance
20.00%	80.00%	13.60%	13.94%	
40.00%	60.00%	15.20%	15.70%	
45.16%	54.84%	15.61%	16.54%	tangency portfolio
60.00%	40.00%	16.80%	19.53%	
80.00%	20.00%	18.40%	24.48%	
100.00%	0.00%	20.00%	30.00%	

Graph shown below.



6. The above graph indicates that the optimal portfolio is the tangency portfolio with expected return approximately 15.6% and standard deviation approximately 16.5%.

7. The proportion of the optimal risky portfolio invested in the stock fund is given by:

$$w_S = \frac{[E(r_S) - r_f] \times \sigma_B^2 - [E(r_B) - r_f] \times Cov(r_S, r_B)}{[E(r_S) - r_f] \times \sigma_B^2 + [E(r_B) - r_f] \times \sigma_S^2 - [E(r_S) - r_f + E(r_B) - r_f] \times Cov(r_S, r_B)}$$

$$= \frac{[(.20 - .08) \times 225] - [(.12 - .08) \times 45]}{[(.20 - .08) \times 225] + [(.12 - .08) \times 900] - [(.20 - .08 + .12 - .08) \times 45]}$$

= 0.4516

$$w_B = 1 - 0.4516$$

= 0.5484

The mean and standard deviation of the optimal risky portfolio are:

$E(r_P)$= (0.4516 × .20) + (0.5484 × .12) = .1561

= 15.61%

$\sigma p = [(0.4516^2 \times 900) + (0.5484^2 \times 225) + (2 \times 0.4516 \times 0.5484 \times 45)]^{1/2}$

= 16.54%

8. The reward-to-volatility ratio of the optimal CAL is:

$$\frac{E(r_p) - r_f}{\sigma_p} = \frac{.1561 - .08}{.1654}$$

.4601 should be .4603 (rounding)

9. a. If you require that your portfolio yield an expected return of 14%, then you can find the corresponding standard deviation from the optimal CAL. The equation for this CAL is:

$$E(r_C) = r_f + \frac{E(r_p) - r_f}{\sigma_P}\sigma_C = .08 + 0.4601\sigma_C$$

.4601 should be .4603 (rounding)

If $E(r_C)$ is equal to 14%, then the standard deviation of the portfolio is 13.03%.

b. To find the proportion invested in the T-bill fund, remember that the mean of the complete portfolio (i.e., 14%) is an average of the T-bill rate and the optimal combination of stocks and bonds (P). Let y be the proportion invested in the portfolio P. The mean of any portfolio along the optimal CAL is:

$$E(r_C) = (1-y)\times r_f + y \times E(r_P) = r_f + y\times[E(r_P) - r_f] = .08 + y\times(.1561 - .08)$$

Setting $E(r_C)$ = 14% we find: y = 0.7881 and (1 − y) = 0.2119 (the proportion invested in the T-bill fund).

To find the proportions invested in each of the funds, multiply 0.7884 times

To find the proportions invested in each of the funds, multiply 0.7884 times the respective proportions of stocks and bonds in the optimal risky portfolio:

Proportion of stocks in complete portfolio = 0.7881 × 0.4516 = 0.3559

Proportion of bonds in complete portfolio = 0.7881 × 0.5484 = 0.4322

10. Using only the stock and bond funds to achieve a portfolio expected return of 14%, we must find the appropriate proportion in the stock fund (w_S) and the appropriate proportion in the bond fund ($w_B = 1 - w_S$) as follows:

$$.14 = .20 \times w_S + .12 \times (1 - w_S) = .12 + .08 \times w_S \Rightarrow w_S = 0.25$$

So, the proportions are 25% invested in the stock fund and 75% in the bond fund. The standard deviation of this portfolio will be:

$$\sigma_P = [(0.25^2 \times 900) + (0.75^2 \times 225) + (2 \times 0.25 \times 0.75 \times 45)]^{1/2} = 14.13\%$$

This is considerably greater than the standard deviation of 13.04% achieved using T-bills and the optimal portfolio.

17. The correct choice is c. Intuitively, we note that since all stocks have the same expected rate of return and standard deviation, we choose the stock that will result in lowest risk. This is the stock that has the lowest correlation with Stock A.

More formally, we note that when all stocks have the same expected rate of return, the optimal portfolio for any risk-averse investor is the global minimum variance portfolio (G). When the portfolio is restricted to Stock A and one additional stock, the objective is to find G for any pair that includes Stock A, and then select the combination with the lowest variance. With two stocks, I and J, the formula for the weights in G is:

$$w_{Min}(I) = \frac{\sigma_J^2 - Cov(r_I, r_J)}{\sigma_I^2 + \sigma_J^2 - 2Cov(r_I, r_J)}$$

$$w_{Min}(J) = 1 - w_{Min}(I)$$

Since all standard deviations are equal to 20%:

$$Cov(r_I, r_J) = \rho\sigma_I\sigma_J = 400\rho \text{ and } w_{Min}(I) = w_{Min}(J) = 0.5$$

This intuitive result is an implication of a property of any efficient frontier, namely, that the covariances of the global minimum variance portfolio with all other assets on the frontier are identical and equal to its own variance. (Otherwise, additional diversification would further reduce the variance.) In this case, the standard deviation of G(I, J) reduces to:

$$\sigma_{Min}(G) = [200 \times (1 + \rho_{IJ})]^{1/2}$$

This leads to the intuitive result that the desired addition would be the stock with the lowest correlation with Stock A, which is Stock D. The optimal portfolio is equally invested in Stock A and Stock D, and the standard deviation is 17.03%.

18. No, the answer to Problem 17 would not change, at least as long as investors are not risk lovers. Risk neutral investors would not care which portfolio they held since all portfolios have an expected return of 8%.

19. Yes, the answers to Problems 17 and 18 would change. The efficient frontier of risky assets is horizontal at 8%, so the optimal CAL runs from the risk-free rate through G. This implies risk-averse investors will just hold Treasury Bills.

CHAPTER 9: THE CAPITAL ASSET PRICING MODEL

1. $E(r_P) = r_f + \beta_P \times [E(r_M) - r_f]$

   $0.18 = 0.06 + \beta_P \times [0.14 - 0.06] \rightarrow \beta_P = \frac{0.120}{0.08} = 1.5$

3. a. False. $\beta = 0$ implies $E(r) = r_f$, not zero.

   b. False. Investors require a risk premium only for bearing systematic (undiversifiable or market) risk. Total volatility includes diversifiable risk.

   c. False. Your portfolio should be invested 75% in the market portfolio and 25% in T-bills. Then:

$$\beta_P = (0.75 \times 1) + (0.25 \times 0) = 0.75$$