

## Chapter 9

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# PRODUCTION

### Boiling Down Chapter 9

If utility has been created for anyone, then production has occurred whether or not the inputs are tangible. The process of generating utility can be spelled out by observing the amount of output that is generated by various combinations of inputs. When all relevant combinations of inputs are related to output we have a production function which is usually described in general terms with the equation  $Q = F(K,L)$ , where K and L are capital and labor inputs, respectively, and Q is the amount of output generated. The Q itself is not the utility created, but the value added to the raw materials that went into the creation process. Utility is then realized as the output is consumed.

Production is divided into two time periods. The short run assumes that capital does not vary from day to day. Labor, can be applied to the fixed capital in varying amounts, creating an entire menu of possible output levels. The idea that one McDonald's hamburger restaurant could employ anywhere from 1 to 100 workers illustrates the short-run decision that a McDonald's manager has to face each week. How many workers to hire will depend on how output responds to differing work force sizes. Figure 9-1 below shows a standard short-run production function that might characterize a McDonald's restaurant for a given day.

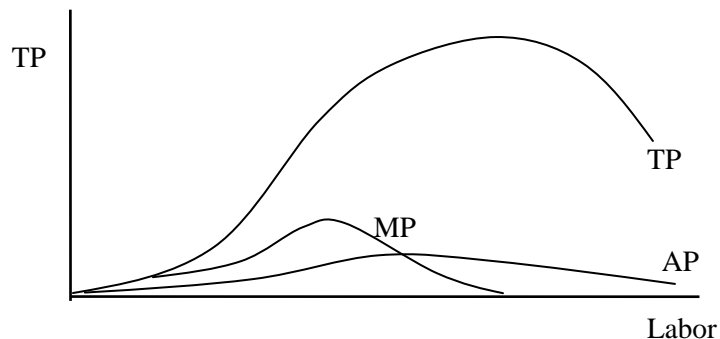


Figure 9-1

Output grows rapidly when the first workers are given more help. As workers are added, the increase in output slows and eventually drops as the workers get in each other's way. The marginal product of labor ( $MP_L$ ) is equal to the slope of the production function ( $MP_L = \text{change in TP} / \text{change in the labor force}$ ), and the average product of labor ( $AP_L$ ) is measured by the slope of a ray from the origin of the production function to the point of total output relevant for the specified labor force ( $AP_L = \text{TP} / \text{total labor force}$ ).

Producing efficiently requires attention to the marginal and average production relationships. As long as the marginal product is greater than average product, adding an

additional worker will increase the average product per worker so profit rises if wages stay constant. Consequently, an additional worker will be employed whenever the marginal product exceeds the average product.

Employment will take place somewhere between the place where marginal product and average product are equal, and where marginal product is 0. On Figure 9-7 in your text the number of persons employed will be between 6 and 8. Later in the text you will see how the wage rate is actually determined, and then a more definitive statement can be made about how many laborers will be employed. Beyond all the graphics, jargon, and algebra, there is a theory building, which leads to the use of just the right mix of inputs in production. The correct production recipe leads to the least cost and the most competitive business operation. This will become clearer when production theory is related to cost in the next chapter.

If the recipe of each production process is correct, then the marginal product of each factor will be identical in all production processes. Otherwise, labor producing a higher marginal product will be bid away from less productive uses until the marginal products are equal. This is the point of Example 9-1 in your text where the marginal product of fish per boat is the important variable to keep in mind when allocating boats.

The long-run time period involves possible changes in both labor and capital. A given output can be produced with many different combinations of labor and capital. If these options are plotted on a graph with labor on one axis and capital on the other, the result is an isoquant. Because more inputs generate more output, input bundles to the northeast of any point on an isoquant will represent more output and a higher isoquant. The slope of an isoquant shows the rate at which one input can be substituted for another in production and that slope is called the marginal rate of technical substitution (MRTS). This MRTS is equal to the ratio of the marginal products of the two inputs. The following example will illustrate the point. If capital has a marginal product of 2 and labor has a marginal product of 1, to keep output constant, it will take 2 laborers to replace 1 unit of capital. Therefore the MRTS and the slope of an isoquant that has labor on the horizontal axis will be  $\frac{1}{2}$ . This relationship is true for any point on the isoquant, but it should be recognized that larger arc movements will involve increasing and decreasing marginal products of the factors so that the marginal product ratio will be only a close approximation of the slope of the chord connecting the two points of the isoquant arc.

Although most production relationships exhibit diminishing returns and therefore isoquants that are concave from above, some factors are perfect substitutes in a production relationship and have linear isoquants. Others do not substitute at all and will have L-shaped isoquants. This is similar to the consumer theory discussion of complements and substitutes in consumption.

When all inputs are increased proportionately, output will increase by some percentage also. If output increases by the same proportion as the input increase, then constant returns to scale exist. If output increases by a smaller percentage, decreasing returns to scale are present. Greater percentage increases of output are called increasing returns to scale. Since factor proportions are not changing in this situation, there is no relationship between diminishing returns to production and decreasing returns to scale. The former is a short-run concept only, and the latter relates only to the long-run possibility of inefficiencies such as administrative layers that communicate poorly.

As in the case of consumer theory, this chapter lends itself to many mathematical applications. The chapter appendix explores some of these without introducing new theoretical ideas.

## Chapter Outline

1. A production function shows the relationship of inputs to output.
  - a. The output involved is the value added to the raw materials.
  - b. In a two-input model the long run is the time period where all inputs are variable and the short run has one input fixed and one variable.
2. The short-run production theory has capital levels fixed.
  - a. Total, average, and marginal products are derived from the total function.
  - b. The marginal product of the variable input will eventually decline as the variable input is increased.
3. The average and marginal product curves are derived from the total product curve.
  - a. If the marginal product curve is above the average product curve, the average curve is rising.
  - b. If the marginal product curve is below the average product curve, the average curve is falling.
  - c. The marginal product curve intersects the average product curve at the peak of the average curve.
  - d. The slope of the total product curve equals the marginal product.
  - e. The marginal-average product relationship is illustrated by the fisherman allocating his boats between two areas of a lake. The marginal product of a resource should be the same in all activities in which it is used.
4. Long-run production theory is depicted with isoquants.
  - a. The slope of the isoquant is the rate at which the inputs can be traded in production while output remains constant. It is also the ratio of the marginal products of the two inputs.
  - b. Perfect substitutes in production have straight-line isoquants and perfect compliments have L-shaped isoquants.
  - c. Long-run production can have increasing, decreasing, or constant returns to scale.
5. The appendix discusses several mathematical extensions and applications of production theory.
  - a. When playing tennis, the optimal percentage of lobs is a point where the lob has a much higher chance of succeeding than a passing shot, but its marginal gain is no greater.
  - b. The production mountain is the construct from which isoquants come.
  - c. The Cobb-Douglas production function is the most commonly used function that exhibits the standard production relationships.
  - d. A Leontief production function illustrates fixed proportions in production.

## Important Terms

production function	effect of technology on production function
intermediate products	average product
value added	marginal product
short run	interior solution
long run	isoquant map
variable input	marginal rate of technical substitution
fixed input	increasing returns to scale
law of diminishing returns	constant returns to scale
total product	decreasing returns to scale

### A Case to Consider

- Megan plans to set up a small computer assembly operation in a village of a poor country that needs jobs. She will have virtually no capital. She has figured that her production function is specified as follows:  $Q = 9L^2 - L^3$  where  $Q$  is equal to the quantity produced, and  $L$  is equal to the number of workers employed. Sketch a total product curve for Megan and then add the average and marginal product curves to your graph. Assume that 7 laborers would be the maximum possible.

L	Q	MP	AP
1			
2			
3			
4			
5			
6			
7			

TP = Q

- When does diminishing returns of labor begin? (Answer this by identifying the point where the marginal product reaches its peak. This can be found by using calculus to find the maximum point on the marginal product curve. Calculus will give a precise answer while the numbers in the chart will give only an approximation because labor moves in full integer increments in the chart rather than in infinitely small changes.)
- If Megan could have all the volunteer labor she wanted, how many would she use if she is a profit maximizer? (No part-time work is possible.) Assume she can sell all the computers she makes without having to lower the price.
- If the price of hiring the input labor was 15 per person, how many laborers should she hire? If she wanted to produce where the average product was at its peak, how many laborers should she hire? (No part-time labor possible.)

### Multiple-Choice Questions

- Which of the following is not considered production activity?
  - Your teacher teaching this class
  - The Chicago Bulls playing a basketball game
  - A car salesman selling a used car
  - A boxer knocking out an opponent
  - All the above are considered production activity.

2. Which is true about the output values of a production function?
  - a. They are conceptually flawed and of marginal use because they do not consider raw materials as an input.
  - b. They represent only the value added of the two inputs that are represented.
  - c. They overlook the role of fixed inputs into production.
  - d. They overlook the role of variable inputs into production.
3. For a given short-run production function,
  - a. technology is assumed to change as capital stock changes.
  - b. technology is assumed to change as the labor input changes.
  - c. technology is considered to be constant for a given production function relationship.
  - d. technology is assumed to change positively until diminishing returns set in and then it changes in the other direction.
4. The production function  $Q = 3KL$  with capital fixed at 2 will be \_\_\_\_\_ and have a slope that is \_\_\_\_\_ the slope of the same function with a fixed capital stock of 1.
  - a. linear, the same as
  - b. linear, twice
  - c. linear, three times
  - d. non-linear, one-third
5. At the point of diminishing returns
  - a. an additional unit of labor will lower the total output of the group.
  - b. the marginal product of labor is equal to the average product of labor.
  - c. the first derivative of the total function is 0.
  - d. the second derivative of the total function is 0.
6. If a ray from the origin is tangent to a typical short-run production function, then
  - a. the marginal product of the variable input is equal to the average product of that input.
  - b. the point of diminishing returns has not yet been reached.
  - c. the marginal product of the variable input is less than the average product of that input.
  - d. the average product of the variable input is at its minimum.
7. If the owner of a service station told a mechanic looking for work that he would not hire another mechanic if the mechanic offered to work for nothing, we can assume that
  - a. the average product of mechanics is 0.
  - b. the average product of mechanics is rising.
  - c. the average product of mechanics is negative.
  - d. the average product of mechanics is falling.
8. If a given input is used in two production processes that exhibit diminishing returns, and its present distribution between the two processes results in a higher marginal product in the first process, then we know that
  - a. the second process should be abandoned completely.
  - b. more of the input should be transferred from the first process to the second process.
  - c. more of the input should be transferred from the second process to the first process.
  - d. without more information, no changes should be made in allocating the input.
9. If, in question 8, the characteristic of diminishing marginal productivity is dropped from the example, then

- a. the second process should be abandoned completely.
  - b. more of the input should be transferred from the first process to the second process.
  - c. some, but not all, of the input should be transferred from the second process to the first process.
  - d. without more information, no changes should be made in allocating the input.
10. Isoquants are concave from above because
- a. of diminishing returns.
  - b. more of both inputs increases output.
  - c. inputs are complements rather than substitutes.
  - d. of all the above.
11. The marginal rate of technical substitution can be measured by
- a. the slope of an isoquant.
  - b. the ratio of the marginal products of the two inputs.
  - c. the amount of capital that must be substituted for a given reduction of labor in a production process in order to keep output from falling.
  - d. all the above.
  - e. none of the above.
12. If you are called into a firm as a consultant and want some production information, which of the following would the firm have the easiest time generating?
- a. The marginal product of capital
  - b. The marginal product of labor
  - c. The average product of labor
  - d. All the above are easy to generate from data taken at one point in time.
13. In a production function with labor and capital where the marginal product of each factor is 0, at the same time it is true that
- a. the two factors are complements in production.
  - b. the two factors are substitutes in production.
  - c. each factor of production detracts from the other in the production process.
  - d. none of the above statements are accurate because both marginal products cannot be 0 at once.
14. Which is a true statement?
- a. Decreasing returns to scale and diminishing returns to production are two ways of stating the same thing.
  - b. Increasing returns to scale is a short-run concept, and diminishing returns to production is a long-run concept.
  - c. Constant returns to scale is a short-run concept, and decreasing returns to scale is a long-run concept.
  - d. All the above are true.
  - e. None of the above are true.
15. Isoquants that are spaced equidistant from each other will
- a. depict constant returns to scale across the entire production surface.
  - b. not show any scale relationships unless there are output numbers put on the isoquants.

- c. show decreasing returns to scale, since equidistant means that they are closer to each other in proportion to the entire distance from the origin as they move out to the northeast.
  - d. be inaccurately drawn.
16. (Appendix) Isoquants come from
- a. vertical slices of the production mountain.
  - b. horizontal slices of the production mountain.
  - c. a slice of the mountain parallel to the labor axis.
  - d. a slice of the mountain parallel to the capital axis.
17. (Appendix) Your text states that people tend to overuse a lob shot in tennis games. Why is this true?
- a. People believe that good lob shots always have a better chance of success than a good passing shot.
  - b. People fail to realize that the marginal gain of one more passing shot is higher than the average gain of the shot.
  - c. There are no diminishing returns for lob shots.
  - d. People enjoy hitting lob shots more than winning.
18. (Appendix) The optimal percentage of lob shots was found by
- a. equating the percentage won of both lobs and passing shots.
  - b. setting the first derivative of the  $P = 30 + 70L - 90L^2$  function equal to 0 and then solving for L.
  - c. equating the slopes of the F(L) and G(L) functions.
  - d. vertically adding the F(L) and G(L) functions and taking the maximum.



4. A daily production function for yo-yo's is  $Q = 12L^{1/2} + 8K^{1/2}$ . Show all your work for the following questions. (Use calculus for this problem where relevant.)
- What are the marginal product equations for each input? (If you have not had calculus get this answer from the answer section and then proceed with the problem.)
  - How many yo-yo's are produced when labor is 9 and capital is 16?
  - What is the slope of the isoquant at the point of production in answer b above? (Hint: The slope of the isoquant is related to the ratio of the marginal products.)
  - If the inputs are quadrupled, are there economies or diseconomies of scale?
  - If the production function in (a) is a short-run function with capital fixed at 16, were there diminishing returns to labor when the 9th worker was hired?
  - Describe the difference between diminishing returns and decreasing returns to scale.
5. (Appendix) A football team averages 3.6 yards a carry and 10 yards per pass. The passing effectiveness depends on how much the defense is looking for the run which

depends on what percentage of the plays are runs. The equation for successful passes as a function of percentage runs is  $Y = 2 + 10R$ . Likewise, runs are more effective when the team is looking for passes so the equation for successful runs as a function of percentage runs is  $Y = .10 - 8R$ . The coach calls running plays 80% of the time despite the complaints of the fans who believe the team would gain more yardage if they used the pass more. Are the fans or the coach right? Show the calculations that prove your answer is right.

- a. What is the average yardage per pass and run at the optimal number of runs?
- b. What do you know without any calculations about the marginal gain per pass and run when the optimal number of running plays is called?
- c. What does this story tell us about the production of yards in a football game or the production of anything? (Answer this in terms of marginal and average product.)

## **ANSWERS TO QUESTIONS FOR CHAPTER 9**

### Case Questions

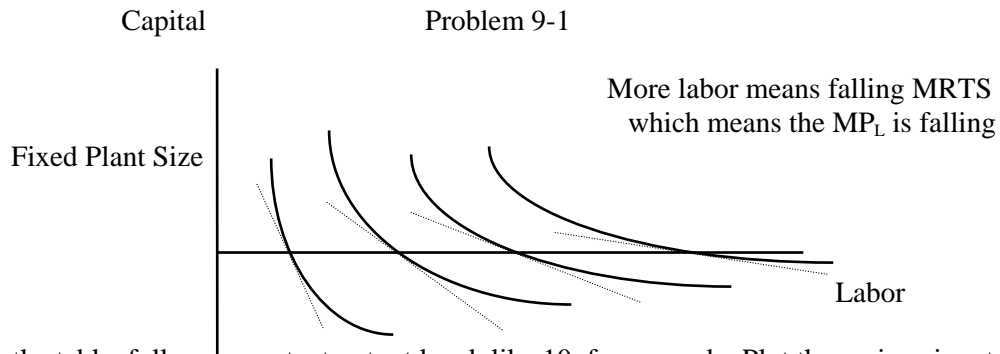
1. Insert 1-7 laborers into the production function to get the total quantity produced for each level of labor. The sequential output will be 8, 28, 54, 80, 100, 108, 98. From this get the  $AP_L$  and the  $MP_L$ . Your graph should look similar to Figure 9-6 in your text.
2. When the first derivative of the marginal product function equals zero, the marginal product function will be at its peak and diminishing returns will begin. This happens at  $L = 3$ . The answer is not precise when read from the chart because both 3 and 4 units of labor show a 26 increase in output. This is because the chart is measuring the increase in output over the entire range of 2 to 3 laborers and 3 to 4 laborers. Calculus measures the change in total product at exactly an output of 3.
3. When total product is maximized the marginal product is zero which is the number Megan should employ if labor is free. This occurs at  $L=6$ . In the table 108 is the maximum. Using calculus, if the marginal product (the first derivative of the total function) is set equal to zero and solved for  $L$  the answer is 6. ( $18L - 3L^2 = 0$  so  $3L(6 - L) = 0$  and  $L = 6$ .)
4. Get the  $AP_L$  and maximize it. The labor amount should be  $L = 4.5$ . If one observes the chart it is evident that average product reaches a peak between the 20 of 4 workers and the 20 of 5 workers. Without calculus one can only estimate the portion of the part time worker that would be hired.

### Multiple-Choice Questions

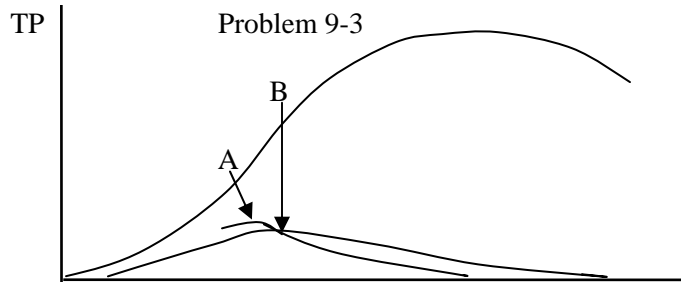
1. e, Any good or service that brings someone utility means productivity has occurred.
2. b, This does not mean that raw materials are irrelevant, but they came from another production process.
3. c, In the short run it is hard to have substantial changes in technology.
4. b, In one case the slope is 3 and in the other it is 6.
5. d, The second derivative of the total product is the first derivative of marginal product.
6. a, If the ray is tangent to the production function, then it is a marginal and average line.
7. d, If the MP is less than zero the average must be falling.
8. c, Always use inputs so that their marginal product is equal in all processes.
9. a, Obviously, the first process will always be better than the second, so use it always.
10. a, Input a can be replaced by a small amount of input b when a is heavily used.
11. d, Check the proofs of these in your text.
12. c, Total product divided by the number of workers is readily available.
13. a, They must be perfect complements and production must be occurring at the bend in the L-shaped isoquant.
14. e, Diminishing returns is a short-run concept and returns to scale is a long-run notion.
15. b, Sequential isoquants have higher output values, but the rate of increase will not be known until the actual values are attached to the isoquants.
16. b, It is helpful to see the isoquants as ridge lines on a mountain of production.
17. b, The interdependence of the events can not be ignored.
18. b, This is the typical maximizing process used before in this text.

### Problems

1. See the sketch labeled Problem 9-1 below.



2. On the table, follow a constant output level, like 10, for example. Plot the various input combinations for each output of 10 shown in the chart. Do this for 5 output levels and your sketch will resemble an isoquant map.
2. a) No because as labor is increased the output does not go up at a decreasing rate.
2. b) There are increasing returns to scale because when all inputs are increased by a given percentage the output goes up more than proportionately.
3. See sketch labeled Problem 9-3 below.



4. a)  $\Delta Q/\Delta L = 6/L^{-1/2}$  and  $\Delta Q/\Delta K = 4/K^{-1/2}$  These come from the derivatives of the total product function with respect to each of the inputs.
  4. b) 68
  4. c) The MRTS, which is the slope of the isoquant, which is also the ratio of the marginal products of the inputs, is 2.
  4. d) Diseconomies of scale, since output does not quadruple.
  4. e) Yes, from the marginal product equations above it is clear that marginal product is declining over the entire range of labor usage.
  4. f) The two are very different. Diminishing returns relates to the effect of the variable input being added to all the fixed factors. Sooner or later the variable input congests the operation and marginal productivity falls. Diseconomies of scale are not as obvious theoretically because they relate to costs of size that happen as all inputs increase together.
5. (Appendix) To find the optimal number of running plays, the production function of yardage must be analyzed. The inputs are running plays and passing plays and the success of each depends on how many runs are attempted. The success of passing plays as a function of running plays is  $Y = 2 + 8R$  where  $Y$  = yards gained and  $R$  = the percent of running plays.

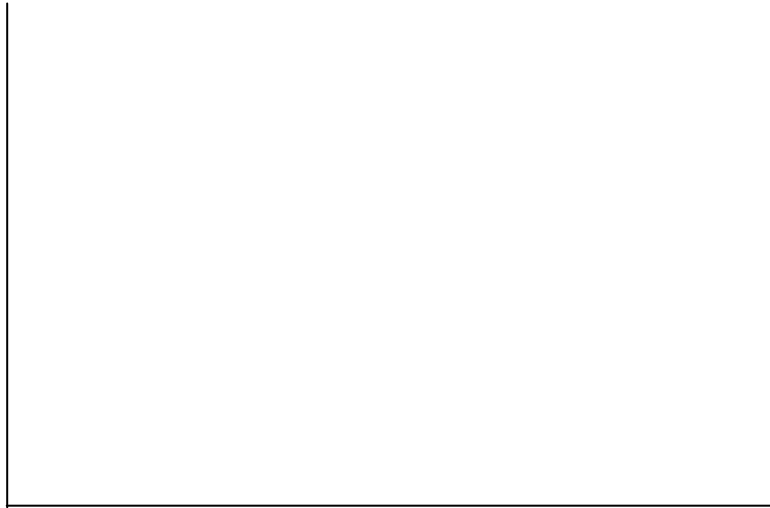
The success of running plays as a function of running plays is  $Y = 10 - 8R$ . If we weight the running plays by the proportion of runs and the passing plays by the proportion of passes we have  $Y = (10 - 8R)(R)$  for runs and  $Y = (1-R)(2 + 10R)$ . To find the total yardage gained as a function of  $R$  the two equations in the line above need to be added and simplified. The result is  $Y = 18R - 18R^2 + 2$ . This shows how the yardage is affected by the percentage of runs. To find the best yardage output this function is maximized by setting its first derivative with respect to  $Y$  equal to zero. The result is that  $Y$  is maximized at  $R = 50$  so the coach should run half the time rather than only 20% of the time.

5. a) By substituting 50% into the input equations the result is that passing yards average 7 and running yards average 6.
5. b) They will be equal.
5. c) In production the marginal return from each input should be the same. The average returns of factors are irrelevant to finding the right mix of inputs.

NOTES:

**HOMEWORK ASSIGNMENT****NAME:** \_\_\_\_\_

1. Draw a set of short run production curves including the total, average and marginal product curves. Identify on the graph the range of labor input that would be most likely relevant if you faced varying wage levels from zero up until you would cease to hire anyone. Identify the point of diminishing returns on your graph.

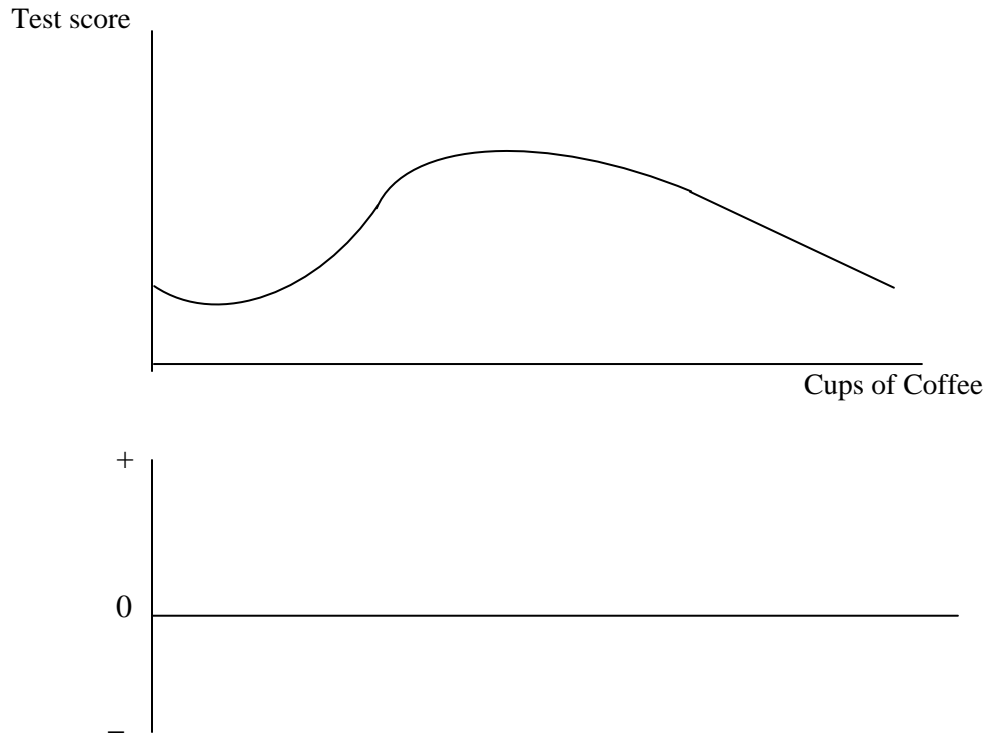


2. Sketch a set of long run isoquants and label them so there are increasing and constant and decreasing returns to production.



3. In order to develop graphing skills relating to total, average and marginal values, take the following somewhat unrealistic sketch of the points you receive on your next test.

The points are related to the number of cups of coffee you drink as you study. Sketch the approximate marginal and average functions on the graph below. Since no numerical values are specified you need only concentrate on the key points where average and marginal values are equal, where they are zero, and where they change directions. For questions 4 and 5 assume that the coffee is free from your roommate.



4. True or False: You would want to drink the amount of coffee that would make the average points gained per cup the greatest. Explain if your answer is true. Go to question five if your answer is false.
  
5. At what point on the graph above would you want to stop drinking coffee? Explain.