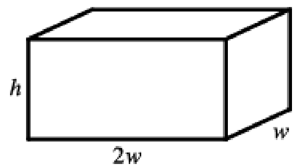
14. Let b be the length of the base of the box and h the height. The volume is $32{,}000 = b^2 h \quad\Rightarrow\quad h = 32{,}000/b^2$.

The surface area of the open box is $S = b^2 + 4hb = b^2 + 4(32{,}000/b^2)b = b^2 + 4(32{,}000)/b$.

So $S'(b) = 2b - 4(32{,}000)/b^2 = 2\left(b^3 - 64{,}000\right)/b^2 = 0 \quad\Leftrightarrow\quad b = \sqrt[3]{64{,}000} = 40$. This gives an absolute minimum since $S'(b) < 0$ if $0 < b < 40$ and $S'(b) > 0$ if $b > 40$. The box should be $40 \times 40 \times 20$.

16.



$V = lwh \quad\Rightarrow\quad 10 = (2w)(w)h = 2w^2h$, so $h = 5/w^2$.

The cost is $10(2w^2) + 6[2(2wh) + 2(hw)] = 20w^2 + 36wh$, so

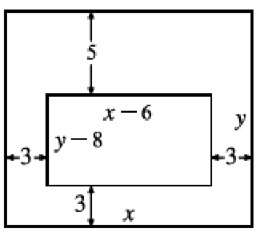
$C(w) = 20w^2 + 36w\left(5/w^2\right) = 20w^2 + 180/w$.

$C'(w) = 40w - 180/w^2 = 40\left(w^3 - \frac{9}{2}\right)/w^2 \quad\Rightarrow\quad w = \sqrt[3]{\frac{9}{2}}$ is the critical number. There is an absolute minimum for C when $w = \sqrt[3]{\frac{9}{2}}$ since $C'(w) < 0$ for $0 < w < \sqrt[3]{\frac{9}{2}}$ and $C'(w) > 0$ for $w > \sqrt[3]{\frac{9}{2}}$.

$$C\left(\sqrt[3]{\tfrac{9}{2}}\right) = 20\left(\sqrt[3]{\tfrac{9}{2}}\right)^2 + \frac{180}{\sqrt[3]{9/2}} \approx \$163.54.$$

20. The distance d from the point $(3, 0)$ to a point $(x, \sqrt{x}\,)$ on the curve is given by $d = \sqrt{(x-3)^2 + (\sqrt{x} - 0)^2}$ and the square of the distance is $S = d^2 = (x-3)^2 + x$. $S' = 2(x-3) + 1 = 2x - 5$ and $S' = 0 \quad\Leftrightarrow\quad x = \frac{5}{2}$. Now $S'' = 2 > 0$, so we know that S has a minimum at $x = \frac{5}{2}$. Thus, the y-value is $\sqrt{\frac{5}{2}}$ and the point is $\left(\frac{5}{2}, \sqrt{\frac{5}{2}}\right)$.

34.



$xy = 900$, so $y = 900/x$. The printed area is

$(x-6)(y-8) = (x-6)(900/x - 8) = 852 - 8x - 5400/x = A(x)$.

$A'(x) = -8 + 5400/x^2 = 0$ when $x^2 = 675 \quad\Rightarrow\quad x = 15\sqrt{3}$. This gives an absolute maximum since $A'(x) > 0$ for $0 < x < 15\sqrt{3}$ and $A'(x) < 0$ for $x > 15\sqrt{3}$. When $x = 15\sqrt{3}$, $y = 20\sqrt{3}$, so the dimensions are $15\sqrt{3} \approx 26.0$ cm and $20\sqrt{3} \approx 34.6$ cm.

60. (a) Let $p(x)$ be the demand function. Then $p(x)$ is linear and $y = p(x)$ passes through $(20, 10)$ and $(18, 11)$, so the slope is $-\frac{1}{2}$ and an equation of the line is $y - 10 = -\frac{1}{2}(x - 20) \quad\Leftrightarrow\quad y = -\frac{1}{2}x + 20$. Thus, the demand is $p(x) = -\frac{1}{2}x + 20$ and the revenue is $R(x) = xp(x) = -\frac{1}{2}x^2 + 20x$.

(b) The cost is $C(x) = 6x$, so the profit is $P(x) = R(x) - C(x) = -\frac{1}{2}x^2 + 14x$. Then $0 = P'(x) = -x + 14 \quad\Rightarrow\quad x = 14$. Since $P''(x) = -1 < 0$, the selling price for maximum profit is $p(14) = -\frac{1}{2}(14) + 20 = \$13$.