EE320 Introductory Mathematical Economics

Quiz 4

1. *(10 points total - 5 points each)*

   a. Find the *total differential* of $y = \underbrace{(2x_1 + 1)}_{u}\underbrace{(x_2^2 - 2)}_{v}\underbrace{(3x_3 - 1)}_{w}$.

Let $u = 2x_1 + 1$, $v = 2x_2^2 - 2$, $w = 3x_3 - 1$.

$\Rightarrow du = 2dx_1$, $dv = 4x_2 dx_2$, $dw = 3dx_3$

$$dy = d(u \cdot v \cdot w) = uv \cdot dw + uw \cdot dv + vw \cdot du$$

$$= (2x_1+1)(2x_2^2-2)(3dx_3) + (2x_1+1)(3x_3-1)(4x_2dx_2)$$
$$+ (2x_2^2-2)(3x_3-1)(2dx_1)$$

$$dy = 3(4x_1x_2^2 - 4x_1 + 2x_2^2 - 2)dx_3 + 4(6x_1x_3 - 2x_1 + 3x_3 - 1)x_2 \cdot dx_2$$
$$+ 2(6x_2^2x_3 - 2x_2^2 - 6x_3 + 2)dx_1$$

   b. Find the *total derivative of y w.r.t. w* when

$$y = f(x, w) = 4w - x^2, \text{ where } x = g(w) = \ln(2w).$$

$$y = f[g(w), w] = 4w - [g(w)]^2 = 4w - [\ln(2w)]^2$$

$$\frac{dy}{dw} = 4 - 2g(w) \cdot g'(w)$$

$$= 4 - 2\ln(2w) \cdot \frac{1}{w}$$

$$\therefore \frac{dy}{dw} = 4 - \frac{2}{w} \cdot \ln(2w)$$

Note.
$g(w) = \ln(2w)$
$g'(w) = \frac{dg}{dw} = \frac{1(2)}{2w} = \frac{1}{w}$

2. *(10 points total - 5 points each)*

a) Given the utility function $U(x, y) = ax^2 + by^2$, where $(a, b > 0)$, find the marginal rate of substitution (MRS) between x and y.

$$MRS_{xy} = \frac{MU_x}{MU_y} = \frac{U_x}{U_y} = \frac{2ax}{2by} = \frac{a}{b} \cdot \frac{x}{y}.$$

b) Suppose that in a macroeconomic model for a closed economy with proportional income tax, the equilibrium national income can be written in a reduced form as

$$Y^* = \frac{C_0 + I_0 + G_0}{1 - b(1-t)},$$

where C_0 = autonomous consumption, I_0 = investment, G_0 = government expenditure; b = marginal propensity to consume $(0 < b < 1)$, and t = income tax rate $(0 < t < 1)$. Determine the comparative-static derivatives $\frac{\partial Y^*}{\partial G_0}$ and $\frac{\partial Y^*}{\partial t}$.

$$\frac{\partial Y^*}{\partial G_0} = \frac{1}{1 - b(1-t)}$$

$$\frac{\partial Y^*}{\partial t} = \frac{(C_0 + I_0 + G_0)b}{[1 - b(1-t)]^2}.$$