Applications of Utility Maximization Problem

- Giffen good
- Subsidy
- Vouchers
- Work & leisure
- Intertemporal consumption

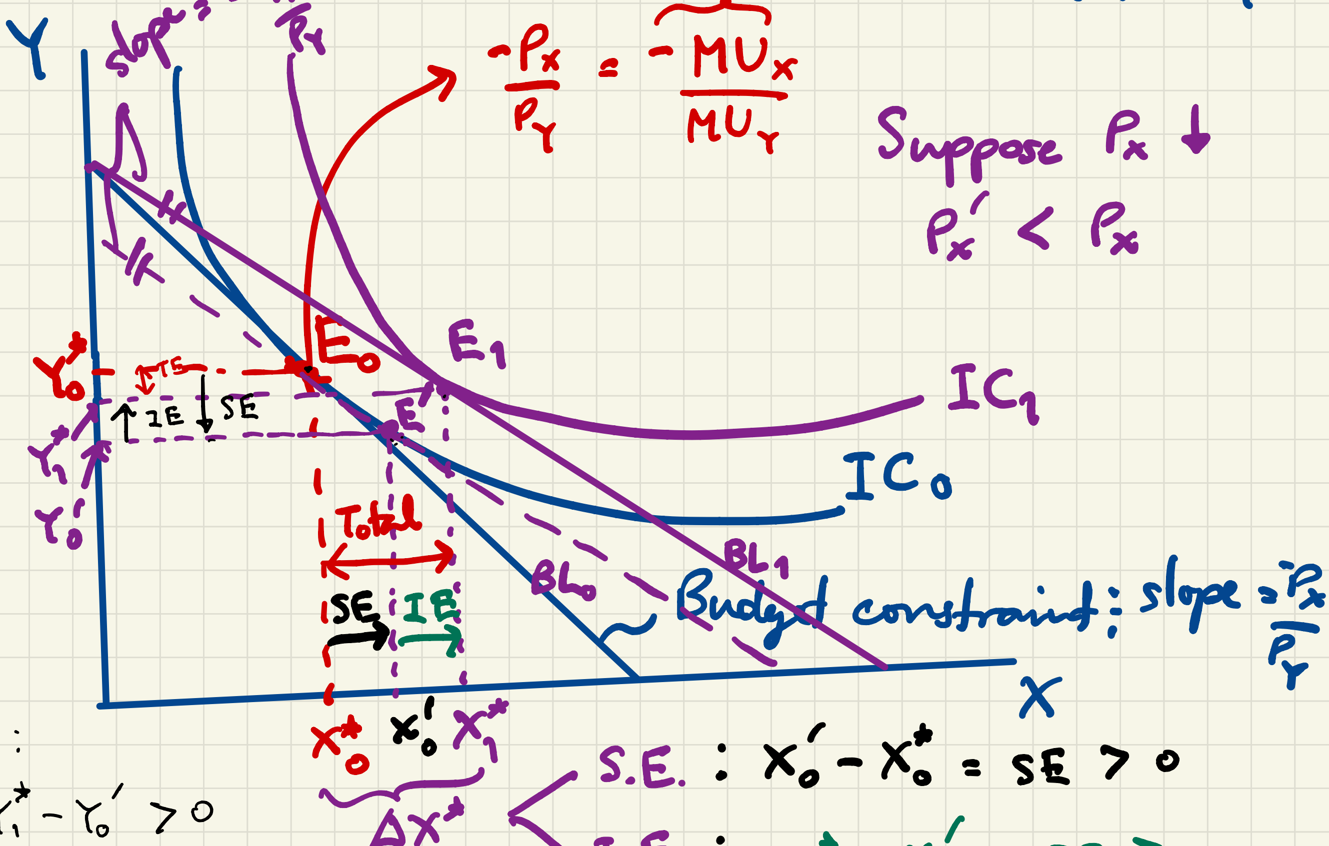
U-max problem → Max $U(X,Y)$

s.t. $P_x \cdot X + P_Y \cdot Y = B$

$$-\frac{P_x}{P_Y} = \overbrace{-\frac{MU_x}{MU_Y}}^{MRS}$$

Suppose $P_x \downarrow$

$P_x' < P_x$



S.E. : $X_0' - X_0^* = SE > 0$

I.E. : $X_1^* - X_0' = IE > 0$

Total effect : $x_1^* - x_0^* > 0$

For Y :

$IE = Y_1^* - Y_0' > 0$

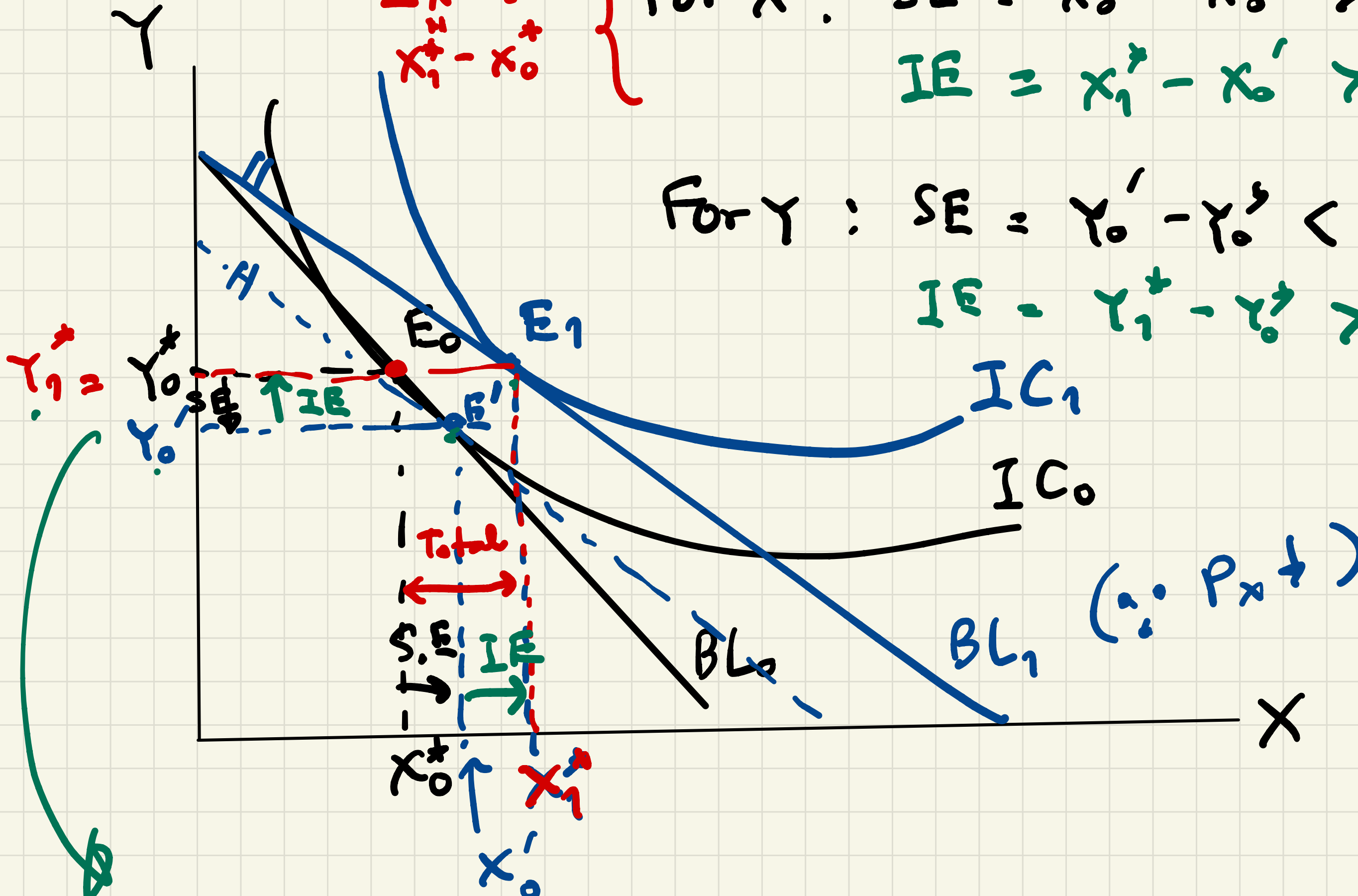
$SE = Y_0' - Y_0^* < 0$

total effect : $Y_1^* - Y_0^* < 0$

$\Delta X^* > 0 = X_1^* - X_0^*$

For X: $SE = X_0' - X_0^* > 0$

$IE = X_1^* - X_0' > 0$



For Y: $SE = Y_0' - Y_0^* < 0$

$IE = Y_1^* - Y_0^* > 0$

$|Y_1^* - Y_0^*| = |Y_0' - Y_0^*| \Rightarrow |S.E| = |I.E.|$

$\Rightarrow \Delta Y^* = 0 \quad (Y_0^* = Y_1^*)$

Application 1: Giffen Good

(income ↑, Q_D ↓)

- Giffen good is a special case of *inferior good*.
- It is a good at which quantity demanded decreases when its price is lower, which is not consistent with the law of demand.
- This is possible when income effect (negative) is greater than substitution effect (positive).
- Example: Suppose there are two goods – potatoes (X) and meat (Y).

↓ Giffen

Let P_X=3, P_Y=4, and B = 120. If P_X decreases to 2, X* will decrease.

$P_X' = 2$

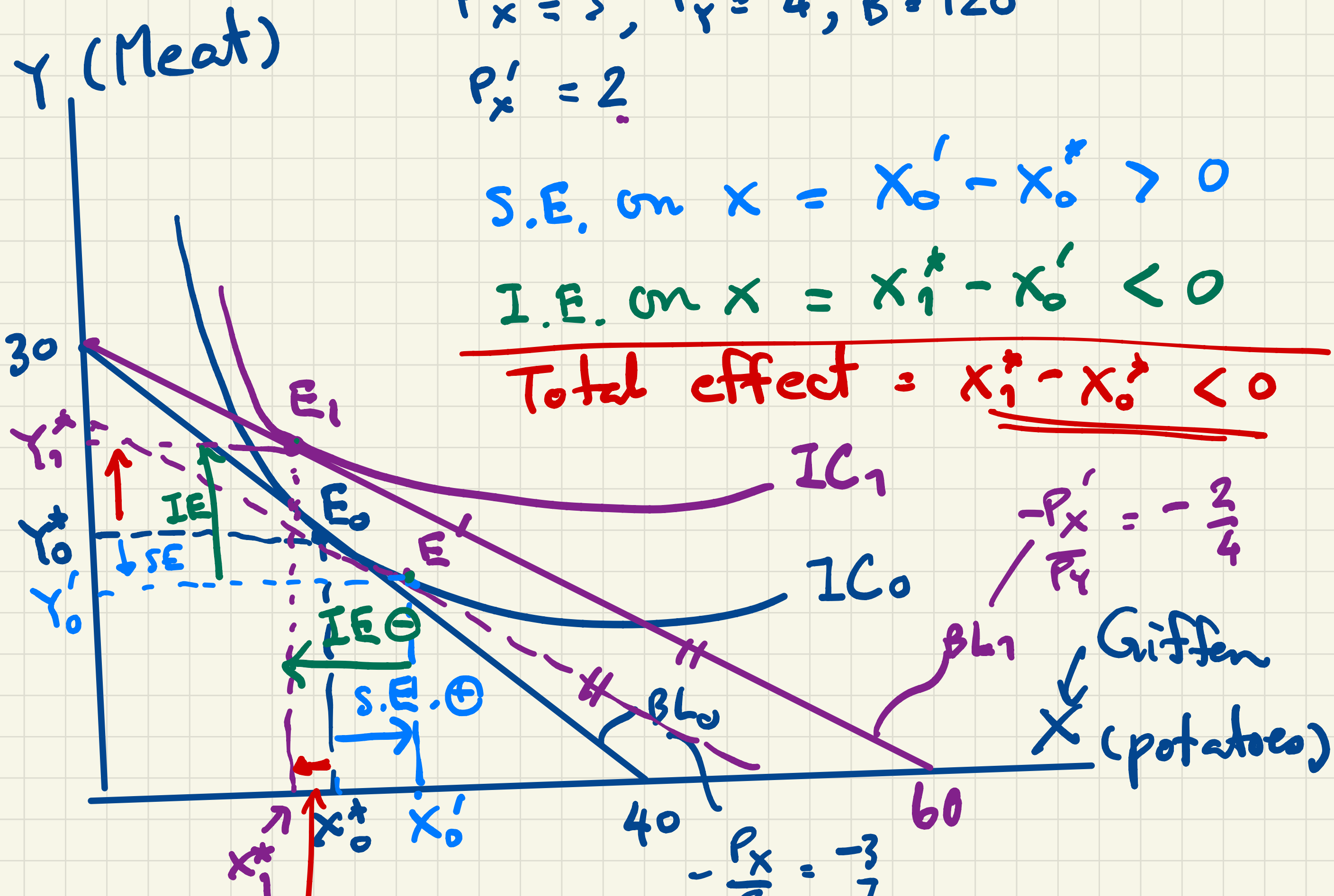
$P_x = 3, \quad P_Y = 4, \quad B = 120$

$P_x' = 2$

S.E. on $X = X_0' - X_0^* > 0$

I.E. on $X = X_1^* - X_0' < 0$

Total effect $= \underline{\underline{X_1^* - X_0^*}} < 0$

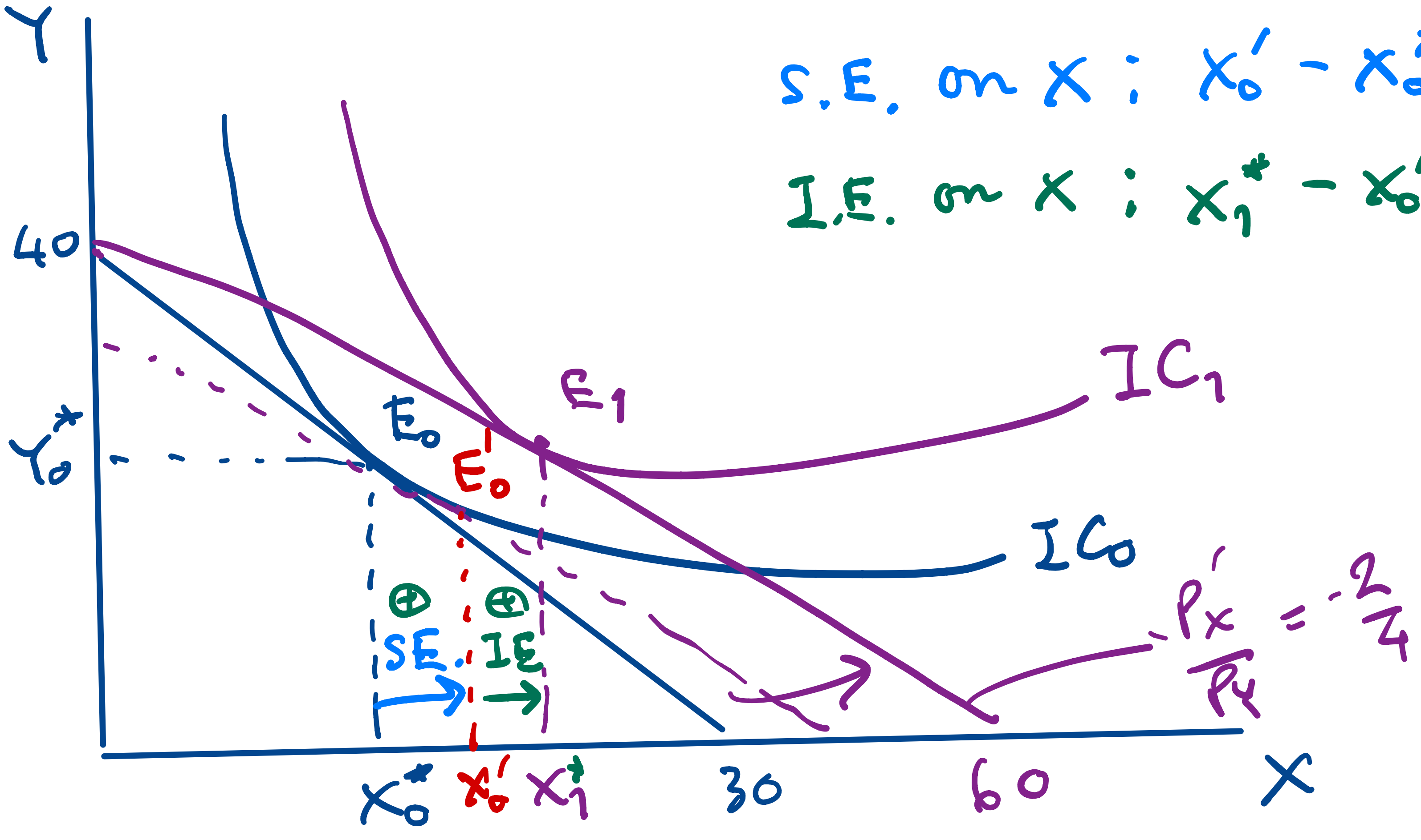


Total effect of $\downarrow P_x$ on X^*

Application 2: Per-Unit Subsidy

Ex: Suppose the gov't gives a $2 per-unit subsidy for good X. Everything else constant.

$P_X = 4, \; P_Y = 3, \; B = 120 \;\Rightarrow\; P_X' = P_X - 2 = 2$



S.E. on X : $X_0' - X_0^* > 0$

I.E. on X : $X_1^* - X_0' > 0$

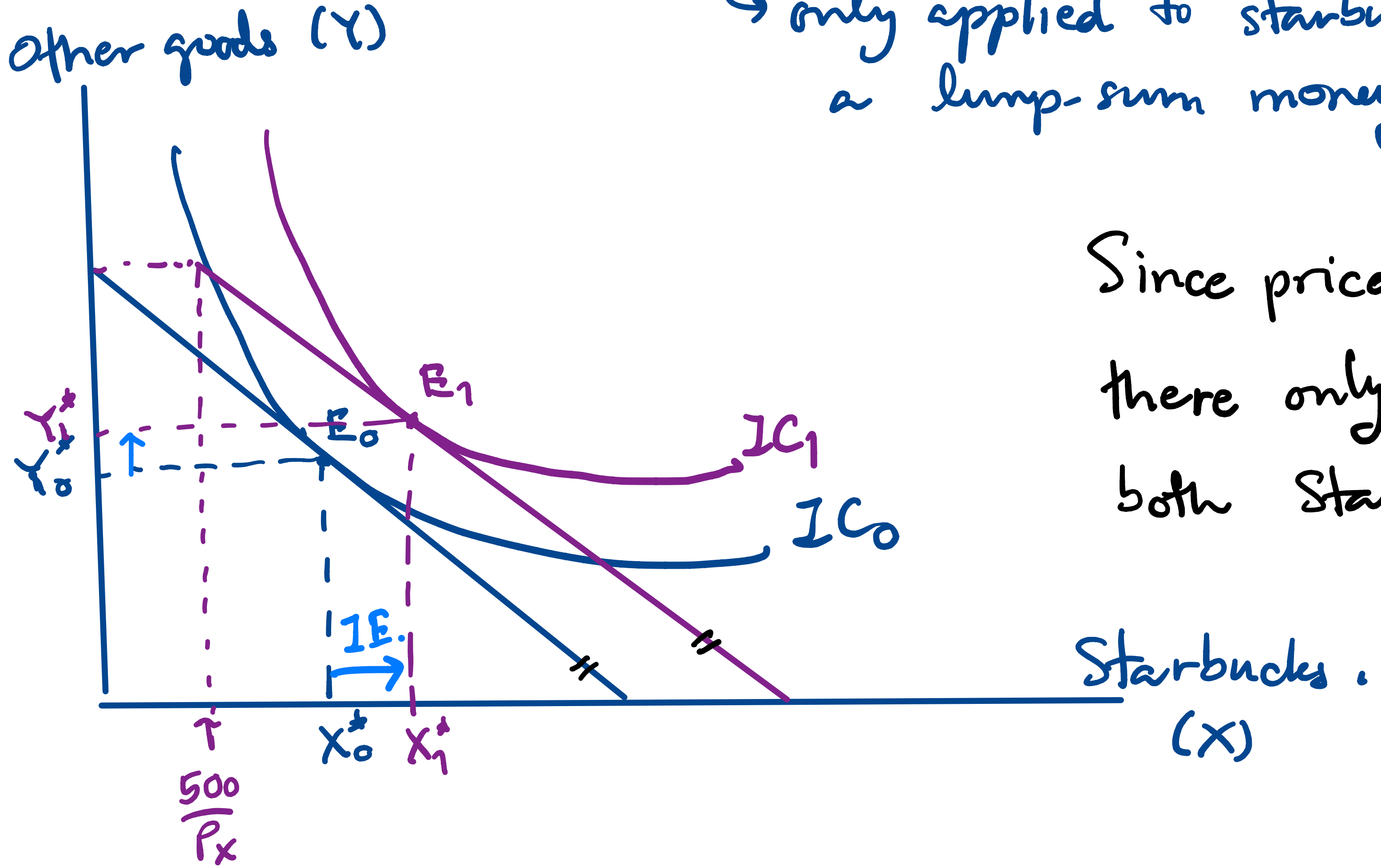
Total effect $= \underbrace{X_1^* - X_0^*}_{+}$

Effect of subsidy is SAME as the effect of price reduction.

Application 3: Voucher

Ex: Suppose you receive a ฿500 starbucks coupon. Everything else constant.

↳ only applied to starbucks only, and it is a lump-sum money (ie. price does not change).



Since price ratio does not change, there only is income effect on both Starbucks and other goods.

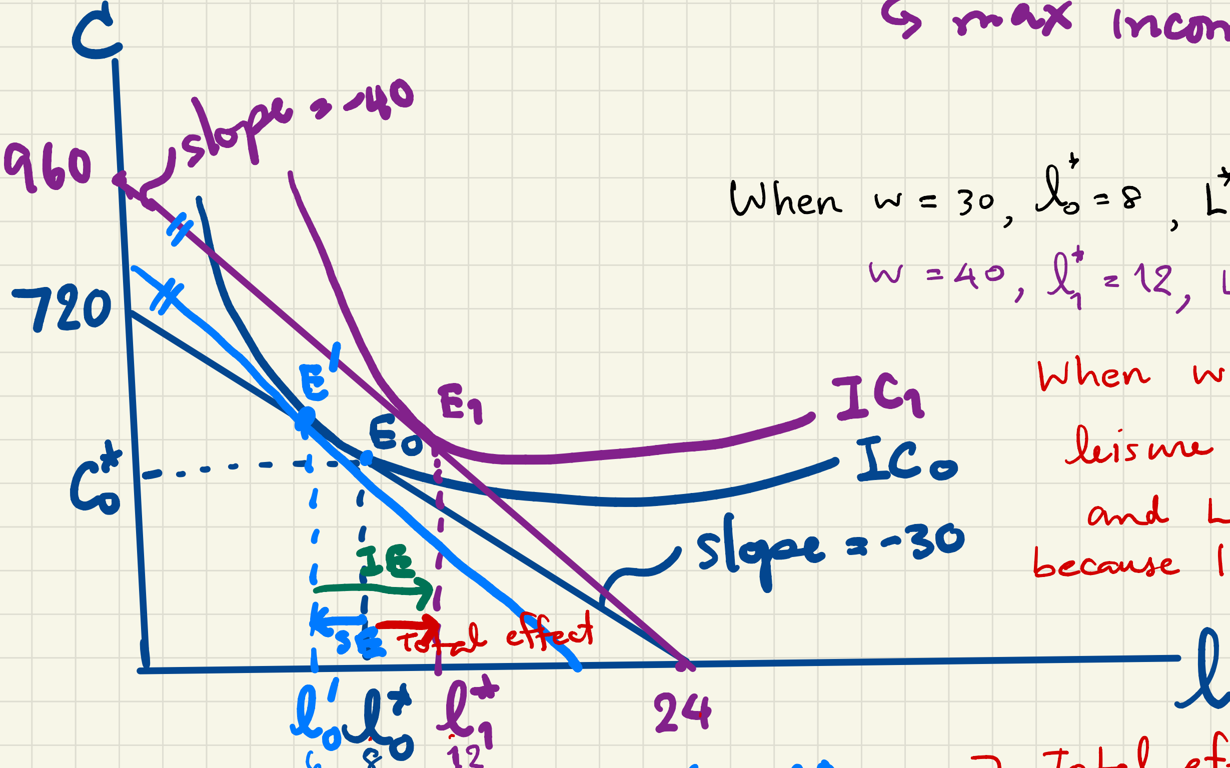
What if wage ↑ from \$30 to \$40/hr?

w $\quad$ w'

→ max income $= w' \times 24$

$= 960$

Case 1 - IE > SE



When $w = 30$, $l_0^* = 8$, $L_0^* = 16$ hrs.

$w = 40$, $l_1^* = 12$, $L_0^* = 12$ hrs.

When $w \uparrow$,

leisure ↑

and Labor ↓.

because $|IE| > |SE|$
$\quad (+) \quad (-)$

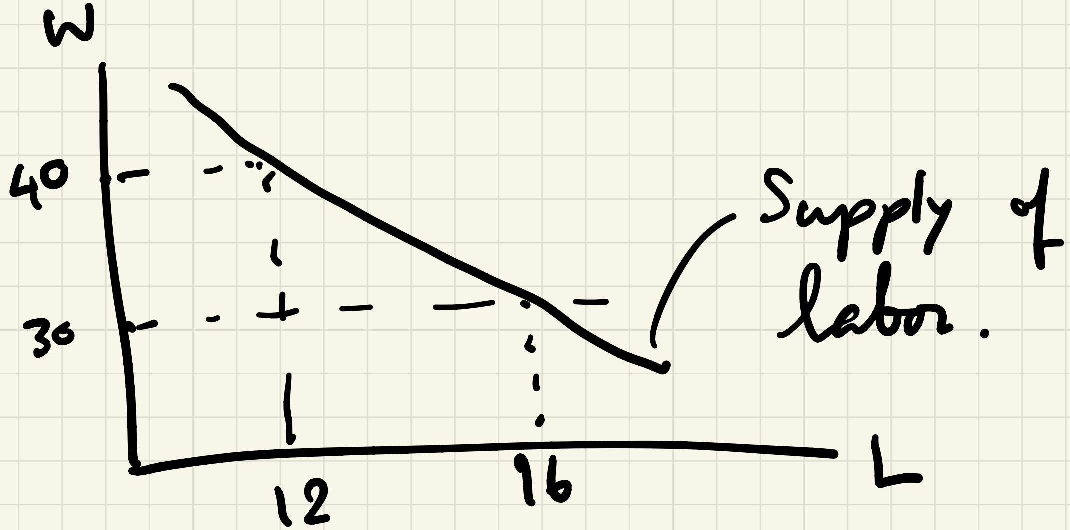
S.E. on leisure $= l_0' - l_0^* < 0$

I.E. on leisure $= l_1^* - l_0' > 0$

Total effect:

$l_1^* - l_0^* > 0$

W↑ , L↓



Supply is downward-sloping because IE > SE.

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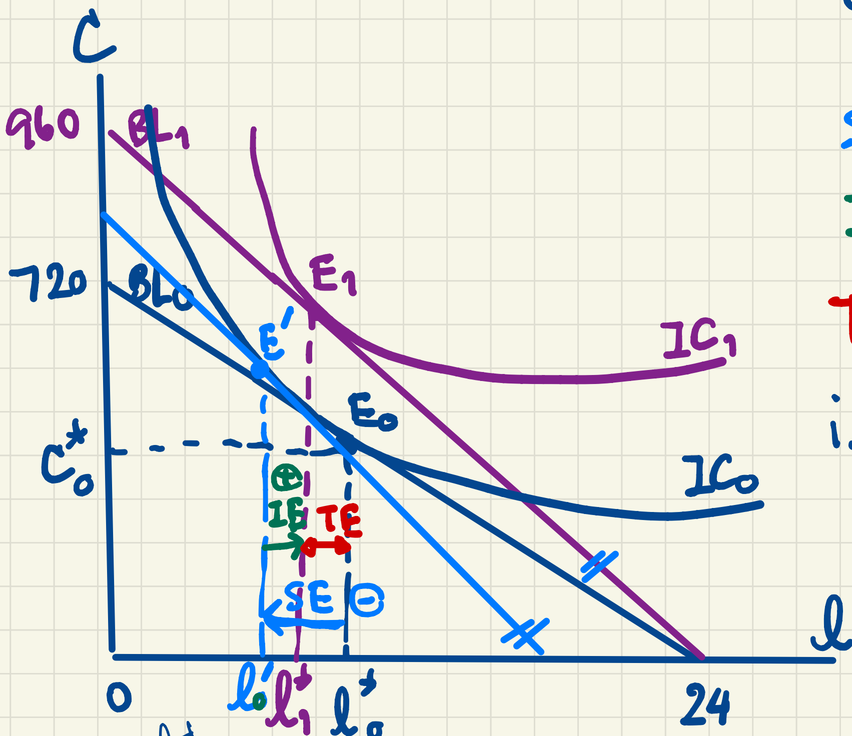
So, higher wage rate results in a reduction in Labor hours and more leisure

Case 2 - IE < SE



$l^* = 6, 8, 12$

$L^* = 18, 16, 12$

Let $w_0 = 30$.

Wage increases to 40, everything else constant.

S.E. on l = $l_0' - l_0^* < 0$

I.E. on l = $l_1^* - l_0' > 0$

Total effect = $l_1^* - l_0^* < 0$

i.e. $L_1^* = 24 - l_1^* > L_0^* = 24 - l_0^*$

∴ When IE (+) > SE (−), a rise in wage rate will lower leisure and increase labor time (L).