Group 7
6304640698
6304640946
6304641191

Group Homework 7 & 8

Semester 2/2021 EE320 Introductory mathematical economics

Due date: May 7th 2022 (before midnight /B.E. moodle).

1. *Theory of firm*

Suppose that production function is given by $Q = f(K, L) = \alpha\sqrt{K} + \beta L$ where K and L are the unit of capital installed and the number of employees hired, respectively. Assume that price of K and L are set equal to "r" and "w", respectively. Consider the following problems.

a) The firm wants to minimize cost and seek for combination of the two factor inputs to produce output level Q_0. Derive the factor inputs demand.

b) Confirm your result with the second order derivative test.

c) State the condition under which demand for capital and labor are both strictly positive.

d) Suppose that the condition required for strictly positive solution holds, derive the long-run optimal cost function, and show that marginal cost function is equal to the LaGrange multiplier of your cost minimization problem.

2. *Theory of consumer*

Consider a household with the utility function given by,

$$U(x, y) = [x^2 + y^2]^{\frac{1}{2}}$$

where x *and* y are two different consumption goods, i.e. good x and good y. Suppose that (i) *the prices for each of the two consumption goods are* p_x *and* p_y *respectively*, and (ii) *household's income is equal to* M. Consider the following problems

a) Calculate the total differential of the utility function.

b) Set up the constrained optimization problem and derive the Marshallian demand function.

c) Does the demand function satisfy *the law of demand*? Mathematically, how do you know that?

d) How does the demand for good y respond to price of good x?

e) What is the numerical value of λ^* when $M = \$300,\ p_x = 1,\ p_y = 1$?

f) Without redoing the optimization problem, what would be the new optimized level of maximum utility when income increases to \$310.

3. Suppose that a monopolist has its marginal cost function given by $MC = 16 + 6q^2$ where q is the amount of output produced. The monopolist faces the market demand function given by $P = 160 - 10q^2$ where P is the price per unit of output. Consider the following problem.

a) Suppose that fixed cost is equal to \$240. Calculate the total and the average cost when q = 9 units.
b) Determine the profit-maximizing level of output for the monopolist. Also, confirm your result by using the second derivative test.
c) Calculate the social welfare under the monopoly environment.
d) Calculate the social welfare loss under the monopoly environment.

4. Suppose the demand and supply curves are $P = \frac{6000}{Q+50}$ and $P = Q + 10$. Find the equilibrium price and quantity, and compute the consumer and producer surplus.

1. *Theory of firm*

Suppose that production function is given by $Q = f(K, L) = \alpha\sqrt{K} + \beta L$ where K and L are the unit of capital installed and the number of employees hired, respectively. Assume that price of K and L are set equal to "r" and "w", respectively. Consider the following problems.

a) The firm wants to minimize cost and seek for combination of the two factor inputs to produce output level Q_0. Derive the factor inputs demand.

Step 1 set objective

objective

$\text{Min } C = wL + rk$

$\text{st } Q_C = \underbrace{\alpha\sqrt{K} + \beta L}_{g(x)}$

Step 2 Lagarn function

$\mathcal{L} = wL + rk + \lambda[Q - \alpha\sqrt{K} - \beta L]$

$wL + rk + Q\lambda - \alpha\sqrt{K}\lambda - \beta L\lambda$

Step 3 FOC

$\mathcal{L}_L = w - \beta\lambda = 0$

$w = \beta\lambda$ — (1)

$\mathcal{L}_K = r - \frac{1}{2}\alpha K^{-\frac{1}{2}}\lambda = 0$

$r = \frac{1}{2}\alpha K^{-\frac{1}{2}}\lambda$ — (2)

$\mathcal{L}_\lambda = Q - \alpha\sqrt{K} + BL = 0$

$Q = \alpha\sqrt{K} + \beta L$ — (3)

$\frac{(1)}{(2)}$; $\frac{w}{r} = \frac{\beta\lambda}{\frac{1}{2}\alpha K^{-\frac{1}{2}}\lambda}$

$\frac{w}{r} = \frac{B}{2\alpha} \cdot K^{\frac{1}{2}}$

$K^{\frac{1}{2}} = \frac{2\alpha w}{r\beta}$

$K = \left(\frac{2\alpha w}{r\beta}\right)^2$ sub in (3)

$Q = \alpha\sqrt{\left(\frac{2\alpha w}{r\beta}\right)^2} + BL$

$Q = \frac{2\alpha^2 w}{r\beta} + \beta L$

$r\beta Q = 2\alpha^2 w + r\beta^2 L$

$r\beta^2 L = r\beta Q - 2\alpha^2 w$

$L = \frac{r\beta Q - 2\alpha^2 w}{r\beta^2}$ #

b) Confirm your result with the second order derivative test.

$$\mathcal{L}_L = w - \beta\lambda \qquad g(x) = \alpha\sqrt{K} + \beta L$$

$$\mathcal{L}_{LK} = 0 \qquad g_L = \beta$$

$$\mathcal{L}_{LL} = 0 \qquad g_L = \frac{1}{2}\alpha K^{-\frac{1}{2}}$$

$$\mathcal{L}_K = r - \frac{1}{2}\alpha K^{-\frac{1}{2}}\lambda$$

$$\mathcal{L}_{KL} = 0$$

$$\mathcal{L}_{KK} = \frac{1}{4}\alpha K^{-\frac{3}{2}}\lambda$$

$$\bar{H} = \begin{bmatrix} 0 & \beta & \frac{1}{2}\alpha K^{-\frac{1}{2}} \\ \beta & 0 & 0 \\ \frac{1}{2}\alpha K^{-\frac{1}{2}} & 0 & \frac{1}{4}\alpha K^{-\frac{3}{2}}\lambda \end{bmatrix}$$

$$\bar{H} = \begin{vmatrix} 0 & \beta & \frac{1}{2}\alpha K^{-\frac{1}{2}} \\ \beta & 0 & 0 \\ \frac{1}{2}\alpha K^{-\frac{1}{2}} & 0 & \frac{1}{4}\alpha K^{-\frac{3}{2}}\lambda \end{vmatrix} \begin{matrix} 0 & \beta \\ \beta & 0 \\ \frac{1}{2}\alpha K^{-\frac{1}{2}} & 0 \end{matrix} \qquad H_1 = \begin{vmatrix} 0 & \beta \\ \beta & 0 \end{vmatrix}$$

$$= -\beta^2 < 0$$

$$= 0 + 0 + 0 - 0 - 0 - \frac{1}{4}\alpha K^{-\frac{3}{2}}\lambda\beta^2$$

$$= -\frac{1}{4}\alpha^{-\frac{3}{2}}\lambda\beta^2 < 0$$

$\therefore$ H_1 and H_2 have the same sign

$\rightarrow$ Min

c) State the condition under which demand for capital and labor are both strictly positive.

$$K = \left(\frac{2\alpha w}{r\beta}\right)^2$$

$$L = \frac{r\beta Q - 2\alpha^2 w}{r\beta^2}$$

find that

$K^2 \rightarrow K$ always positive

L positive when

$$\frac{\alpha\beta Q - 2\alpha^2 w}{r\beta^2} > 0$$

$$\alpha\beta Q - 2\alpha^2 w > 0$$

$$\alpha\beta Q > 2\alpha^2 w$$

$$Q > \frac{2\alpha^2 w}{\alpha\beta}$$

$$Q > \frac{2\alpha w}{\beta} \quad \#$$

d) Suppose that the condition required for strictly positive solution holds, derive the long-run optimal cost function, and show that marginal cost function is equal to the LaGrange multiplier of your cost minimization problem.

$\frac{dc}{dQ}$

$$C = WL + RK$$

$$K = \left(\frac{2\alpha w}{r\beta}\right)^2$$

$$L = \frac{r\beta Q - 2\alpha^2 w}{r\beta^2}$$

$$C = w\left[\frac{r\beta Q - 2\alpha^2 w}{r\beta^2}\right] + r\left[\frac{2\alpha w}{r\beta}\right]^2$$

$$C = \frac{r\beta w Q}{r\beta^2} - \frac{2\alpha^2 w^2}{r\beta^2} + r\left[\frac{2\alpha w}{r\beta}\right]^2$$

$$\text{Marginal cost} = \frac{dc}{dQ}$$

$$\frac{dc}{dQ} = \frac{r\beta w}{r\beta^2} = \frac{w}{\beta}$$

$$\therefore MC = \frac{w}{\beta}$$

from (a) $w = \beta\lambda$ — (1)

$$\lambda = \frac{w}{\beta}$$

2. *Theory of consumer*

Consider a household with the utility function given by,

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where x *and* y are two different consumption goods, i.e. good x and good y. Suppose that (i) *the prices for each of the two consumption goods are* p_x *and* p_y *respectively*, and (ii) *household's income is equal to M.* Consider the following problems

a) Calculate the total differential of the utility function.

$$\Delta U = \frac{\partial U}{\partial x} \cdot \Delta x + \frac{\partial U}{\partial y} \cdot \Delta y$$

step ① find $\frac{\partial U}{\partial x}$

$$\frac{\partial U}{\partial x} = \frac{1}{2}(x^2 + y^2)^{-\frac{1}{2}}(2x)$$

$$= x(x^2+y^2)^{-\frac{1}{2}}$$

step ② find $\frac{\partial U}{\partial y}$

$$\frac{\partial U}{\partial y} = \frac{1}{2}(x^2+y^2)^{-\frac{1}{2}}(2y)$$

$$= y(x^2+y^2)^{-\frac{1}{2}}$$

step ③ $x(x^2+y^2)^{-\frac{1}{2}} \cdot \Delta x + y(x^2+y^2)^{-\frac{1}{2}} \cdot \Delta y$ #

b) Set up the constrained optimization problem and derive the Marshallian demand function.

Marshallian

Max U st. Budget

Hick

Min Budget st. U

↳ $M = P_x X + P_y Y$

$$\text{Max} \quad U = [x^2+y^2]^{\frac{1}{2}}$$
$$\text{st} \quad M = P_x X + P_y Y$$

$$\mathcal{L} = [x^2+y^2]^{\frac{1}{2}} + \lambda[M - P_x X - P_y Y]$$

FOC ;

$$\mathcal{L}_x = \frac{1}{2}[x^2+y^2]^{-\frac{1}{2}}[2x] - P_x\lambda = 0$$

$$\frac{1}{2}[x^2+y^2]^{-\frac{1}{2}}[2x] = P_x\lambda \quad — ①$$

$$\mathcal{L}_y = \frac{1}{2}[x^2+y^2]^{\frac{1}{2}}[2y] - P_y\lambda = 0$$

$$\frac{1}{2}[x^2+y^2]^{\frac{1}{2}}[2y] = P_y\lambda \quad — ②$$

$$\mathcal{L}_\lambda = M - P_x X - P_y Y = 0$$

$$M = P_x X + P_y Y \quad — ③$$

$$\frac{①}{②} ; \quad \frac{\frac{1}{2}[x^2+y^2]^{-\frac{1}{2}}[2x]}{\frac{1}{2}[x^2+y^2]^{-\frac{1}{2}}[2y]} = \frac{P_x\lambda}{P_y\lambda}$$

$$\frac{x}{y} = \frac{P_x}{P_y}$$

$$x = \frac{P_x Y}{P_y} \quad \text{แทน ③}$$

$$M = P_x\left[\frac{P_x Y}{P_y}\right] + P_y Y$$

$$P_y M = P_x^2 Y + P_y^2 Y$$

$$= [P_x^2 + P_y^2]y$$

$$y = \frac{P_y M}{[P_x^2 + P_y^2]}$$

$$x = \frac{P_x M}{[P_x^2 + P_y^2]}$$

demand function.

$P\uparrow \rightarrow Q\downarrow$

c) Does the demand function satisfy *the law of demand*? Mathematically, how do you know that?

$$x = \frac{P_x M}{[P_x^2 + P_y^2]}$$

$$\frac{dx}{dP_x} = \frac{[P_x^2+P_y^2]M - P_x M[2P_x]}{[P_x^2+P_y^2]^2}$$

$$= \frac{MP_x^2 + MP_y^2 - 2MP_x^2}{[P_x^2+P_y^2]^2}$$

$$= \frac{MP_y^2 - MP_x^2}{[P_x^2+P_y^2]^2} = \frac{M[P_y^2 - P_x^2]}{[P_x^2+P_y^2]^2}$$

⊖ when $P_x > P_y \rightarrow$ law of demand

⊕ when $P_y > P_x \rightarrow$ not law of demand

$$y = \frac{P_y M}{[P_x^2 + P_y^2]}$$

$$\frac{dy}{dP_y} = \frac{[P_x^2+P_y^2]M - P_y M[2P_y]}{[P_x^2+P_y^2]^2}$$

$$= \frac{MP_x^2 + MP_y^2 - 2MP_y^2}{[P_x^2+P_y^2]^2}$$

$$= \frac{MP_x^2 - MP_y^2}{[P_x^2+P_y^2]^2} = \frac{M[P_x^2 - P_y^2]}{[P_x^2+P_y^2]^2}$$

d) How does the demand for good y respond to price of good x?

$$y = \frac{P_y M}{[P_x^2 + P_y^2]}$$

$$\frac{dy}{dP_x} = \frac{[P_x^2+P_y^2]1 - P_y M[2P_x]}{[P_x^2+P_y^2]^2}$$

$$= \frac{-P_y M 2P_x}{[P_x^2+P_y^2]^2} < 0$$

$$= P_x\uparrow \rightarrow y\downarrow$$

e) What is the numerical value of λ^* when $M = \$300,\ p_x = 1,\ p_y = 1$?

$$\frac{1}{2}[x^2+y^2]^{-\frac{1}{2}}[2x] = P_x\lambda \quad -①$$

λ

$$y = \frac{P_y M}{[P_x^2+P_y^2]} = \frac{300(1)}{[1^2+1^2]} = 150$$

$$x = \frac{P_x M}{[P_x^2+P_y^2]} = \frac{300(1)}{[1^2+1^2]} = 150$$

$$\frac{1}{2}[150^2+150^2]^{-\frac{1}{2}}[2(150)] = 1\lambda$$

$$\lambda = 0.7071$$

f) Without redoing the optimization problem, what would be the new optimized level of maximum utility when income increases to \$310.

λ tell that if constrain change how much objective change , $\lambda = 0.7$

so if M increase 1 unit U increase 0.7
M increase 10 unit U increase $0.7 \times 10 = 7$ unit

3. Suppose that a monopolist has its marginal cost function given by $MC = 16 + 6q^2$ where q is the amount of output produced. The monopolist faces the market demand function given by $P = 160 - 10q^2$ where P is the price per unit of output. Consider the following problem.

a) Suppose that fixed cost is equal to \$240. Calculate the total and the average cost when q = 9 units.

b) Determine the profit-maximizing level of output for the monopolist. Also, confirm your result by using the second derivative test.

c) Calculate the social welfare under the monopoly environment.

d) Calculate the social welfare loss under the monopoly environment.

(a) $TC = \int MC \, dq$

$= \int 16 + 6q^2 \, dq$

$= 16q + \frac{6q^3}{3} + C$ $\quad q = 9 \quad FC = 240$

$TC = 16(9) + \frac{6(9)^3}{3} + 240$

$= 144 + 1458 + 240$

$= 1842$ #

$ATC = \frac{1842}{9} = 204.67$ #

b) Determine the profit-maximizing level of output for the monopolist. Also, confirm your result by using the <u>second derivative test</u>.

$$\text{Profit} = TR - TC$$

$$= P \cdot Q - 16q + \frac{6q^3}{3} + 240$$

$$= (160 - 10Q^2)Q - 16Q - 2Q^3 - 240$$

$$= 160Q - 10Q^3 - 16Q - 2Q^3 - 240$$

$$= 144Q - 12Q^3 - 240$$

$$\frac{d\pi}{dQ} = 144 - 36Q^2 = 0$$

$$144 = 36Q^2$$

$$Q^2 = 4$$

$$Q = 2$$

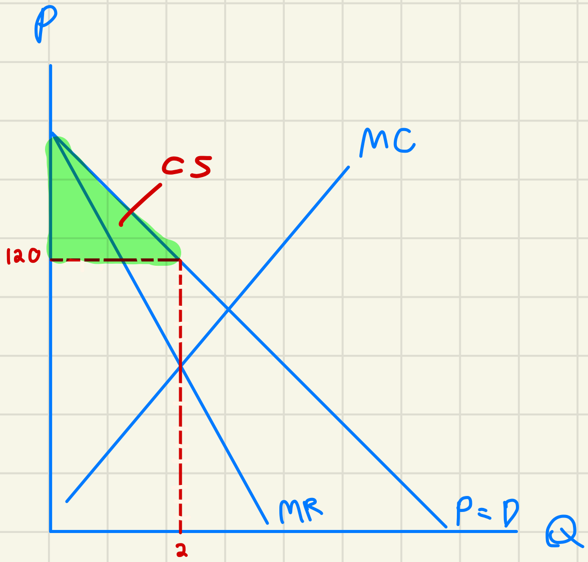
Second derivative

$$\frac{d^2\pi}{dq^2} = -72Q \qquad Q = 2$$

$$= -144 < 0 \qquad \text{max}$$

c) Calculate the social welfare under the monopoly environment.

Social welfare = CS + PS



$P = 120 \quad Q = 2$

$$CS = \int_0^2 D \, dq - P \cdot Q$$

$$= \int_0^2 160 - 10Q^2 \, dQ - P.Q$$

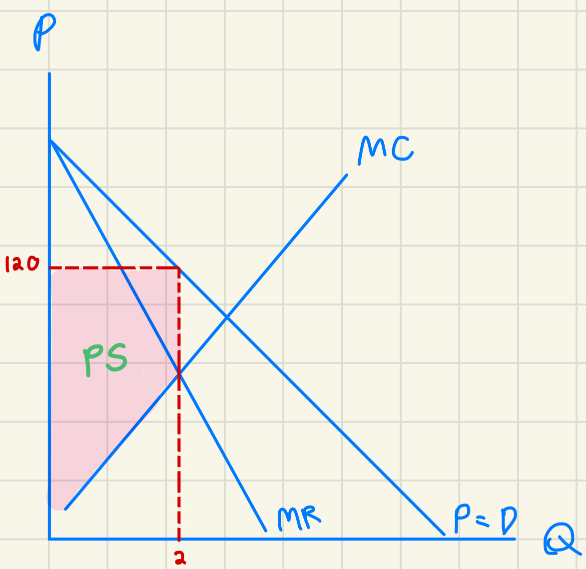
$$= 160Q - \frac{10Q^3}{3} \Big|_0^2 - P.Q$$

$$= 160(2) - \frac{10(2)^3}{3} - P.Q$$

$$= 320 - \frac{80}{3} - (120 \times 2)$$

$$320 - \frac{80}{3} - 240$$

$$= 53.33$$



$$PS = P.Q - \int S \, dq$$

$$= 240 - \int_0^2 16 + 6Q^2 \, dQ$$

$$= 240 - \left(16Q + 2Q^3 \Big|_0^2\right)$$

$$= 240 - 16(2) + 2(2)^3$$

$$= 240 - 32 - 16$$

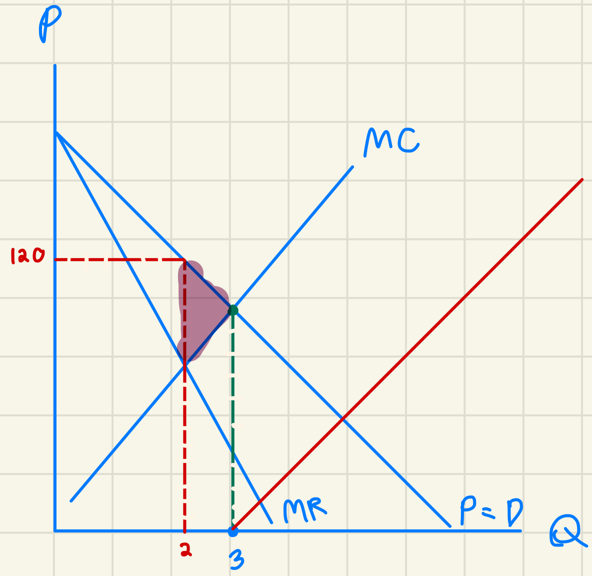
$$= 192$$

$$PS + CS = 192 + 53.33$$

$$= 245.33$$

d) Calculate the social welfare loss under the monopoly environment.

Welfare loss = dead weight loss



step 1 $\quad P = MC$

$$16 + 6Q^2 = 160 - 10Q^2$$

$$16Q^2 = 144$$

$$Q^2 = 9$$

$$Q = 3$$

step2 $\displaystyle\int_2^3 d \, dQ - \int_2^3 s \, dQ$

$$\int_2^3 160 - 10Q^2 \, dQ - \int_2^3 16 + 6Q^2 \, dQ$$

$$\int_2^3 144 - 16Q^2 \, dQ$$

$$= 144Q - \frac{16Q^3}{3} \Big|_2^3$$

$$= \left[144(3) - \frac{16(3)^3}{3}\right] - \left[144(2) - \frac{16(2)^3}{3}\right]$$

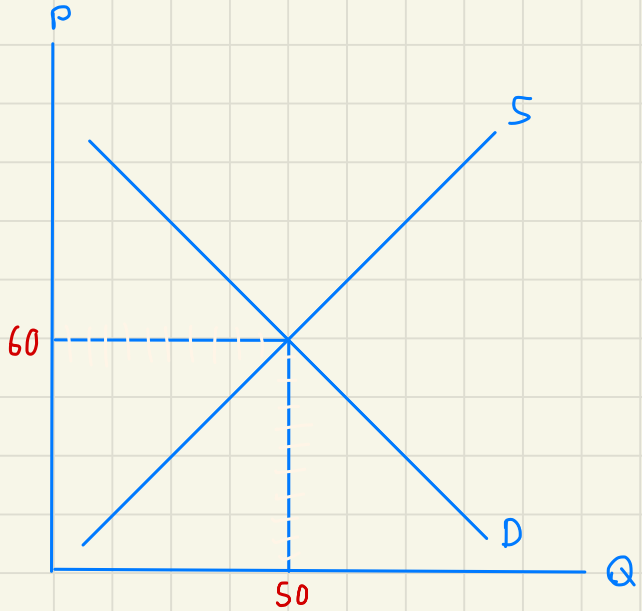
$$= (432 - 144) - (288 - 42.67)$$

$$= 288 - 245.33$$

$$= 42.67$$

4. Suppose the demand and supply curves are $\overset{D}{P} = \frac{6000}{Q+50}$ and $\overset{S}{P} = Q + 10$. Find the equilibrium price and quantity, and compute the consumer and producer surplus.

$$CS = \int_0^Q D \, dQ - P \cdot Q \qquad PS = P \cdot Q - \int_0^Q S \, dQ$$



Step 1 $\quad P, Q$

$D = S$

$$\frac{6000}{Q+50} = Q + 10$$

$$6000 = (Q+10)(Q+50)$$
$$6000 = Q^2 + 60Q + 500$$
$$0 = Q^2 + 60Q - 5500$$
$$= (Q+110)(Q-50)$$
$$Q = 50$$
$$P = 60$$

$$\int \frac{1}{x} \, dx = \ln x$$

Step 2 $\quad PS = PQ - \int S \, dQ$

$$PS = (50)(60) - \int_0^{50} Q + 10 \, dQ$$
$$= 3000 - \frac{Q^2}{2} + 10Q \Big|_0^{50}$$
$$= 3000 - \left[\frac{50^2}{2} + 10(50)\right]$$
$$= 3000 - (1250 + 500)$$
$$= 3000 - 1750$$
$$PS = 1250$$

Step 3 : $CS = \int D \, dQ - P \cdot Q$

$$CS = \int_0^{50} \frac{6000}{Q+50} \, dQ - 3000$$
$$= 6000 \int_0^{50} \frac{1}{Q+50} \, dQ - 3000$$
$$= 6000 \ln(Q+50) \Big|_0^{50} - 3000$$
$$= 6000\left[\ln(100) - \ln(50)\right] - 3000$$
$$= 4{,}158 - 3000$$
$$= 1{,}158$$

5. Let $MR = 25 - 5x - 2x^2$ and $MC = 10 - 3x - x^2$, where x is the unit of output. Assume that fixed cost is \$7. Determine the level of production that contributes to maximum profit and determine the level of maximized profit.

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5. Let $MR = 25 - 5x - 2x^2$ and $MC = 10 - 3x - x^2$, where x is the unit of output. Assume that fixed cost is $7. Determine the level of production that contributes to maximum profit and determine the level of maximized profit.

<u>Normal</u>

max profit : $MR = MC$

$\text{profit} = TR - TC$

$TR = \int MR\, dx$

$TC = \int MC\, dx$

<u>Perfect price discriminate</u>

Q หาจาก $P = MC$

$\max \pi = TR - TC$

$TR = \int P\, dx$

$TC = \int MC\, dx$

Max profit : $MR = MC$

$$25 - 5x - 2x^2 = 10 - 3x - x^2$$
$$25 - 10 = 5x - 3x + 2x^2 - x^2$$
$$15 = 2x + x^2$$
$$0 = x^2 + 2x - 15$$
$$x = (x+5)(x-3)$$
$$x = 3, -5$$
$$x = 3$$

$$TC = \int MC\, dx$$
$$= \int 10 - 3x - x^2\, dx$$
$$= 10x - \frac{3x^2}{2} - \frac{x^3}{3} + c$$

$$TC(0) = 0 - 0 - 0 + c = 7$$
$$c = 7$$

$$\therefore TC = 10x - \frac{3x^2}{2} - \frac{x^3}{3} + 7 \quad \#$$

$$TR = \int MR\, dx$$
$$= \int 25 - 5x - 2x^2\, dx$$
$$= 25x - \frac{5x^2}{2} - \frac{2x^3}{3} + c$$

$$TR(0) = 0$$
$$c = 0$$

$$\therefore TR = 25x - \frac{5x^2}{2} - \frac{2x^3}{3} \quad \#$$

Profit $= TR - TC \qquad X = 3$

$$= \left[25(3) - \overset{34.5}{\frac{5(3)^2}{2}} - \frac{2(3)^3}{3}\right] - \left[\overset{14.5}{10(3)} - \frac{3(3)^2}{2} - \frac{3^3}{3} + 7\right]$$

$$= 75 - \frac{45}{2} - \frac{54}{3} - 30 + \frac{27}{2} + \frac{27}{3} - 7$$

$$= 75 - 22.5 - 18 - 30 + 13.5 + 9 - 7$$

Profit $= 20$ ##