

Assignment 2 Simultaneous Equations Model

Due: 1/9/2020

Demand and Supply Equations

$$\ln S_t = \beta_{10} + \beta_{11} \ln P_{Dt} + \beta_{12} \ln P_{X2t} + \beta_{13} \ln P_{X3t} + \beta_{14} \ln P_{X4t} + \varepsilon_{1t} \quad (1)$$

$$\ln D_t = \beta_{20} + \beta_{21} \ln P_{Dt} + \beta_{22} \ln GDP_t + \varepsilon_{2t} \quad (2)$$

where: S_t = Domestic Supply at time t

D_t = Domestic Demand at time t

P_{Dt} = Domestic Price at time $t = P_{Mt} + T_t$

T_t = Tariff at time t

P_{X2t} = Price of Input 2 at time t

P_{X3t} = Price of Input 3 at time t

P_{X4t} = Price of Input 4 at time t

GDP_t = Gross Domestic Product (Representing Income) at time t

Eq. 1: $S_t = D_t$

Endogenous variables in this system include S_t , D_t , and P_{Dt}

Exogenous variables in this system include P_{X2t} , P_{X3t} , P_{X4t} , and GDP_t

From Data Assignment 2.dta:

1. State reduce form model of these system models.
2. Estimate reduce form model using OLS and prediction of the endogenous variables.
3. Estimate structural form using predicted endogenous variables as independent variables in the structural form model.
4. Estimate the structural models of these system equations using OLS, 2SLS, 3SLS, and 4SLS. Concerning on the asymptotic property, which model is the most appropriated model? Why?
5. What do β_{21} and β_{22} mean?

1. Equil condition: $S_t = D_t$

$$\ln P_{Dt} = \frac{\beta_{20} + \beta_{22} \ln GDP_t + \varepsilon_{2t} - \ln D_t}{\beta_{21}}$$

$$(1.1) \quad \ln S_t = \beta_{10} + \beta_{11} \left[\frac{\beta_{20} + \beta_{22} \ln GDP_t + \varepsilon_{2t} - \ln D_t}{\beta_{21}} \right] + \beta_{12} \ln P_{x2t} + \beta_{13} \ln P_{x3t} + \beta_{14} \ln P_{x4t} + \varepsilon_{1t}$$

$$\left(1 + \frac{\beta_{11}}{\beta_{21}}\right) \ln S_t = \beta_{10} + \frac{\beta_{11}}{\beta_{21}} \beta_{20} + \frac{\beta_{11}}{\beta_{21}} \beta_{22} \ln GDP_t + \beta_{12} \ln P_{x2t} + \beta_{13} \ln P_{x3t} + \beta_{14} \ln P_{x4t} + \varepsilon_{1t} + \frac{\beta_{11}}{\beta_{21}} \varepsilon_{2t}$$

$$\ln S_t = \frac{\beta_{21} \beta_{10}}{\beta_{11} + \beta_{21}} + \frac{\beta_{11} \beta_{20}}{\beta_{11} + \beta_{21}} + \frac{\beta_{11} \beta_{22}}{\beta_{11} + \beta_{21}} \ln GDP_t + \frac{\beta_{21} \beta_{12}}{\beta_{11} + \beta_{21}} \ln P_{x2t} + \frac{\beta_{21} \beta_{13}}{\beta_{11} + \beta_{21}} \ln P_{x3t} + \frac{\beta_{21} \beta_{14}}{\beta_{11} + \beta_{21}} \ln P_{x4t} + \frac{\beta_{21}}{\beta_{11} + \beta_{21}} \varepsilon_{1t} + \frac{\beta_{11}}{\beta_{11} + \beta_{21}} \varepsilon_{2t}$$

$$\ln S_t = \underbrace{\frac{\beta_{11} \beta_{20} + \beta_{10} \beta_{21}}{\beta_{11} + \beta_{21}}}_{\pi_{10}} + \underbrace{\frac{\beta_{21} \beta_{12}}{\beta_{11} + \beta_{21}}}_{\pi_{11}} \ln P_{x2t} + \underbrace{\frac{\beta_{21} \beta_{13}}{\beta_{11} + \beta_{21}}}_{\pi_{12}} \ln P_{x3t} + \underbrace{\frac{\beta_{21} \beta_{14}}{\beta_{11} + \beta_{21}}}_{\pi_{13}} \ln P_{x4t} + \underbrace{\frac{\beta_{11} \beta_{22}}{\beta_{11} + \beta_{21}}}_{\pi_{14}} \ln GDP_t + \underbrace{\frac{\beta_{21} \varepsilon_{1t} + \beta_{11} \varepsilon_{2t}}{\beta_{11} + \beta_{21}}}_{\omega_{1t}}$$

$$\ln S_t = \pi_{10} + \pi_{11} \ln P_{x2t} + \pi_{12} \ln P_{x3t} + \pi_{13} \ln P_{x4t} + \pi_{14} \ln GDP_t + \omega_{1t} \quad - (1)$$



$$(1.2) \quad \ln P_{Dt} = \frac{\beta_{10} + \beta_{12} \ln P_{x2t} + \beta_{13} \ln P_{x3t} + \beta_{14} \ln P_{x4t} + \varepsilon_{1t} - \ln S_t}{\beta_{11}}$$

$$\ln D_t = \beta_{20} + \beta_{21} \left[\frac{\beta_{10} + \beta_{12} \ln P_{x2t} + \beta_{13} \ln P_{x3t} + \beta_{14} \ln P_{x4t} + \varepsilon_{1t} - \ln S_t}{\beta_{11}} \right] + \beta_{22} \ln GDP_t + \varepsilon_{2t}$$

$$\left(\frac{\beta_{11} + \beta_{21}}{\beta_{11}} \right) \ln D_t = \beta_{20} + \frac{\beta_{10} \beta_{21}}{\beta_{11}} + \frac{\beta_{12} \beta_{21}}{\beta_{11}} \ln P_{x2t} + \frac{\beta_{13} \beta_{21}}{\beta_{11}} \ln P_{x3t} + \frac{\beta_{14} \beta_{21}}{\beta_{11}} \ln P_{x4t} + \beta_{22} \ln GDP_t + \frac{\beta_{21}}{\beta_{11}} \varepsilon_{1t} + \varepsilon_{2t}$$

$$\ln D_t = \underbrace{\frac{\beta_{11} \beta_{20} + \beta_{10} \beta_{21}}{\beta_{11} + \beta_{21}}}_{\pi_{20}} + \underbrace{\frac{\beta_{12} \beta_{21}}{\beta_{11} + \beta_{21}}}_{\pi_{21}} \ln P_{x2t} + \underbrace{\frac{\beta_{13} \beta_{21}}{\beta_{11} + \beta_{21}}}_{\pi_{22}} \ln P_{x3t} + \underbrace{\frac{\beta_{14} \beta_{21}}{\beta_{11} + \beta_{21}}}_{\pi_{23}} \ln P_{x4t} + \underbrace{\frac{\beta_{11} \beta_{22}}{\beta_{11} + \beta_{21}}}_{\pi_{24}} \ln GDP_t + \underbrace{\frac{\beta_{21} \varepsilon_{1t} + \beta_{11} \varepsilon_{2t}}{\beta_{11} + \beta_{21}}}_{\omega_{2t}}$$

$$\ln D_t = \pi_{20} + \pi_{21} \ln P_{x2t} + \pi_{22} \ln P_{x3t} + \pi_{23} \ln P_{x4t} + \pi_{24} \ln GDP_t + \omega_{2t} \quad - (2)$$

2.

```
. tsset obs
      time variable:  obs, 1986 to 2007
              delta:  1 unit
```

```
. g pd=pm+t
```

```
. g lnst=ln(st)
```

```
. g lndt=ln(dt)
```

```
. g lnpd=ln(pd)
```

```
. g lnp2=ln(px2)
```

```
. g lnp3=ln(px3)
```

```
. g lnp4=ln(px4)
```

```
. g lngdp=ln(gdp)
```

```
. reg lnst lnp2 lnp3 lnp4 lngdp
```

Source	SS	df	MS	Number of obs	=	22
-----+-----				F(4, 17)	=	37.32
Model	4.64569724	4	1.16142431	Prob > F	=	0.0000
Residual	.529104674	17	.031123804	R-squared	=	0.8978
-----+-----				Adj R-squared	=	0.8737
Total	5.17480192	21	.246419139	Root MSE	=	.17642

lnst	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]
-----+-----					
lnp2	-.4503744	.1515961	-2.97	0.009	-.7702142 -.1305347
lnp3	-.9242052	.2783356	-3.32	0.004	-1.511442 -.3369685
lnp4	-.3883793	.4222332	-0.92	0.371	-1.279214 .5024549
lngdp	.3438812	.1913463	1.80	0.090	-.0598242 .7475865
_cons	24.65741	5.309757	4.64	0.000	13.4548 35.86002
-----+-----					

```
. predict lnsthat
(option xb assumed; fitted values)
```

```
. reg lndt lnp2 lnp3 lnp4 lngdp
```

Source	SS	df	MS	Number of obs	=	22
-----+-----				F(4, 17)	=	26.43
Model	3.4026552	4	.850663799	Prob > F	=	0.0000
Residual	.54721789	17	.032189288	R-squared	=	0.8615
-----+-----				Adj R-squared	=	0.8289
Total	3.94987309	21	.188089195	Root MSE	=	.17941

lndt	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]
-----+-----					

lnpx2		-.4887365	.1541691	-3.17	0.006	-.8140049	-.1634682
lnpx3		-.7243134	.2830597	-2.56	0.020	-1.321517	-.1271097
lnpx4		-.577921	.4293997	-1.35	0.196	-1.483875	.3280333
lngdp		.1265855	.194594	0.65	0.524	-.2839719	.5371429
_cons		27.18614	5.399879	5.03	0.000	15.79339	38.57889

```
. predict lndthat
(option xb assumed; fitted values)
```

```
. reg lnpg lnpx2 lnpx3 lnpx4 lngdp
```

Source	SS	df	MS	Number of obs	=	22
-----+-----				F(4, 17)	=	6.76
Model	.17707359	4	.044268398	Prob > F	=	0.0019
Residual	.111247189	17	.006543952	R-squared	=	0.6142
-----+-----				Adj R-squared	=	0.5234
Total	.288320779	21	.013729561	Root MSE	=	.08089

lnpd	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
lnpx2	.1318015	.0695123	1.90	0.075	-.0148567	.2784596
lnpx3	.0939842	.127627	0.74	0.472	-.1752851	.3632535
lnpx4	.4939641	.1936093	2.55	0.021	.0854842	.9024439
lngdp	.1632779	.0877392	1.86	0.080	-.0218357	.3483914
_cons	2.87652	2.434717	1.18	0.254	-2.260283	8.013322

```
. predict lnpdhat
(option xb assumed; fitted values)
```

3.

```
. reg lnst lnpdhat lnpx2 lnpx3 lnpx4
```

Source	SS	df	MS	Number of obs	=	22
				F(4, 17)	=	37.32
Model	4.64569773	4	1.16142443	Prob > F	=	0.0000
Residual	.529104183	17	.031123775	R-squared	=	0.8978
				Adj R-squared	=	0.8737
Total	5.17480192	21	.246419139	Root MSE	=	.17642

Inst	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
lnpdhat	2.106112	1.171903	1.80	0.090	-.3663879	4.578612
lnpx2	-.727963	.1840856	-3.95	0.001	-1.11635	-.3395762
lnpx3	-1.122146	.2824139	-3.97	0.001	-1.717988	-.5263052
lnpx4	-1.428722	.4751381	-3.01	0.008	-2.431176	-.4262679
_cons	18.59912	8.546622	2.18	0.044	.5673274	36.63092

```
. reg lndt lnpdhat lngdp
```

Source	SS	df	MS	Number of obs	=	22
Model	3.26129847	2	1.63064924	F(2, 19)	=	44.99
Residual	.688574614	19	.036240769	Prob > F	=	0.0000
Total	3.94987309	21	.188089195	R-squared	=	0.8257
				Adj R-squared	=	0.8073
				Root MSE	=	.19037

lnpdhat	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
lnpdhat	-2.574157	.5697943	-4.52	0.000	-3.76675	-1.381563
lngdp	.5212927	.1344816	3.88	0.001	.2398194	.802766
_cons	35.93498	7.189835	5.00	0.000	20.88648	50.98347

4.

```
. reg3 (lnst lnpd lnpx2 lnpx3 lnpx4) (lndt lnpd lngdp), ols
```

Multivariate regression

Equation	Obs	Parms	RMSE	"R-sq"	F-Stat	P
lnst	22	4	.1652258	0.9103	43.14	0.0000
lndt	22	2	.1391259	0.9069	92.53	0.0000

	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
lnst						
lnpd	-1.111835	.4515147	-2.46	0.019	-2.027549	-.1961207
lnpx2	-.4189546	.1431634	-2.93	0.006	-.7093034	-.1286059
lnpx3	-.9424196	.2585266	-3.65	0.001	-1.466736	-.4181034
lnpx4	-.521346	.3441643	-1.51	0.139	-1.219344	.1766516
_cons	41.4946	3.661911	11.33	0.000	34.0679	48.9213
lndt						
lnpd	-2.181329	.2946999	-7.40	0.000	-2.779008	-1.58365
lngdp	.5776586	.0887536	6.51	0.000	.397658	.7576593
_cons	31.03578	3.761201	8.25	0.000	23.40771	38.66385

```
. reg3 (lnst lnpd lnpx2 lnpx3 lnpx4) (lndt lnpd lngdp), 2sls nodfk inst(lnpx2 lnpx3 lnpx4 lngdp)
```

Two-stage least-squares regression

Equation	Obs	Parms	RMSE	"R-sq"	F-Stat	P
lnst	22	4	.329951	0.6424	13.81	0.0000
lndt	22	2	.1454858	0.8982	89.20	0.0000

	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
lnst						
lnpd	2.10611	1.926677	1.09	0.282	-1.801371	6.013591
lnpx2	-.7279628	.3026471	-2.41	0.021	-1.34176	-.114166
lnpx3	-1.122146	.464304	-2.42	0.021	-2.063798	-.180494
lnpx4	-1.428722	.7811544	-1.83	0.076	-3.012977	.1555325
_cons	18.59914	14.05113	1.32	0.194	-9.897873	47.09616
lndt						
lnpd	-2.574157	.4046743	-6.36	0.000	-3.394875	-1.75344
lngdp	.5212921	.0955104	5.46	0.000	.327588	.7149961
_cons	35.93499	5.106302	7.04	0.000	25.57893	46.29105

Endogenous variables: lnst lnpd lndt

Exogenous variables: lnpx2 lnpx3 lnpx4 lngdp

```
. reg3 (lnst lnpd lnpx2 lnpx3 lnpx4) (lndt lnpd lngdp), 3sls inst(lnpx2 lnpx3 lnpx4 lngdp)
```

Three-stage least-squares regression

Equation	Obs	Parms	RMSE	"R-sq"	chi2	P
lnst	22	4	.2963642	0.6266	57.47	0.0000
lndt	22	2	.135203	0.8982	178.41	0.0000

	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]	
lnst						
lnpd	2.171576	1.926095	1.13	0.260	-1.603501	5.946652
lnpx2	-.7990055	.2985983	-2.68	0.007	-1.384247	-.2137635
lnpx3	-1.329743	.4560002	-2.92	0.004	-2.223487	-.4359989
lnpx4	-1.171403	.775654	-1.51	0.131	-2.691657	.348851
_cons	17.84948	14.04122	1.27	0.204	-9.670808	45.36976
lndt						
lnpd	-2.574157	.4046743	-6.36	0.000	-3.367304	-1.78101
lngdp	.5212921	.0955104	5.46	0.000	.3340951	.708489
_cons	35.93499	5.106302	7.04	0.000	25.92682	45.94316

Endogenous variables: lnst lnpd lndt

Exogenous variables: lnpx2 lnpx3 lnpx4 lngdp

```
. reg3 (lnst lnpd lnpx2 lnpx3 lnpx4) (lndt lnpd lngdp), 3sls ireg3 inst(lnpx2 lnpx3 lnpx4 lngdp)
```

Iteration 1: tolerance = .1059484

Iteration 2: tolerance = .04569793

```
Iteration 3:  tolerance = .01846611
Iteration 4:  tolerance = .00725496
Iteration 5:  tolerance = .00281814
Iteration 6:  tolerance = .00108981
Iteration 7:  tolerance = .00042072
Iteration 8:  tolerance = .00016231
Iteration 9:  tolerance = .0000626
Iteration 10: tolerance = .00002414
Iteration 11: tolerance = 9.310e-06
Iteration 12: tolerance = 3.590e-06
Iteration 13: tolerance = 1.384e-06
Iteration 14: tolerance = 5.339e-07
```

Three-stage least-squares regression, iterated

Equation	Obs	Parms	RMSE	"R-sq"	chi2	P
lnst	22	4	.3022006	0.6117	54.83	0.0000
lndt	22	2	.135203	0.8982	178.41	0.0000

		Coef.	Std. Err.	z	P> z	[95% Conf. Interval]	
lnst							
	lnpd	2.212666	2.005956	1.10	0.270	-1.718936	6.144268
	lnpx2	-.8435967	.3049354	-2.77	0.006	-1.441259	-.2459342
	lnpx3	-1.460044	.4623671	-3.16	0.002	-2.366267	-.5538216
	lnpx4	-1.009892	.7998393	-1.26	0.207	-2.577548	.557764
	_cons	17.37893	14.61488	1.19	0.234	-11.26571	46.02357
lndt							
	lnpd	-2.574157	.4046743	-6.36	0.000	-3.367304	-1.78101
	lngdp	.5212921	.0955104	5.46	0.000	.3340951	.708489
	_cons	35.93499	5.106302	7.04	0.000	25.92682	45.94316

Endogenous variables: lnst lnpd lndt

Exogenous variables: lnpx2 lnpx3 lnpx4 lngdp

Concerning on asymptotic property, the 3SLS model is the most appropriated model, because the 3SLS model has the lowest standard error comparing to OLS, 2SLS, and I3SLS.

5.

β_{21} is the price elasticity of demand

β_{22} is the income elasticity of demand