B.E. International Program

Faculty of Economics, Thammasat University

Final Examination: 1/2014

Subject: MA 217 Calculus for Social Sciences 2

Date: Saturday 13 December 2014 Time: 9.00 – 12.00 hrs.

1.

(a) Find the minimum value of the function $f(x,y) = x^2 + y^2$ subjected to $x \ge 0$ (or $-x \le 0$), use the **Lagrange Multiplier Method**. Explain why your answer is the minimum value of the function.

(b) Fine the optimum value of the function $f(x,y) = x^2 + y^2$ subjected to $2xy = 3$ and $x \ge 0$. Is it possible to use matrix to solve for critical points and why?

(7 marks)

2. Given a linear system

$$\begin{aligned} x_1 + 2x_2 + 3x_3 &= a \\ 4x_1 + 5x_2 + 6x_3 &= b, \\ 7x_1 + 8x_2 + 9x_3 &= c \end{aligned}$$

(a) Is it possible for the linear system to have a unique solution, infinite number of solutions or no solution? Answer all three cases with reasons.

(b) If $a = b = c = 1$, find the solution. If the system has infinite number of solutions, give the answer in vector form.

(7 marks)

3. Consider a linear system

$$\begin{aligned} ax + by &= e \\ cx + dy &= f \end{aligned}$$ where a, b, c, d, e and f are constants,

(a) Determine the relationship of a, b, c and d that make the system has a unique solution.

(b) Find the unique solution in term of a, b, c, d, e and f. In addition, show that your answer is correct.

(8 marks)

4. Given $\underline{\mathbf{A}} = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \end{bmatrix}$

(a) Find $\underline{\mathbf{A}}^{-1}$ and show that your answer is correct.

(b) If $\underline{\mathbf{A}}^{-1}\underline{\mathbf{x}} = \begin{bmatrix} -1 \\ 2 \\ -3 \\ 4 \end{bmatrix}$, find $\underline{\mathbf{x}}$.

(c) Given $\underline{\mathbf{B}}^{\mathrm{T}} - 2\underline{\mathbf{I}} = 2\underline{\mathbf{A}}^{-1}$, determine $\underline{\mathbf{B}}$.

(12 marks)

5. Given $\underline{\mathbf{A}} = \begin{bmatrix} 2 & 4 & 2 & 1 \\ 4 & 3 & 0 & -1 \\ -6 & 0 & 2 & 0 \\ 0 & 1 & 1 & a \end{bmatrix}$

(a) Use **the row operations and co-factor expansion method reducing down to 2x2 matrix** (cross-over calculation for determinant of matrix 3x3 is not allowed.), determine $\det \underline{\mathbf{A}}$ in term of a. In addition, determine the value of a that will make $\underline{\mathbf{A}}$ non-invertable.

(b) If $a = 2$, calculate $\det \underline{\mathbf{A}}$.

(c) Given $\underline{\mathbf{B}} = \begin{bmatrix} 3 & 1 & 5 & -24 \\ 0 & 4 & 1 & -6 \\ 0 & 0 & 25 & 4 \\ 0 & 0 & 0 & -1 \end{bmatrix}$, determine $\det \underline{\mathbf{B}}$ and $\det\left(-\frac{1}{2}\underline{B}^{\mathrm{T}}\right)$.

(d) Given $\underline{\mathbf{C}} = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 10 \end{bmatrix}$, $\underline{\mathbf{A}}^2 = 21\underline{\mathbf{B}}^3\underline{\mathbf{C}}^{-1}\underline{\mathbf{D}}$ and $a = 2$. Find $\det \underline{D}$

(10 marks)

6. Determine this integral $I = \int_{\frac{1}{2}}^{1} \int_{\frac{1}{x}}^{3-2x} dydx$.

(3 marks)

7. Determine this integral $I = \int_0^{\ln 5}\int_0^{\ln 4}\int_0^{\ln 3} e^{-x-y-z}\, dzdxdy$.

(4 marks)

8. Determine this integral $I = \int_1^2 \int_1^z \int_0^y (x+y+z)\, dxdydz$.

(4 marks)

Formulas

Differentiation

We assume that u is a differentiable function of x.

$\frac{d}{dx}(c) = 0$	$\frac{dy}{dx} = \frac{dy}{du}\cdot\frac{du}{dx}$
$\frac{d}{dx}\left(x^n\right) = n\,x^{n-1}$	$\frac{d}{dx}\left(u^n\right) = n\,u^{n-1}\frac{du}{dx}$
$\frac{d}{dx}\left[c\ f(x)\right] = c\,f'(x)$	$\frac{d}{dx}(\ln u) = \frac{1}{u}\frac{du}{dx}$
$\frac{d}{dx}\left[f(x)\pm g(x)\right] = f'(x)\pm g'(x)$	$\frac{d}{dx}\left(e^u\right) = e^u\frac{du}{dx}$
$\frac{d}{dx}\left[f(x)g(x)\right] = f(x)g'(x) + f'(x)g(x)$	$\frac{d}{dx}(\log_b u) = \frac{1}{(\ln b)u}\cdot\frac{du}{dx}$
$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)f'(x) - f(x)g'(x)}{\left[g(x)\right]^2}$	$\frac{d}{dx}\left(a^u\right) = a^u(\ln a)\frac{du}{dx}$

Integration

$\int k dx = kx + C$	$\int\left[f(x)\pm g(x)\right]dx = \int f(x)\,dx \pm \int g(x)\,dx$
$\int x^n dx = \frac{x^{n+1}}{n+1} + C,\ n\neq -1$	$\int[u(x)]^n du = \frac{u^{n+1}}{n+1} + C$
$\int e^x dx = e^x + C$	$\int e^u du = e^u + C$
$\int kf(x)dx = k\int f(x)dx$	$\int\frac{1}{u}du = \ln\lvert u\rvert + C,\ u\neq 0$

Integration by parts

$$\int f(x)g'(x)dx = f(x)g(x) - \int f'(x)g(x)dx$$

$$\int u\,dv = uv - \int v\,du$$

Numerical Solutions

1.

(a) $f(0,0) = 0$

(b) $f\left(\sqrt{\frac{3}{2}},\sqrt{\frac{3}{2}}\right) = 3$, no

2.

(a) Unique > not possible, Infinite > possible, No solution > possible

(b) $\underline{x} = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix} C; C \in \Re$

3.

(a) $|\underline{A}| \neq 0 \neq ad - bc$

(b) $x = \frac{de - bf}{ad - bc}, y = \frac{af - ce}{ad - bc}$

4.

(a) $\underline{A}^{-1} = \begin{bmatrix} -1 & 0 & 0 & 1 \\ 0 & 0 & 1 & -1 \\ 0 & 1 & -1 & 1 \\ 1 & -1 & 1 & -1 \end{bmatrix}$

(b) $\underline{x} = \begin{bmatrix} 1 \\ -1 \\ 2 \\ 0 \end{bmatrix}$

(c) $\underline{B} = \begin{bmatrix} 0 & 0 & 0 & 2 \\ 0 & 2 & 2 & -2 \\ 0 & 2 & 0 & 2 \\ 2 & -2 & 2 & 0 \end{bmatrix}$

5.

(a) $|\underline{A}| = 16a - 18$

(b) $|\underline{A}| = 14$

(c) $|\underline{B}| = -300$, $\left|-\frac{1}{2}\underline{B}^{T}\right| = -\frac{75}{4}$

(d) Undefined, sizes of matrixes are not compatible.

6. $I = \frac{3}{4} + \ln\left(\frac{1}{2}\right) = 0.05685$

7. $I = \frac{2}{5}$

8. $I = \frac{5}{2}$