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EE325 Introductory Econometrics, Semester 1/2019 (Section 046402)

Due Date: Thursday 27th February 2020 by 09.30 via Assignment Submission in Moodle.

Instruction: Do all questions with your own handwriting and your own attempt.

Use 4 decimal places for numerical answers

1. In Table 1. X_i is total econometrics exam point (total points are 100) and Y_i is GPA of each BE student.

Table 1

Student	Y_i	X_i
1	2.8	63
2	3.4	72
3	3.0	78
4	3.5	81
5	3.6	87
6	3.0	75
7	2.7	75
8	3.7	90

1.1 Now consider the two-variable model $Y_i = \beta_1 + \beta_2 X_i + u_i,\ u_i \sim NIID(0, \sigma^2)$
Use OLS to find the estimator of $\beta_1 and \beta_2$. Interpret the regression.

1.2 Find $\hat{Y}_i \quad and\ \hat{u}_i$ and show that $\sum_{i=1}^{n} \hat{u}_i \approx 0$

1.3 Find $var(\hat{u}_i), var(\hat{\beta}_1), and\ var(\hat{\beta}_2)$

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2. Data is listed in the table

X_i	Y_i
10	0
12	2
14	5
16	6
18	7
22	10
24	10
26	15
28	16
30	20

2.1 From the simple regression model $Y_i = \beta_1 + \beta_2 X_i + u_i, u_i \sim NIID(0, \sigma^2)$

Find estimators of β_1 and β_2 from the OLS method and interpret the meaning.

2.2 Find the value of $\hat{Y}_i$ and $\hat{u}_i$. Show that $\sum \hat{u}_i \approx 0$

2.3 Plot graph and draw regression line. Does the line pass $(\bar{X}, \bar{Y})$?

2.4 If $X_i = 18$, what is the predicted Y?

2.5 Find $var(\hat{u}_i), var(\hat{\beta}_1), var(\hat{\beta}_2)$

3. Consider the below regression function: consider the two-variable model

$\boldsymbol{Y_i = \beta_1 + \beta_2 X_i + u_i,\ u_i \sim NIID(0, \sigma^2)}$

Find an OLS estimator of β_1. Then, provide a proof that this is an unbiased estimator. Please state the assumption(s) of CLRM when used (pages 66-75 in Gujarati).

"Practice makes Perfect."

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Student	Y_i	X_i	$(x_i - \bar{x})$	$(y_i - \bar{y})$	$x_i y_i$
1	2.8	63	-14.625	-0.4125	6.0328125
2	3.4	72	-5.625	0.1875	-1.0546875
3	3.0	78	0.375	-0.2125	-0.0796875
4	3.5	81	3.375	0.2875	0.9703125
5	3.6	87	9.375	0.3875	3.6328125
6	3.0	75	-2.625	-0.2125	0.5578125
7	2.7	75	-2.625	-0.5125	1.3453125
8	3.7	90	12.375	0.4875	6.0328125
Σ	25.7	621			Σ = 17.4375

$$\boxed{\begin{aligned}\bar{y} &= 3.2125\\ \bar{x} &= 77.625\\ \Sigma x_i^2 &= 511.875\end{aligned}}$$

1.1 $\hat{\beta}_1 = \bar{y} - \hat{\beta}_2 \bar{x}$

$$\hat{\beta}_1 = 3.2125 - 0.0340659\,(77.625)$$
$$= 0.568132$$

$$\hat{y}_i = 0.568132 + 0.0340659\,(x_i)$$

$$\hat{\beta}_2 = \frac{\Sigma (x_i - \bar{x})(y_i - \bar{y})}{\Sigma (x_i - \bar{x})^2}$$
$$= \frac{17.4375}{511.875} = 0.0340659$$

with the exam score increases by 1 unit, GPA will increase by 0.0340659 units. With no score on exam, student will get GPA equal to 0.568132

1.2

$\hat{y}_1 = 2.714284$, $\hat{u}_1 = 0.085716$ $\quad$ $\hat{y}_2 = 3.020877$, $\hat{u}_2 = 0.379123$ $\quad$ $\hat{y}_3 = 3.225452$, $\hat{u}_3 = -0.225452$ $\quad$ $\hat{y}_4 = 3.32765$, $\hat{u}_4 = 0.17235$ $\quad$ $\hat{y}_5 = 3.532045$, $\hat{u}_5 = 0.067955$ $\quad$ $\hat{y}_6 = 3.123255$, $\hat{u}_6 = -0.123255$

$\hat{y}_7 = 3.123255$, $\hat{u}_7 = -0.423255$ $\quad$ $\hat{y}_8 = 3.634243$, $\hat{u}_8 = 0.065757$ $\quad$ $\Sigma \hat{u}_i = 0.001061$

1.3 $$\mathrm{Var}(\hat{u}_i) = \frac{\Sigma u_i^2}{n-2} = \frac{0.4348931}{6} = 0.0724822 = \sigma^2$$

$$\mathrm{Var}(\hat{\beta}_1) = \frac{\Sigma X_i^2}{n \Sigma x_i^2}\sigma^2 = \frac{48717}{8(511.875)}(0.0724822) = 0.862299$$

$$\mathrm{Var}(\hat{\beta}_2) = \frac{\sigma^2}{\Sigma x_i^2} = \frac{0.0724822}{511.875} = 0.0001416$$

2.1 $\hat{\beta}_1 = \bar{y} - \hat{\beta}_2 \bar{x}$
$$= 9.1 - (17.909091)$$
$$= -8.809091$$

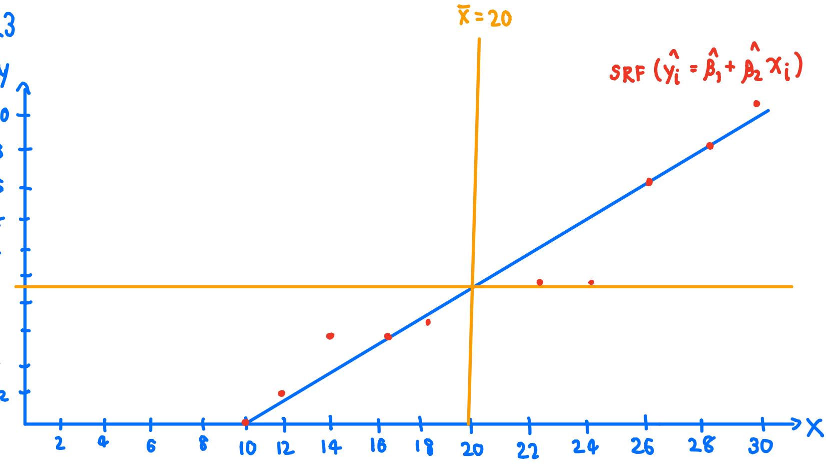
$$\hat{\beta}_2 = \frac{\Sigma x_i y_i}{\Sigma x_i^2}$$
$$= \frac{394}{440} = 0.895455$$

$$\hat{y}_i = -8.8091 + 0.8955\,(x_i)$$

2.2

$\hat{y}_1 = 0.1459$	$\hat{u}_1 = -0.1459$
$\hat{y}_2 = 1.9369$	$\hat{u}_2 = 0.0631$
$\hat{y}_3 = 3.7279$	$\hat{u}_3 = 1.2721$
$\hat{y}_4 = 5.5189$	$\hat{u}_4 = 0.4811$
$\hat{y}_5 = 7.3099$	$\hat{u}_5 = -0.3099$
$\hat{y}_6 = 10.8919$	$\hat{u}_6 = -0.8919$
$\hat{y}_7 = 12.6829$	$\hat{u}_7 = -2.6829$
$\hat{y}_8 = 14.4739$	$\hat{u}_8 = 0.5261$
$\hat{y}_9 = 16.2649$	$\hat{u}_9 = -0.2649$
$\hat{y}_{10} = 18.0559$	$\hat{u}_{10} = 1.9441$
	$\Sigma\hat{u}_i = -0.009 \approx 0$

2.3



$$\hat{y}_i = -8.8091 + 0.8955(20) = 9.1009 \approx 9.1 = \bar{y}$$

2.4 $x_i = 18$, $\hat{y}_i = 7.3099$ from 2.2

2.5

$$Var(\hat{u}_i) = \sigma^2 = \frac{\Sigma\hat{u}_i^2}{n-2} = \frac{14.09092}{8} = 1.761365$$

$$Var(\hat{\beta}_2) = \frac{\sigma^2}{\Sigma x_i^2} = \frac{1.761365}{440} = 0.004$$

$$Var(\hat{\beta}_1) = \frac{\Sigma X_i^2}{n} \cdot \frac{\sigma^2}{\Sigma x_i^2} = \frac{444}{10}(0.004) = 1.776$$

3. As $\hat{\beta}_1$ is an estimator of β_1, $E(\hat{\beta}_1)$ should be equal to β_1

$$\hat{\beta}_1 = \bar{y} - \hat{\beta}_2\bar{X}, \quad \hat{\beta}_2 = \frac{\Sigma x_i y_i}{\Sigma x_i^2} \qquad \text{let } k = \frac{x_i}{\Sigma x_i^2}$$

$$\hat{\beta}_1 = \bar{y} - \Sigma k y_i \bar{X} \quad \text{and} \quad y_i = \beta_1 + \beta_2 X_i + u_i$$

$$\hat{\beta}_1 = \frac{\Sigma y_i}{n} - \Sigma k Y_i \bar{X} = \Sigma\left(\frac{1}{n} - k\bar{X}\right) y_i$$

$$= \Sigma\left(\frac{1}{n} - k\bar{X}\right)(\beta_1 + \beta_2 X_i + u_i)$$

$$= \Sigma\left(\frac{\beta_1}{n} + \frac{\beta_2 X_i}{n} + \frac{u_i}{n} - k\bar{X}\beta_1 - k\bar{X}\beta_2 X_i - k\bar{X}u_i\right)$$

$$= \Sigma\frac{\beta_1}{n} + \beta_2\bar{X} + \frac{\Sigma u_i}{n} - \beta_2\bar{X} - \bar{X}\Sigma k_i u_i$$

take $E(\cdot) \rightarrow E(\hat{\beta}_1) = E(\beta_1) + \bar{X}E(\Sigma k_i u_i) \rightarrow$ assumption 3 : zero mean value of disturbance
treat X as given
$E(u_i | x_i)$

$$E(\hat{\beta}_1) = E(\beta_1) + \bar{X}\Sigma k_i \underbrace{\cancel{E(u_i)}}_{0}$$

$$E(\hat{\beta}_1) = \beta_1$$

$\hat{\beta}_1$ is an unbiased estimator of actual β_1

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