Chapter 13
Oligopoly

Solutions to Problems

8. A homogeneous products duopoly faces a market demand function given by *P* = 300 - 3*Q*, where *Q* = *Q*1 + *Q*2. Both firms have a constant marginal cost *MC* = 100.
a) What is Firm 1's profit-maximizing quantity, given that Firm 2 produces an output of 50 units per year? What is Firm 1's profit-maximizing quantity when Firm 2 produces 20 units per year?
b) Derive the equation of each firm's reaction curve and then graph these curves.
c) What is the Cournot equilibrium quantity per firm and price in this market?
d) What would the equilibrium price in this market be if it were perfectly competitive?
e) What would the equilibrium price in this market be if the two firms colluded to set the monopoly price?
f) What is the Bertrand equilibrium price in this market?
g) What are the Cournot equilibrium quantities and industry price when one firm has a marginal cost of 100 but the other firm has a marginal cost of 90?
h) Assuming that Firm 1 is the Stackelberg leader, find the Stackelberg equilibrium quantities for each firm and the market price.

a) With two firms, demand is given by $P = 300 - 3Q_1 - 3Q_2$. If $Q_2 = 50$, then $P = 300 - 3Q_1 - 150$ or $P = 150 - 3Q_1$. Setting $MR = MC$ implies

$$150 - 6Q_1 = 100$$
$$Q_1 = 8.33$$

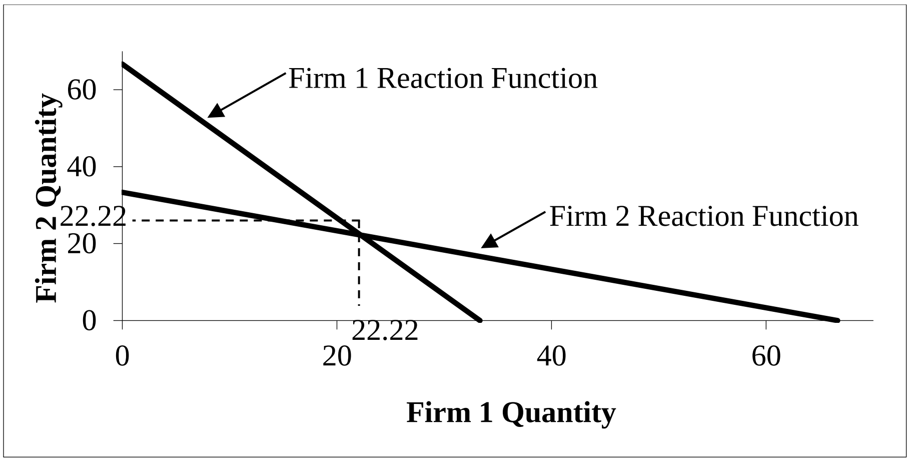
If $Q_2 = 20$, then $P = 240 - 3Q_1$. Setting $MR = MC$ implies

$$240 - 6Q_1 = 100$$
$$Q_1 = 23.33$$

b) For Firm 1, $P = (300 - 3Q_2) - 3Q_1$. Setting $MR = MC$ implies

$$(300 - 3Q_2) - 6Q_1 = 100$$
$$Q_1 = 33.33 - 0.5Q_2$$

Since the marginal costs are the same for both firms, symmetry implies $Q_2 = 33.33 - 0.5Q_1$. Graphically, these reaction functions appear as



c) Because of symmetry, in equilibrium both firms will choose the same level of output. Thus, we can set $Q_1 = Q_2$ and solve

$$Q_2 = 33.33 - 0.5Q_2$$
$$Q_2 = 22.22$$

Since both firms will choose the same level of output, both firms will produce 22.22 units. Price can be found by substituting the quantity for each firm into market demand. This implies price will be $P = 300 - 3(44.44) = 166.67$.

d) If this market were perfectly competitive, then equilibrium would occur at the point where $P = MC = 100$.

e) If the firms colluded to set the monopoly price, then

$$300 - 6Q = 100$$
$$Q = 33.33$$

At this quantity, market price will be $P = 300 - 3(^{200}\!/_{6}) = 200$.

f) If the firms acted as Bertrand oligopolists, the equilibrium would coincide with the perfectly competitive equilibrium of $P = 100$.

g) Suppose Firm 1 has $MC = 100$ and Firm 2 has $MC = 90$. For Firm 1, $P = (300 - 3Q_2) - 3Q_1$. Setting $MR = MC$ implies

$$(300 - 3Q_2) - 6Q_1 = 100$$
$$Q_1 = 33.33 - 0.5Q_2$$

For Firm 2, $P = (300 - 3Q_1) - 3Q_2$. Setting $MR = MC$ implies

$$(300 - 3Q_1) - 6Q_2 = 90$$
$$Q_2 = 35 - 0.5Q_1$$

Solving these two reaction functions simultaneously yields $Q_1 = 21.11$ and $Q_2 = 24.44$. With these quantities, market price will be $P = 163.36$.

h) Suppose Firm 1 is the leader. Firm 2 will maximize its profit by setting its quantity according to the best response: $Q_2 = 33.33 - 0.5Q_1$. Firm 1 knows this, so it maximizes its profit by setting MC = MR while taking into account that $Q_2 = 33.33 - 0.5Q_1$.

$TR_1 = PQ_1 = (300 - 3Q_1 - 100 + 1.5Q_1)Q_1 = (200 - 1.5Q_1)Q_1$
$MR_1 = 200 - 3Q_1$
$MR_1 = MC: \quad 200 - 3Q_1 = 100$
$Q_1 = 33.33$
$Q_2 = 33.33 - 0.5(33.33) = 0.5(33.33)$

18. Suppose that the industry consists of a dominant firm, Braeutigam Cobalt (BC), which has a constant marginal cost equal to $40 per unit. The market demand for cobalt is given by Q = 200 – P. There are nine other fringe producers, each of whom has a marginal cost curve *MC* = 40 + 10*q*, where *q* is the output of a typical fringe producer. Assume there are no fixed costs for any producer.
a) What is the supply curve of the competitive fringe?
b) What is BC's residual demand curve?
c) Find BC's profit-maximizing output and price. At this price, what is BC's market share?

a) The supply curve of the competitive fringe is the horizontal summation of the marginal cost curves (supply curves) for the individual firms. Since $MC = 40 + 10q$ for each firm, $q = 0.1P - 4$ is the supply curve for an individual firm, so long as $P > 40$. For $P \leq 40$, fringe supply is zero. Summing these for the 9 fringe producers in this market implies

$$S_F = \begin{cases} 0 & P \leq 40 \\ 0.9P - 36 & P > 40 \end{cases}$$

b) For $P \leq 40$, fringe supply is zero so residual demand is equal to market demand. For $P > 40$, residual demand is the horizontal difference between the fringe supply and market demand. Thus, residual demand is

$$Q_R = \begin{cases} 200 - P & P \leq 40 \\ 236 - 1.9P & P > 40 \end{cases}$$

c) For $P > 40$, the inverse residual demand curve is $P = (10/19)(236 - Q_R)$, so the associated marginal revenue curve is $MR = (10/19)(236 - 2Q_R)$. BC maximizes profit by equating MR to its $MC = 40$:

$$(10/19)(236 - 2Q_R) = 40$$
$$Q_R = 80$$

Using the residual demand curve, BC's profit-maximizing price is $P = (10/19)(236 - 80) = 82.11$ and total market demand is $Q = 200 - 82.11 = 117.89$. Thus, BC's market share is $80/117.89 = 0.68$, or 68 percent.

24. RyanAir and EasyJet both fly between London and Marrakech. . Their demand curves for EasyJet and RyanAir are given, respectively, by $Q_E = 500 - 2P_E + P_R$ and $Q_R = 500 - 2P_R + P_E$. Q_E and Q_R stand for the number of passengers per day for EasyJet and RyanAir, respectively. The marginal cost of each carrier is £10 per passenger.
a) If EasyJet sets a price of £100, what is the equation of RyanAir's demand curve and marginal revenue curve? What is RyanAir's profit-maximizing price when Easyjet sets a price of £100?
b) Derive the equations for EasyJet's and RyanAir's price reaction curves.
c) What is the Bertrand equilibrium in this market?

a) If EasyJet sets a price of $100, we can plug this price into RyanAir's demand curve to get RyanAir's perceived demand curve.

$Q_R = 500 - 2P_R + 100$
$P_R = 300 - 0.5Q_R$

To find RyanAir's profit-maximizing price set $MR = MC$.

$MR_R = 300 - Q_R$
$300 - Q_R = 10$
$Q_R = 290$
$P_R = 155$

b) We need to find the best response functions for both players.

For EasyJet, set $MR_E = MC_E = 10$

$Q_E = 500 - 2P_E + P_R$
$P_E = 250 - 0.5Q_E + 0.5P_R$

$TR_E = P_EQ_E = (250 - 0.5Q_E + 0.5P_R)Q_E$.
$MR_E = 250 - Q_E + 0.5P_R$

$MR_E = MC_E = 10$
$250 - Q_E + 0.5P_R = 10$
$Q_E = 240 + 0.5P_R$

$Q_E = 500 - 2P_E + P_R$
$500 - 2P_E + P_R = 240 + 0.5P_R$
$260 + 0.5P_R = 2P_E$

BR_E: $P_E = 130 + 0.25P_R$

Since the two firms have face a similar demand and MC, we can conclude that
BR_R: $P_R = 130 + 0.25P_E$

As an alternative, you can find BR by setting the derivative of $Profit_{Easy}$ w.r.t. P_{Easy} equal to zero. We can use this ONLY when AC = MC.

$Profit_E = TR_E – TC_E = P_EQ_E – AC_EQ_E = (P_E – AC_E)Q_E$
$Profit_E = (P_E – 10)(500 – 2P_E + P_R)$

$dProfit/dP_E = 0$ >>> $(P_E – 10)(-2) + (500 – 2P_E + P_R)(1) = 0$

$500 – 2P_E + P_R = 2P_E – 20$
$520 + P_R = 4P_E$
$P_E = 130 + 0.25P_R$ which is the same as the MR = MC approach above.

c) The Bertrand equilibrium will occur where these price reaction functions intersect. Substituting the expression for P_R into the expression for P_E implies

$$P_E = 130 + 0.25(130 + 0.25P_E)$$
$$P_E = P_R = 185.71$$

Chapter 10
Monopsony

Solutions to Problems

26. A coal mine operates with a production function *Q* = *L*/2, where *L* is the quantity of labor it employs and *Q* is total output. The firm is a price taker in the output market, where the price is currently 32. The firm is a monopsonist in the labor market, where the supply curve for labor is *w* = 4*L*.
a) What is the monopsonist's marginal expenditure function, ME_L?
b) Calculate the monopsonist's optimal quantity of labor. What wage rate must the monopsonist pay to attract this quantity of labor?
c) What would be the w* and L* if the factor market were competitive?

a) For this monopsonist

$$ME_L = w + L\frac{\Delta w}{\Delta L}$$
$$ME_L = 4L + L(4)$$
$$ME_L = 8L$$

b) The monopsonist will maximize profit at the point where $MRP_L = ME_L$, where

$$MRP_L = P\frac{\Delta Q}{\Delta L}$$

In this example, $\Delta Q/\Delta L = 0.5$, so $MRP_L = 0.5P$. Since $P = 32$, $MRP_L = 16$. Now setting $MRP_L = ME_L$ implies

$$16 = 8L$$
$$L = 2$$

At this quantity of labor, $w = 4L = 8$.

c) In a competitive labor market, $w = MRP_L$. So the competitive supply of labor satisfies $4L = 16$ or $L = 4$, with $w = 4L = 16$.