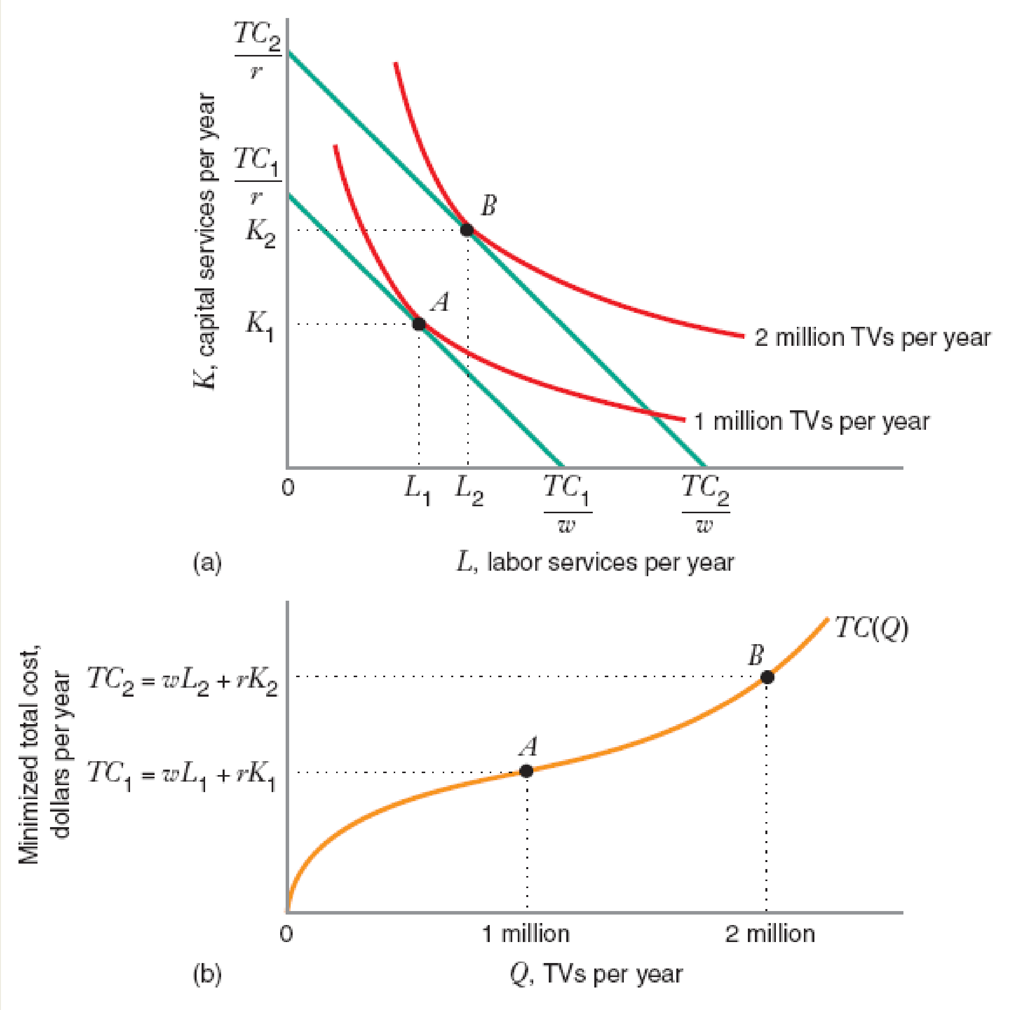
Topic 7

Cost Curves
(Chapter 8)

Long Run Cost Functions



As output increases from 1 million to 2 million, with input prices (w, r) constant, cost-minimizing input combination moves from TC_1 to TC_2 which gives the TC(Q) curve.

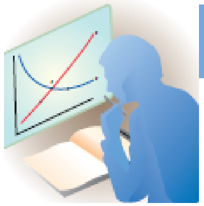
Long Run Cost Functions

Definition: The **long run total cost (LRTC) function** relates minimized total cost to output, Q, and to the factor prices (w and r).

$TC(Q,w,r) = wL^*(Q,w,r) + rK^*(Q,w,r)$

Where: L^* and K^* are the long-run input demand functions.

Long Run Cost Functions



LEARNING-BY-DOING EXERCISE 8.1

Finding the Long-Run Total Cost Curve from a Production Function

Let's return again to the production function $Q = 50\sqrt{LK}$ that we introduced in Learning-By-Doing Exercise 7.2.

Problem

(a) How does minimized total cost depend on the output Q and the input prices w and r for this production function?

(b) What is the graph of the long-run total cost curve when $w = 25$ and $r = 100$?

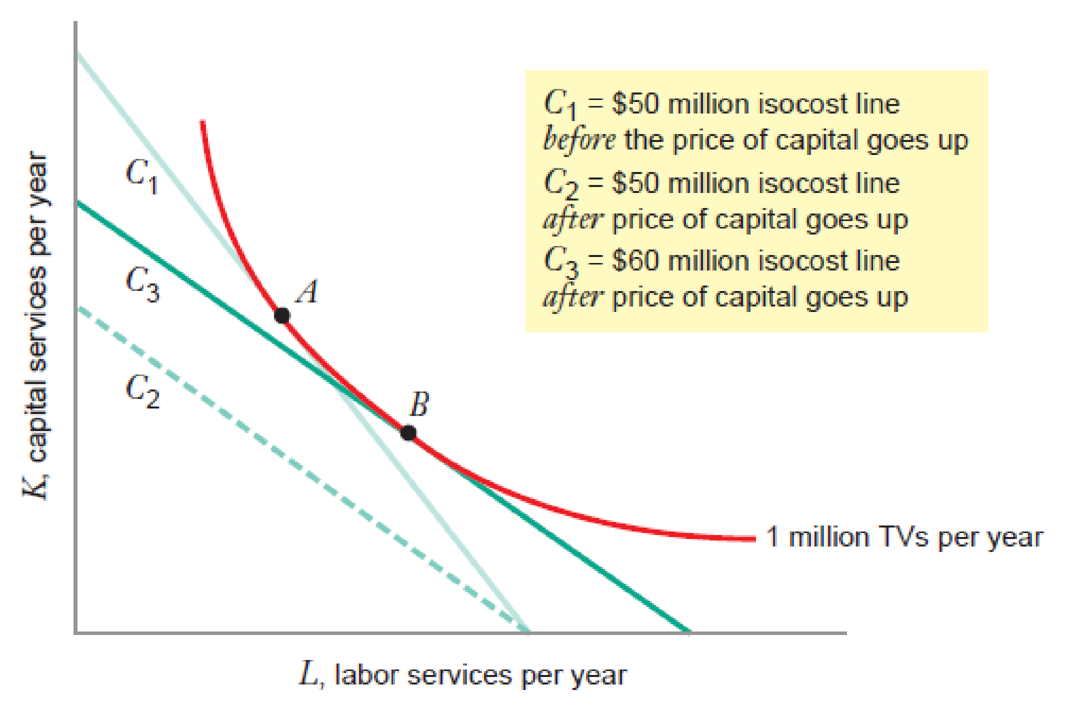
Long Run Total Cost Curve

- A **movement** along the TC curve is a result from changing output.
- As we produce more, we face higher TC.

- What happens when input prices changes?
- **The TC curve will shift**.

- Let's consider two cases:
  - When r (price of K) rises.
  - When both r and w rises equally.

Long Run TC Curve – Higher r



Given higher r, we opt for a cheaper input, L.

FIGURE 8.3 How a Change in the Price of Capital Affects the Optimal Input Combination and Long-Run Total Cost for a Producer of Television Sets The firm's long-run total cost increases after the price of capital increases. The isocost line moves from C_1 to C_3 and the cost-minimizing input combination shifts from point *A* to point *B*.

C_2 = $50m BUT few K can be bought, so C_2 is not sufficient for producing 1m TVs.

C_3 is sufficient to produce the required output.

Long Run TC Curve – Higher r

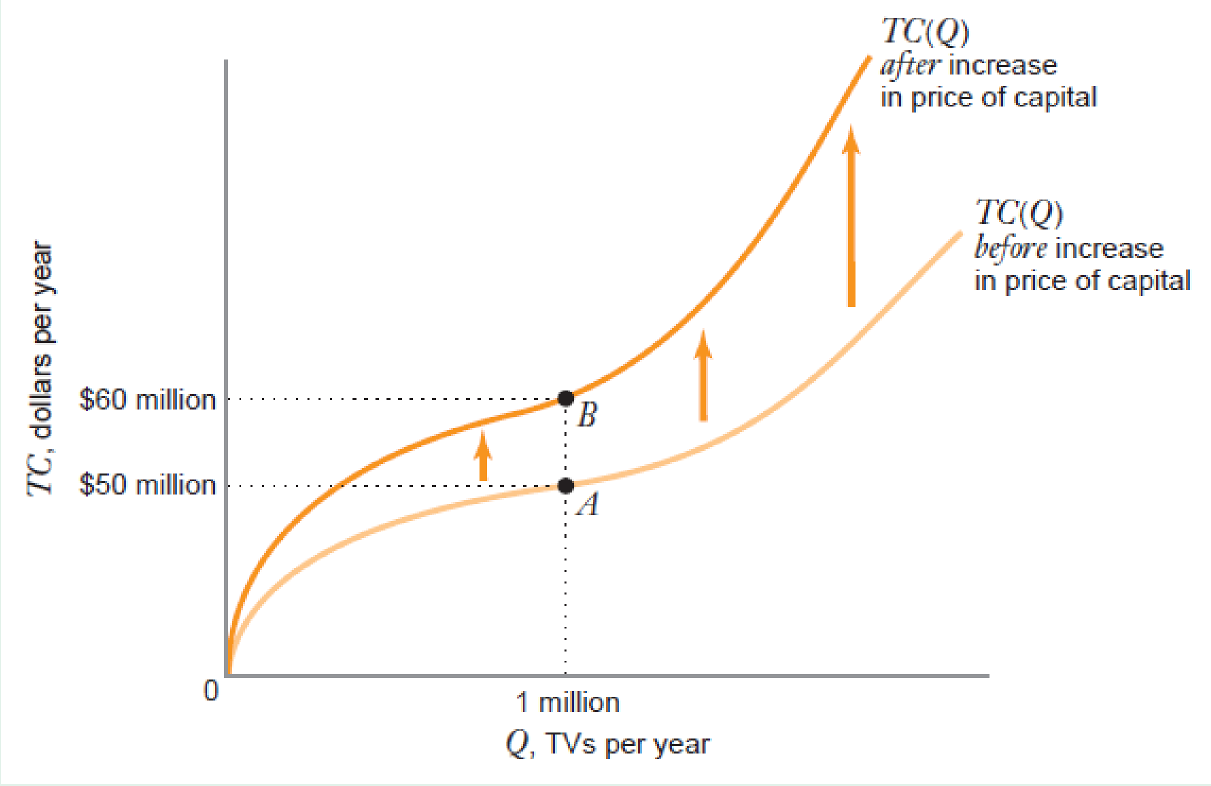
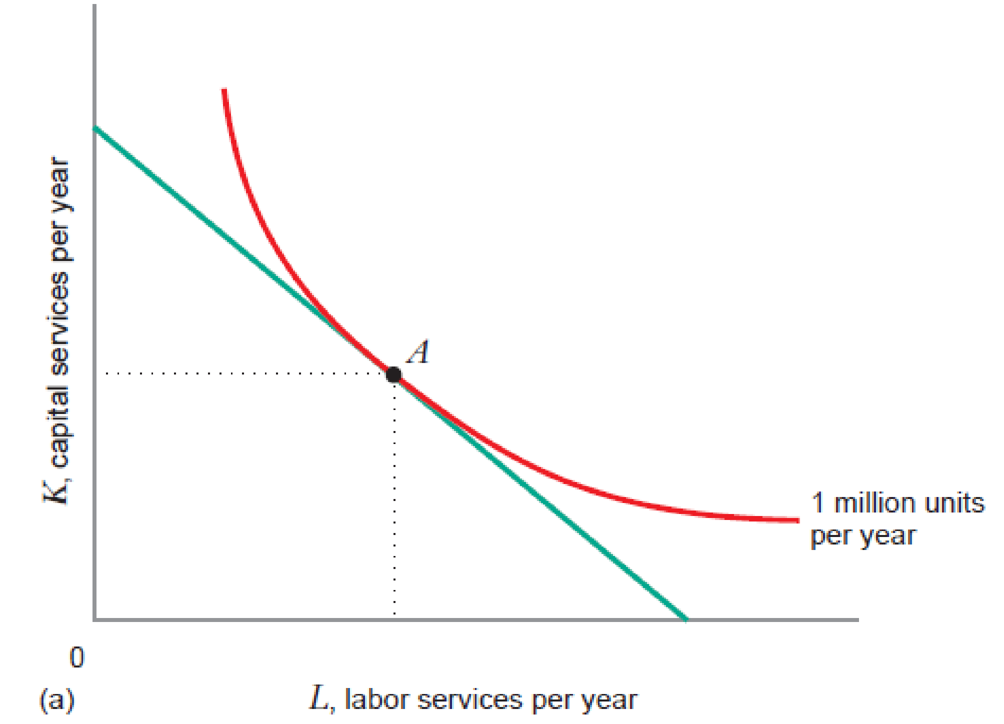


FIGURE 8.4 How a Change in the Price of Capital Affects the Long-Run Total Cost Curve for a Producer of Television Sets
An increase in the price of capital causes the long-run total cost curve *TC*(*Q*) to rotate upward. Points *A* and *B* correspond to the cost-minimizing input combinations in Figure 8.3.

With higher r, producing 1m TVs now requires $60m, instead of $50m

Long Run TC Curve – Higher r & w



<u>When both r and w rise in the same proportion</u>, the cost-minimizing K* and L* do not change.

We cannot opt for a cheaper input because both input prices rise.

Note that the TC will rise in the same proportion.

Long Run TC Curve – Higher r & w

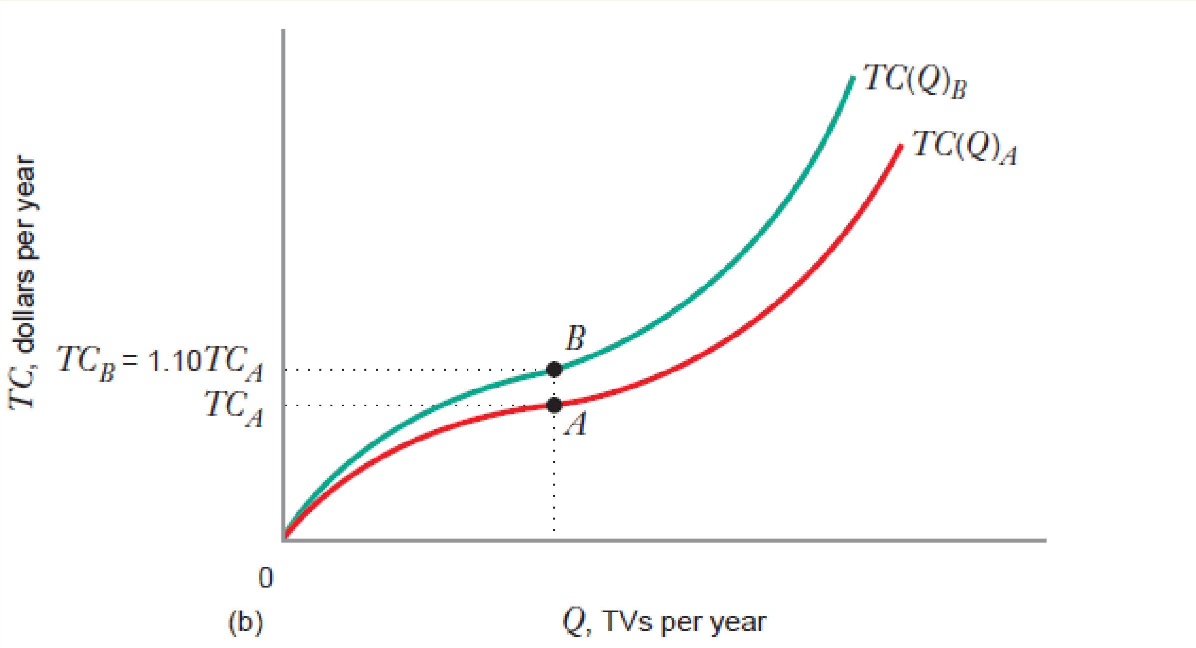


FIGURE 8.5 How a Proportionate Change in the Prices of All Inputs Affects the Cost-Minimizing Input Combination and the Total Cost Curve
The price of each input increases by 10 percent. Panel (a) shows that the cost-minimizing input combination remains the same (at point *A*), because the slope of the isocost line is unchanged. Panel (b) shows that the total cost curve shifts up by the same 10 percent.

Here, r and w rise by 10%.

TC also rises by 10%.

Long-Run AC and MC Functions

Definition: The **LR Average Cost (AC) function** is the long run total cost function divided by output, Q. That is, the LRAC function tells us the firm's cost per unit of output.

$$AC(Q,w,r) = TC(Q,w,r)/Q$$

Definition: The **LR Marginal Cost (MC) function** measures the rate of change of total cost as output varies, holding constant input prices. It is also the slope of TC(Q).

$$MC(Q) = \Delta TC(Q)/\Delta Q$$

Long-Run AC and MC Functions

LEARNING-BY-DOING EXERCISE 8.2

Deriving Long-Run Average and Marginal Cost Curves from a Long-Run Total Cost Curve

In Learning-By-Doing Exercise 8.1 we derived the equation for the long-run total cost curve for the production function $Q = 50\sqrt{LK}$ when the price of labor L is $w = 25$ and the price of capital K is $r = 100$: $TC(Q) = 2Q$.

Problem What are the long-run average and marginal cost curves associated with this long-run total cost curve?

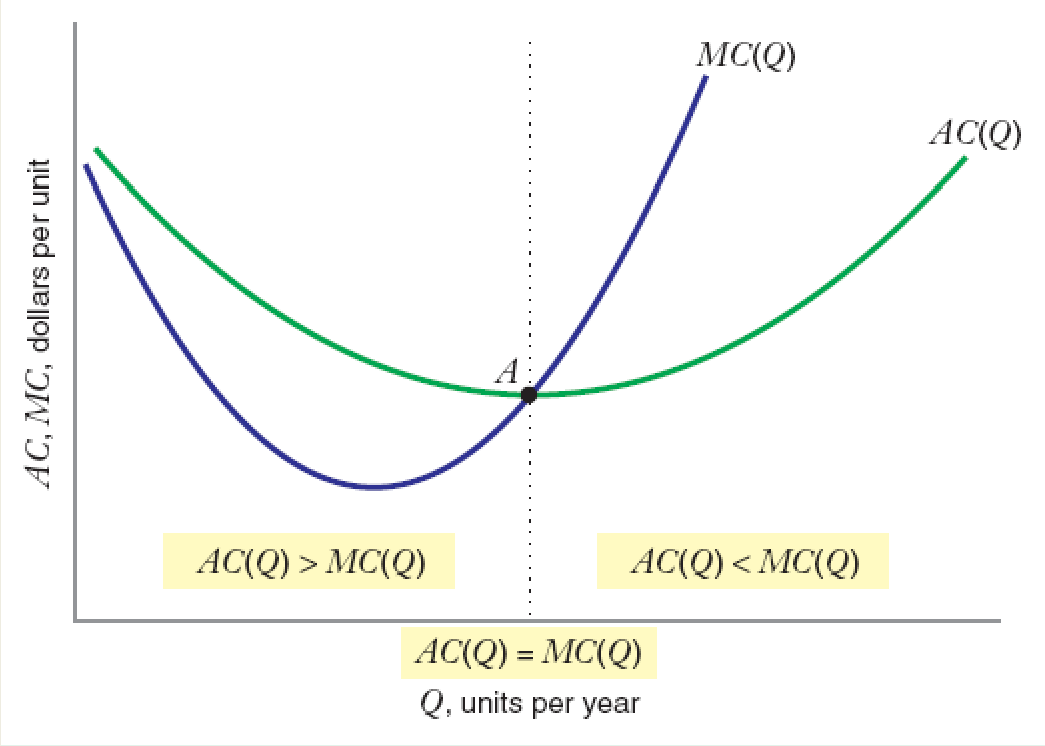
Long-run marginal cost is the slope of the long-run total cost curve. With $TC(Q) = 2Q$, the slope of the long-run total cost curve is 2, and thus $MC(Q) = 2$. Long-run marginal cost also does not depend on Q. Its graph is the same horizontal line.

This exercise illustrates a general point. Whenever the long-run total cost is a straight line (as in Figure 8.2), long-run average and long-run marginal cost curves will

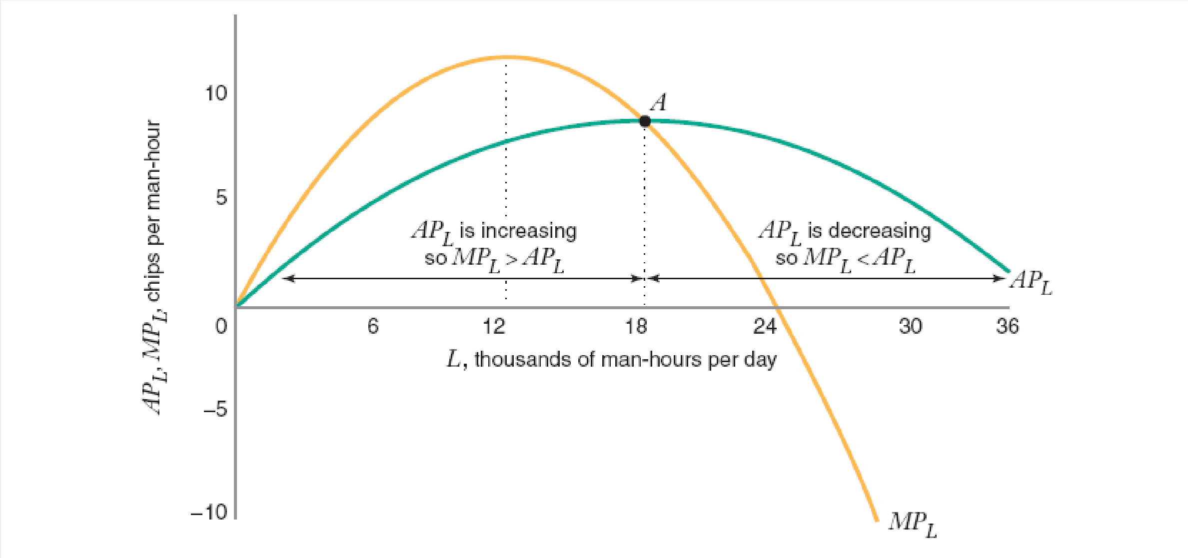
Relationship between AC and MC

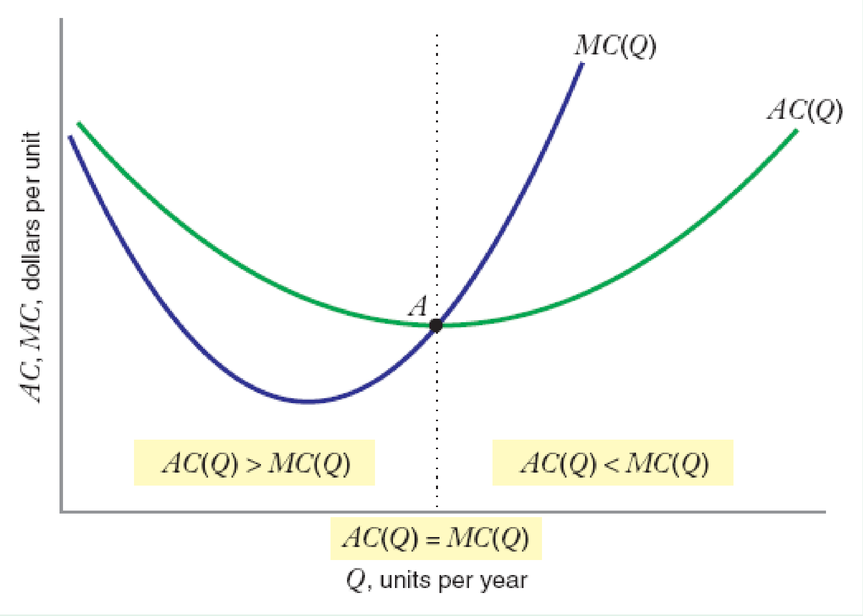
Suppose that w and r are fixed.

- If MC(Q) > AC(Q), AC(Q) increases in Q.
- If MC(Q) < AC(Q), AC(Q) decreases in Q.
- If MC(Q) = AC(Q), AC(Q) is flat with respect to Q, and is minimized.



ANALOGY: AP & MP vs AC & MC





Economies & Diseconomies of Scale

Definition: If LRAC decreases as output rises, all else equal, the LRAC function exhibits economies of scale.

Definition: if LRAC increases as output rises, all else equal, the LRAC function exhibits diseconomies of scale.

Definition: The smallest quantity at which the LRAC curve attains its minimum point is called the minimum efficient scale (MES).

Economies & Diseconomies of Scale

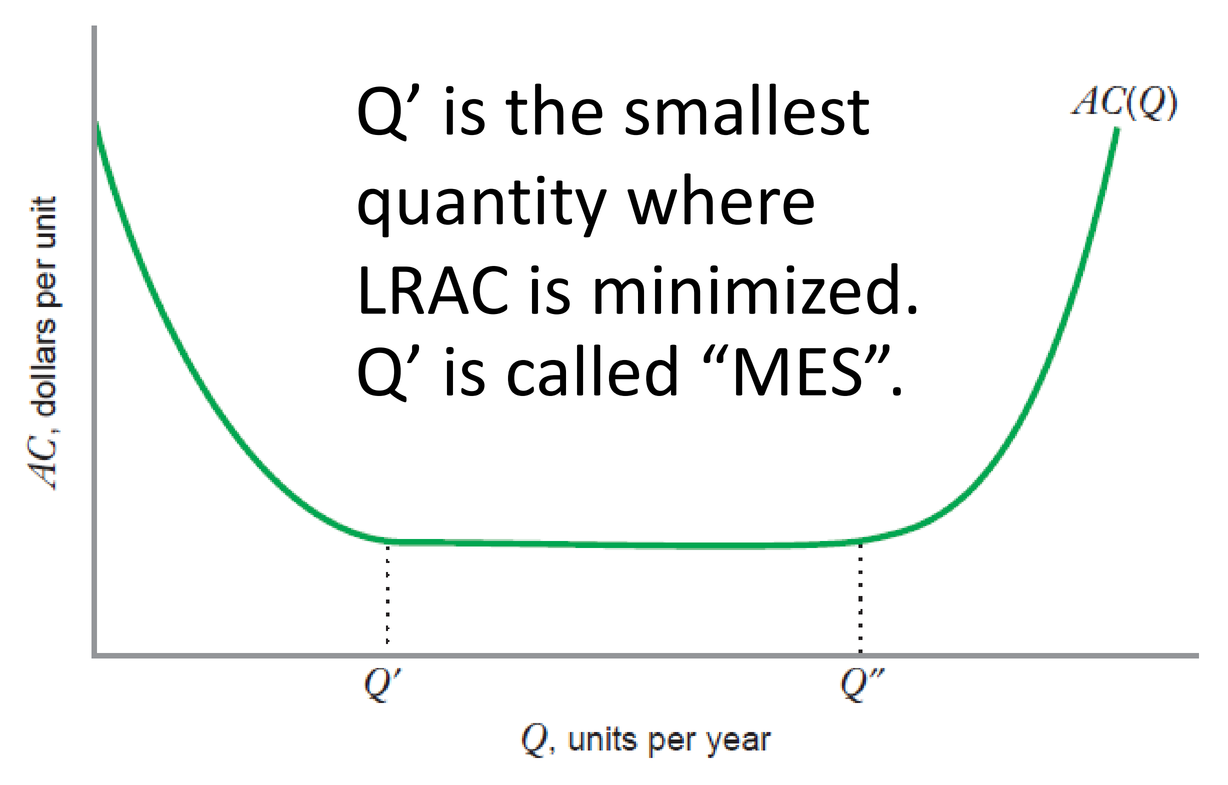


FIGURE 8.11 **Economies and Diseconomies of Scale for a Typical Real-World Average Cost Curve** There are economies of scale for outputs less than *Q*′. Average costs are flat between and *Q*′ and *Q*″ and there are diseconomies of scale thereafter. The output level *Q*′ is called the minimum efficient scale.

From 0 to Q', we have "**economies of scale**".
From Q'' onwards, we have "**diseconomies of scale**".

Economies & Diseconomies of Scale

Economies of Scale (lower AC when Q increases)

- Specialization / Division of Labor
- Quantity Discount / Bulk Buying
- Indivisible Inputs (e.g. large machine)
- Cost-Saving

Diseconomies of Scale (higher AC when Q increases)

- Managerial Diseconomies
- Coordination Problems
- Principal-Agent Problem
- Cost-Dissaving

Returns to Scale & Economies of Scale

- When the production function exhibits **increasing returns to scale**, the LRAC function exhibits **economies of scale** so that AC(Q) decreases with Q, all else equal.

- When the production function exhibits **decreasing returns to scale**, the LRAC function exhibits **diseconomies of scale** so that AC(Q) increases with Q, all else equal.

- When the production function exhibits **constant returns to scale**, **the LRAC function is flat**: it neither increases nor decreases with output.

Returns to Scale & Economies of Scale

Relationship between Economies of Scale and Returns to Scale

	Production Function		
	$Q = L^2$	$Q = \sqrt{L}$	$Q = L$
Labor requirements function	$L = \sqrt{Q}$	$L = Q^2$	$L = Q$
Long-run total cost	$TC = w\sqrt{Q}$	$TC = wQ^2$	$TC = wQ$
Long-run average cost	$AC = w/\sqrt{Q}$	$AC = wQ$	$AC = w$
How does long-run average cost vary with Q?	Decreasing	Increasing	Constant
Economies/diseconomies of scale?	Economies of scale	Diseconomies of scale	Neither
Returns to scale	Increasing	Decreasing	Constant

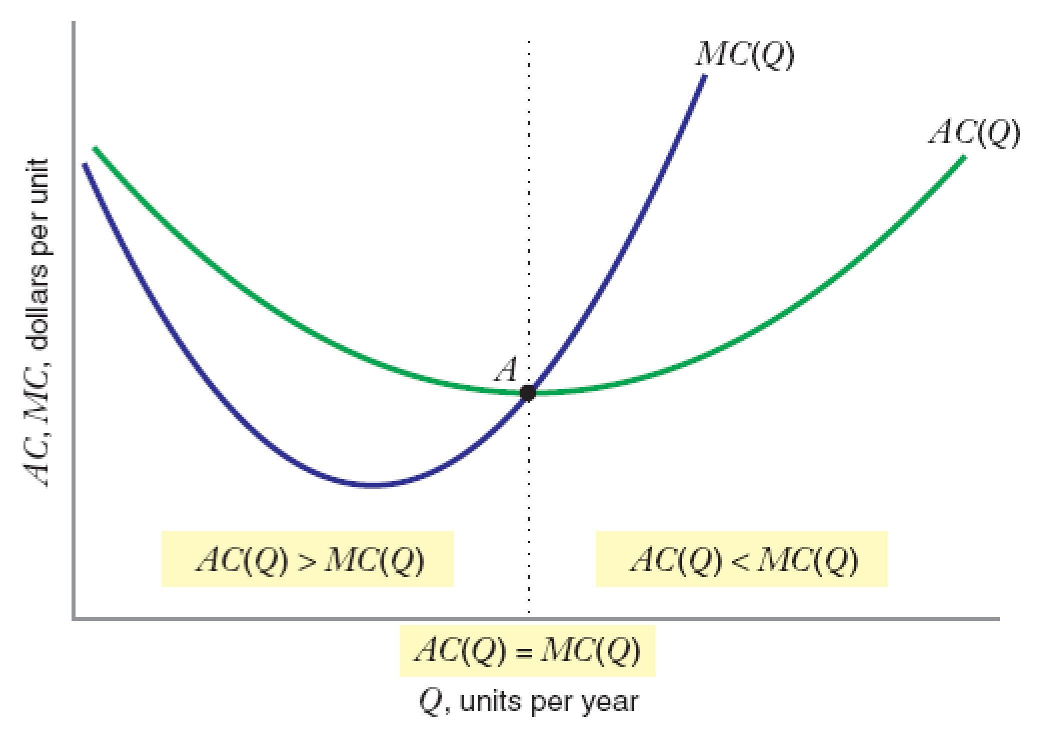
Output Elasticity of Total Cost

Definition: the **output elasticity of total cost**, $\varepsilon_{TC,Q}$ measures the responsiveness of total cost when output changes.

$\varepsilon_{TC,Q}$ = $(\Delta TC/TC)(\Delta Q/Q) = (\Delta TC/\Delta Q)/(TC/Q) =$ **MC/AC**

Value of $\epsilon_{TC,Q}$	*MC* Versus *AC*	How *AC* Varies as *Q* Increases	Economies/ Diseconomies of Scale
$\epsilon_{TC,Q} < 1$	$MC < AC$	Decreases	Economies of scale
$\epsilon_{TC,Q} > 1$	$MC > AC$	Increases	Diseconomies of scale
$\epsilon_{TC,Q} = 1$	$MC = AC$	Constant	Neither

Output Elasticity of Total Cost



Value of $\epsilon_{TC,Q}$	MC Versus AC	How AC Varies as Q Increases	Economies/ Diseconomies of Scale
$\epsilon_{TC,Q} < 1$	$MC < AC$	Decreases	Economies of scale
$\epsilon_{TC,Q} > 1$	$MC > AC$	Increases	Diseconomies of scale
$\epsilon_{TC,Q} = 1$	$MC = AC$	Constant	Neither

Short Run Cost Functions

Definition: The short run total cost (SRTC) function tells us the minimized total cost of producing a given quantity, when (at least) one input is fixed.

Definition: The total variable cost function (TVC) is the minimized sum of expenditures on variable inputs from the short-run cost-minimizing input combinations.

Definition: The total fixed cost function (TFC) is constant and equal to the cost of the fixed input(s).

$$\mathbf{STC(Q,K_0) = TVC(Q,K_0) + TFC(Q,K_0) = wL + rK_0}$$

Where: K_0 is the fixed input and w and r are fixed

Short Run Cost Functions

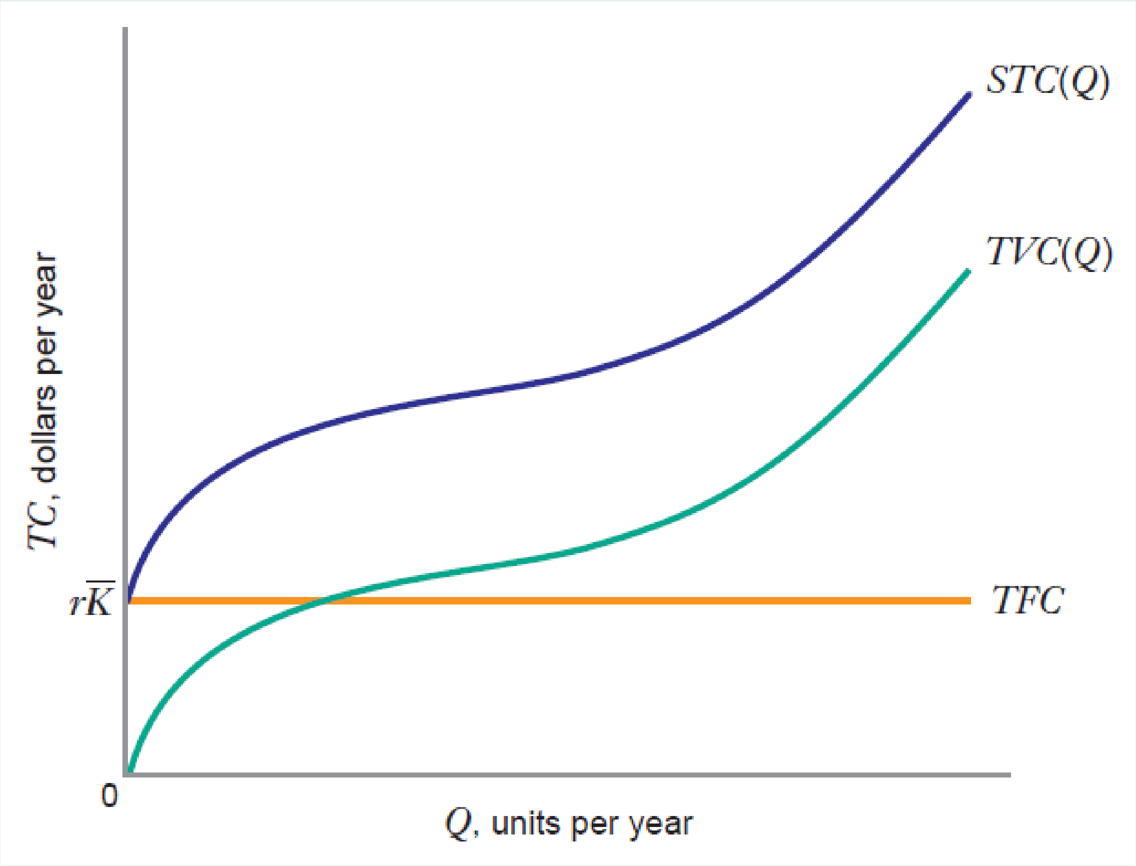


FIGURE 8.13 Short-Run Total Cost Curve The short-run total cost curve $STC(Q)$ is the sum of the total variable cost curve $TVC(Q)$ and the total fixed cost curve TFC. Total fixed cost is equal to the cost $r\overline{K}$ of the fixed capital services.

Short Run Cost Functions

LEARNING-BY-DOING EXERCISE 8.3

Deriving a Short-Run Total Cost Curve

Let us return to the production function in Learning-By-Doing Exercises 7.2, 7.4, 7.5, and 8.1, $Q = 50\sqrt{LK}$.

Problem What is the short-run total cost curve for this production function when capital is fixed at a level $\overline{K}$ and the input prices of labor and capital are $w = 25$ and $r = 100$, respectively?

Relationship between LR and SR Costs

The firm can minimize costs at least as well in the long run as in the short run because it is "**less constrained**".

Hence, **the short run total cost curve lies everywhere above the long run total cost curve.**

However, some inputs combination solves both SR cost-minimization and LR cost-minimization. At such inputs, SRTC = LRTC.

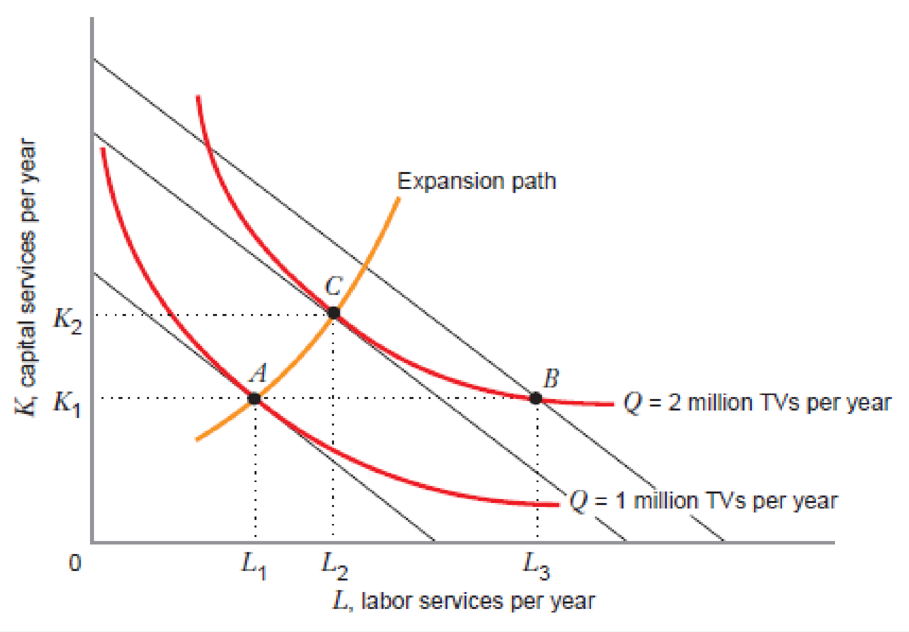


FIGURE 8.14 Total Costs Are Generally Higher in the Short Run than in the Long Run Initially, the firm produces 1 million TVs per year and operates at point *A*, which minimizes cost in both the long run and the short run, if the firm's usage of capital is fixed at K_1. If *Q* is increased to 2 million TVs per year, and capital remains fixed at K_1 in the short run, the firm operates at point *B*. But in the long run, the firm operates at point *C*, on a lower isocost line than point *B*.

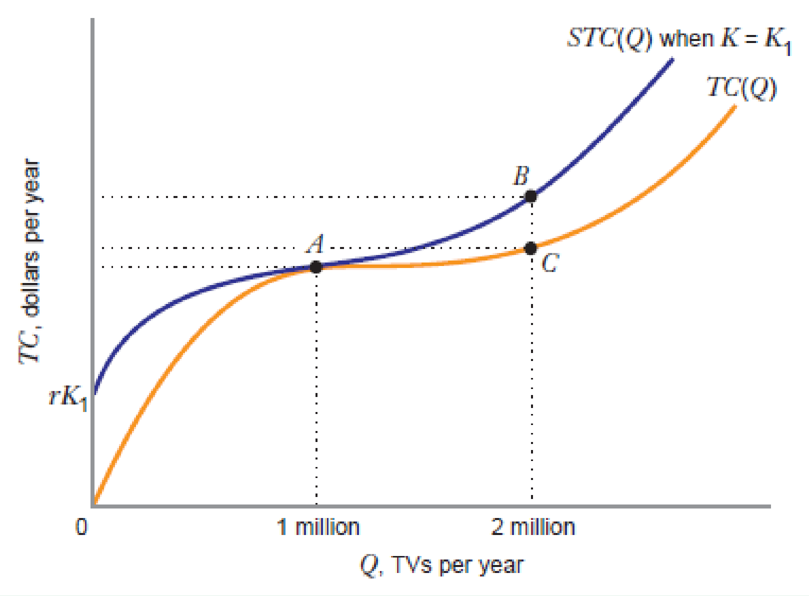


FIGURE 8.15 Relationship between Short-Run and Long-Run Total Cost Curves When the quantity of capital is fixed at K_1, *STC*(*Q*) is always above *TC*(*Q*), except at point *A*. Point *A* solves both the long-run and the short-run cost-minimization problem when the firm produces 1 million TVs per year.

Short-Run AC and MC Functions

Definition: The **SR Average Cost (SRAC) function** is the SRTC function divided by output. The SRAC function tells us the firm's short run cost per unit of output.

$$SRAC(Q,K_0) = SRTC(Q,K_0)/Q$$

Definition: The **SR Marginal Cost (SRMC) function** measures the rate of change of SRTC as output varies, holding constant input prices and fixed inputs. It is also the slope of SRTC(Q).

$$SRMC(Q,K_0) \quad = \Delta SRTC(Q,K_0)/\Delta Q$$

Short-Run AC and MC Functions

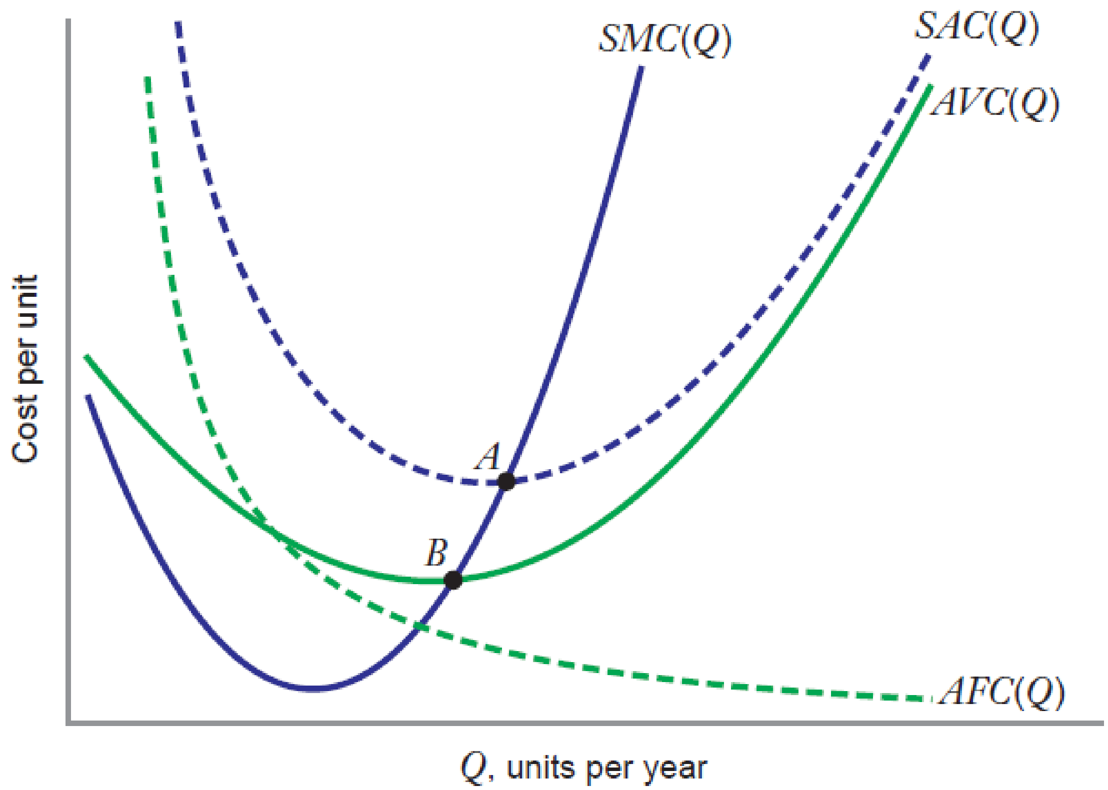
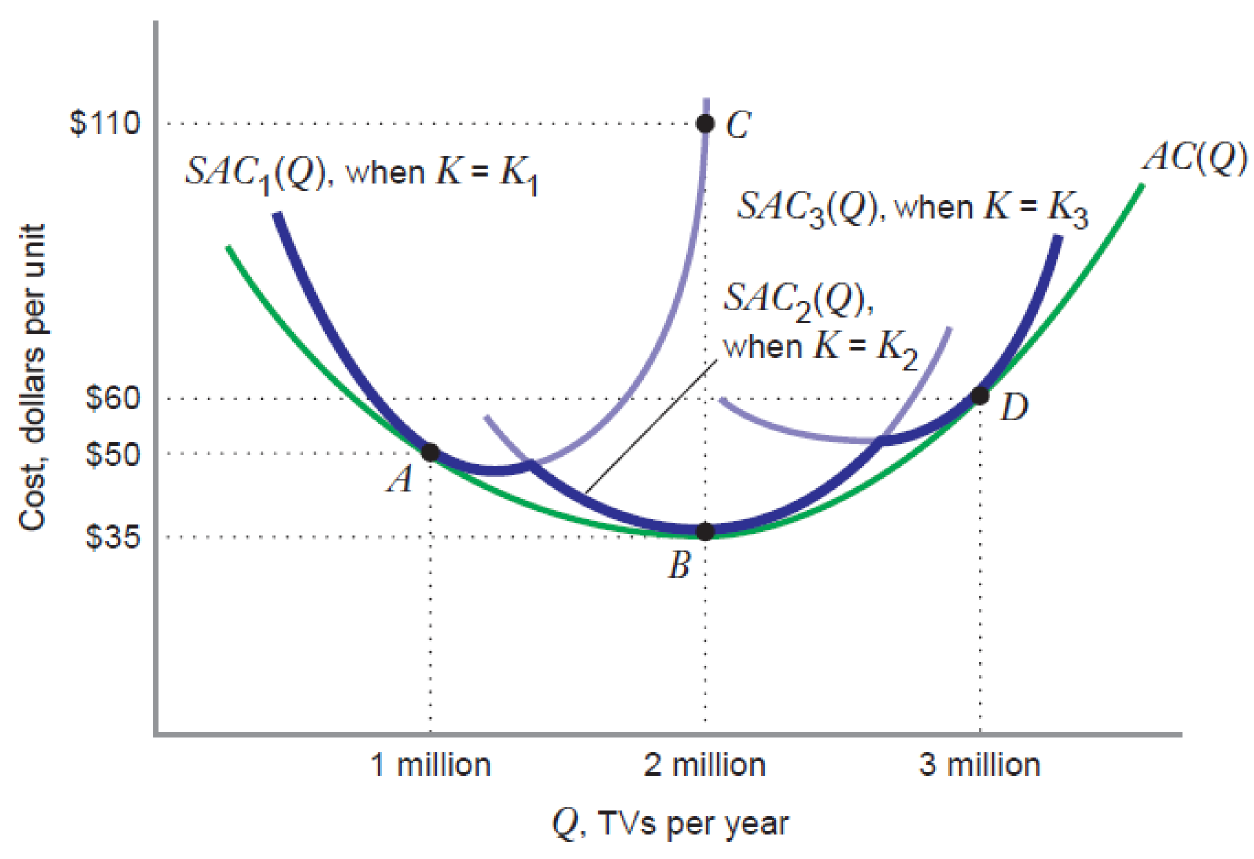


FIGURE 8.16 **Short-Run Marginal and Average Cost Curves**
The short-run average cost curve *SAC*(*Q*) is the vertical sum of the average variable cost curve *AVC*(*Q*) and the average fixed cost curve *AFC*(*Q*). The short-run marginal cost curve *SMC*(*Q*) intersects *SAC*(*Q*) at point *A* and *AVC*(*Q*) at point *B*, where each is at a minimum.

Relationship between LRAC and SRAC

Recall that the SRTC curve lies everywhere above the LRTC curve. Thus, SRAC also lies above LRAC.

FIGURE 8.17 The Long-Run Average Cost Curve as an Envelope Curve
The short-run average cost curves $SAC_1(Q)$, $SAC_2(Q)$, and $SAC_3(Q)$, lie above the long-run average cost curve $AC(Q)$ except at points A, B, and D. This shows that short-run average cost is always greater than long-run average cost except at the level of output for which a plant size (K_1, K_2, or K_3) is optimal. Point C shows where the firm would operate in the short run if it produced 2 million TV sets per year with capital remaining fixed at K_1. If the figure included progressively more short-run curves, the dark scalloped lower boundary of the short-run curves would smooth out and ultimately coincide with the long-run curve.



Relationship between LRAC and SRAC

At Q where SRAC = LRAC, we also have SRMC = LRMC.

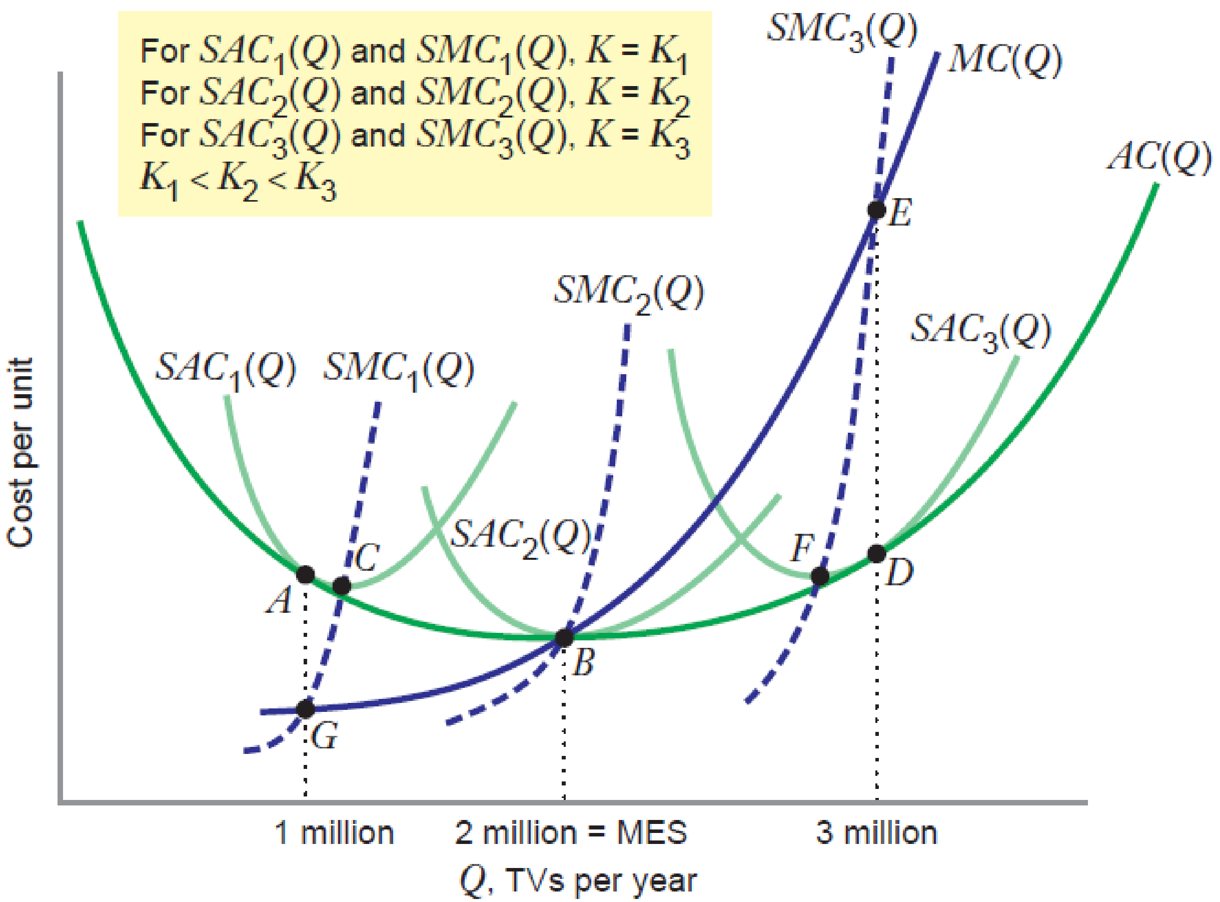


FIGURE 8.18 The Relationship between the Long-Run Average and Marginal Cost Curves and the Short-Run Average and Marginal Cost Curves When the firm's short-run and long-run average costs are equal, its short-run and long-run marginal costs must also be equal.

Economies of Scope

Economies of Scope – a production characteristic in which the total cost of producing given quantities of two goods in the same firm is less than the total cost of producing those quantities in two single-product firms.

$$\mathbf{TC(Q_1, Q_2) < TC(Q_1, 0) + TC(0, Q_2)}$$

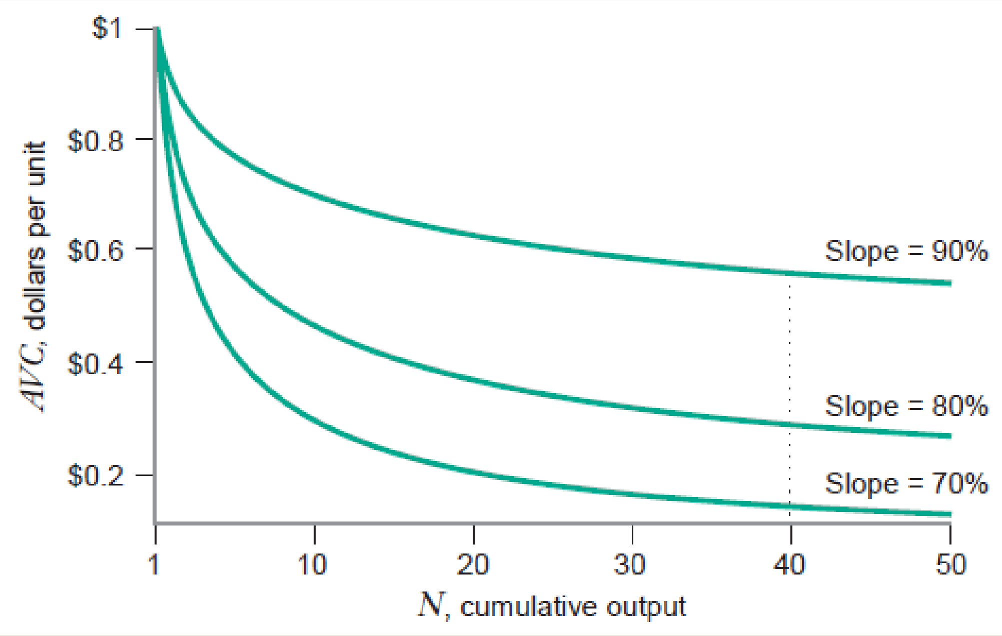
Ex1: A satellite owner hosts movie and news channels.
Ex2: Coke makes another soft drink, using its factory.

Stand-alone Costs – the cost of producing a good in a single-product firm, e.g. $TC(Q_1, 0)$ or $TC(0, Q_2)$.

Economies of Experience

Economies of Experience – cost advantages that result from accumulated experience or learning-by-doing.

Econ. of Scale is usually great in capital-intensive production.
Econ. of EXP is usually great in labor-intensive production.



Experience Curve:
a relationship between AVC and cumulative production volume.

Smaller slope = Lower AVC

Estimating Cost Functions

Total Cost Function – a mathematical relationship that shows how TC vary with factors that influence it.

Cost Driver – A factor that influences or “drives” total or average costs. These factors usually include Q, w, r, and material costs.

Economists then use “regression analysis” to estimate the magnitude of how these factors affect TC.