

$$\int_1^2 \int_0^1 \frac{x}{y^2} e^{\frac{x}{y}} dx dy = \int_1^2 \left( \frac{1}{y^2} \underbrace{\int_0^1 x e^{\frac{x}{y}} dx}_I \right) dy$$

$$I = \int_0^1 x e^{\frac{x}{y}} dx$$

$$f(x) = x \quad g'(x) = e^{\frac{x}{y}}$$

$$f'(x) = 1 \quad g(x) = y e^{\frac{x}{y}}$$

$$I = \left[ x y e^{\frac{x}{y}} \right]_{x=0}^{x=1} - y \int_0^1 e^{\frac{x}{y}} dx$$

$$= y e^{\frac{1}{y}} - y \left[ y e^{\frac{x}{y}} \right]_{x=0}^{x=1}$$

$$= y e^{\frac{1}{y}} - y^2 \left[ e^{\frac{1}{y}} - 1 \right]$$

$$\mathcal{I} = \int_1^2 \frac{1}{y^2} \left( y e^{\frac{1}{y}} - y^2 (e^{\frac{1}{y}} - 1) \right) dy$$

$$= \int_1^2 \left( \underbrace{\frac{1}{y} e^{\frac{1}{y}}}_{I_2} - e^{\frac{1}{y}} + 1 \right) dy$$

$$I_2 = \int_1^2 \frac{1}{y} e^{\frac{1}{y}} dy \quad u = \frac{1}{y} = y^{-1}$$

$$\frac{du}{dy} = -\frac{1}{y^2}$$

$$dy = -y^2 du$$

$$= \int \frac{1}{y} e^u y^2 du$$

$$= \int y e^u du = \int \frac{1}{u} e^u du$$

"sub doesn't work"

Try by parts

$$\int e^{\frac{1}{y}} dy$$

$$f = e^{\frac{1}{y}}$$

$$f' = -\frac{1}{y^2} e^{\frac{1}{y}}$$

$$g' = 1$$

$$g = y$$

$$= y e^{\frac{1}{y}} - \int -\frac{1}{y} e^{\frac{1}{y}} dy$$

$$\int_1^2 \frac{1}{y} e^{\frac{1}{y}} dy - y e^{\frac{1}{y}} - \int \frac{1}{y} e^{\frac{1}{y}} dy$$
$$+ \int 1 dy$$

$$= -2e^{\frac{1}{2}} + e^1 + (2-1)$$

$$= 1 + e - 2e^{\frac{1}{2}}$$

OR

$$\int_1^2 \int_0^1 \frac{x}{y^2} e^{\frac{x}{y}} dx dy = \int_0^1 \int_1^2 \left( \frac{x}{y^2} e^{\frac{x}{y}} dy \right) dx$$

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$I_1$

$$u = \frac{x}{y} \Rightarrow \frac{du}{dy} = -\frac{x}{y^2} \Rightarrow dy = \frac{y^2}{x} du$$

$$\begin{aligned} I_1 &= \int_1^2 \frac{x}{y^2} e^u \left( \frac{-y^2}{x} \right) du \\ &= - \int_{y=1}^{y=2} e^u du = \left[ -e^{\frac{x}{y}} \right]_{y=1}^{y=2} \\ &= -e^{\frac{x}{2}} + e^x \end{aligned}$$

$$\begin{aligned} \int_0^1 (-e^{\frac{x}{2}} + e^x) dx &= \left[ -2e^{\frac{x}{2}} \right]_0^1 + \left[ e^x \right]_0^1 \\ &= -2e^{\frac{1}{2}} + 2 + e - 1 \\ &= -2e^{\frac{1}{2}} + e + 1 \end{aligned}$$