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EE 325 Section 1 (Aj.Wanwiphang) Homework Assignment 1

Due date: 31 January 2020 before 11pm

*** Please submit this assignment on Moodle. For those who work on paper, please scan or submit the pictures of your work.***

1. Find the answers following questions (please also show your calculation)

   a. $\sum_{i=1}^{5}(a+bx_i) = \sum_{i=1}^{5} a + \sum_{i=1}^{5} bx_i = 5a + b\sum_{i=1}^{5} x_i$

   b. $\sum_{y=0}^{5} f(x+y) = f(x+0) + f(x+1) + f(x+2) + f(x+3) + f(x+4) + f(x+5)$

   c. $\sum_{i=1}^{10} i^2 = 1^2+2^2+3^2+4^2+5^2+6^2+7^2+8^2+9^2+10^2 = 385$

   d. $\sum_{x=1}^{2}\sum_{y=2}^{3}(2x+y) = \sum_{x=1}^{2} (2x+2)+(2x+3) = [(2)(1)+2] + [(2)(2)+3]$
   $= 11$

2. Given X is discrete random variable. The probability distribution function (PDF) of this variable is shown in the table

X	-2	-1	0	1	2	3	4
$f(x)$	0.5b 0.0625	b 0.125	2.25b 0.28125	2b 0.25	1.5b 0.1875	0.5b 0.0625	0.25b 0.03125

** when b is constant number

   a. Find the value of b

   $\sum f(x) = 1$

   $0.5b + b + 2.25b + 2b + 1.5b + 0.5b + 0.25b = 1 \rightarrow b = \frac{1}{8}$ or 0.125

   b. Find the answer for $P(X \le 2)$

   $0.0625 + 0.125 + 0.28125 + 0.25 + 0.1875$

   $= 0.90625$

   c. Find the answer for $P(-2 \le X \le 3)$

   $1 - P(X=4) = 1 - 0.03125$

   $= 0.96875$

   d. Find the answer for $P(X \ge 1)$

   $0.25 + 0.1875 + 0.0625 + 0.03125$

   $= 0.53125$

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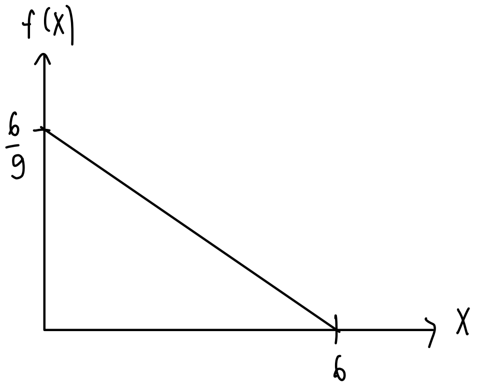
3. Given X is continuous random variable. The probability distribution function (PDF) of this variable is

$$f(x) = -\frac{1}{9}x + \frac{6}{9}, 0 \le x \le 3$$

a. Plot graph for $f(x)$



b. Find the answer for $P(1 \le X \le 3)$

$$\int_1^3 -\frac{1}{9}x + \frac{6}{9}\,dx = \left[\frac{-1}{9(2)}x^2 + \frac{6x}{9}\right]_1^3 = \left(\frac{-\cancel{9}}{\cancel{18}\,2} + \frac{\cancel{18}^{\,2}}{\cancel{9}}\right) - \left(\frac{-1}{18} + \frac{\cancel{6}^{\,2}}{\cancel{9}_{\,3}}\right)$$

$$= \frac{3}{2} - \frac{11}{18}$$

$$= \frac{8}{9}$$

c. Find the answer for $P(X \ge 2)$

$$\int_2^3 -\frac{1}{9}x + \frac{6}{9}\,dx = \left[-\frac{1}{18}x^2 + \frac{6x}{9}\right]_2^3 = \left(-\frac{1}{2} + 2\right) - \left(-\frac{4}{18} + \frac{12}{9}\right)$$

$$= \frac{3}{2} - \frac{20}{18}$$

$$= \frac{7}{18}$$

d. Find the expected value of X

$$E(X) = \int_0^3 x f(x)\,dx$$

$$= \int_0^3 -\frac{1}{9}x^2 + \frac{6}{9}x\,dx$$

$$= \left[\frac{-1}{27}x^3 + \frac{6}{18}x^2\right]_0^3$$

$$= -\frac{27}{27} + \frac{54}{18}$$

$$= 2$$

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4. Let random variable X be the outcome of throwing one dice and random variable Y be the outcome of tossing one coin. Coin has two sided that has valued 1 and 0.

   a. Construct the joint probability distribution function (PDF) table of X and Y

x/y	1	2	3	4	5	6	
0	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{6}{12}$
1	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{6}{12}$
	$\frac{2}{12}$	$\frac{2}{12}$	$\frac{2}{12}$	$\frac{2}{12}$	$\frac{2}{12}$	$\frac{2}{12}$	

   b. Find the marginal probability distribution function (PDF) of X

$\frac{2}{12}$

   c. Find the marginal probability distribution function (PDF) of Y

$\frac{6}{12}$

   d. Find the conditional probability distribution function (PDF) of X given Y is equal to 1

$$p(x|y=1) = \frac{p(x=x, y=1)}{p(y=1)} = \frac{p(x=1, y=1)}{p(y=1)} + \frac{p(x=2, y=1)}{p(y=1)} + \ldots \frac{p(x=6, y=1)}{p(y=1)}$$
$$= \left(\frac{1}{12} \times 2\right) 6 = 1$$

   e. Find the expected value of X given Y is equal to 1

$$E(X|Y=1) = \sum x f(x) = \frac{(x)(p(x=x, y=1))}{p(y=1)} = \frac{(1)(p(x=1, y=1))}{p(y=1)} + \frac{(2)(p(x=2, y=1))}{p(y=1)} + \ldots + \frac{(6)(p(x=6, y=1))}{p(y=1)}$$
$$= \frac{1}{6} + \frac{1}{3} + \frac{1}{2} + \frac{2}{3} + \frac{5}{6} + 1 = \frac{21}{6}$$

   f. Find the variance of X given Y is equal to 1

$$var(x|y=1) = E(x^2) - [E(x)]^2$$
$$= \sum x^2 f(x) - \left(\frac{21}{6}\right)^2$$
$$= (1)^2\left(\frac{1}{12}\times 2\right) + (2)^2\left(\frac{1}{12}\times 2\right) + (3)^2\left(\frac{1}{12}\times 2\right) + (4)^2\left(\frac{1}{12}\times 2\right) + (5^2)\left(\frac{1}{12}\times 2\right) + (6^2)\left(\frac{1}{12}\times 2\right) - \frac{441}{36}$$
$$= \left(\frac{1}{12}\times 2\right)(91) - \frac{441}{36} = \frac{546}{36} - \frac{441}{36} = \frac{35}{12}$$

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5. If X_1, X_2, X_3 is a random sample from a population with mean u and variance σ^2. X_1, X_2, X_3 are not independent dependent

$$Cov(X_1, X_2) = Cov(X_1, X_3) = Cov(X_2, X_3) = \frac{1}{4}\sigma^2$$

$\bar{X}$ is estimator used to estimate mean value. $\bar{X} = \frac{1}{3}(X_1 + X_2 + X_3)$

Find $E(\bar{X})$ and $\mathrm{var}(\bar{X})$

$$E(\bar{x}) = E\left(\frac{1}{3}\sum_{i=1}^{3} x_i\right)$$

$$= E\left[\left(\frac{1}{\cancel{3}}\right)(\cancel{3}x_i)\right]$$

$$= E(x_i) = \mu$$

$$var(\bar{x}) = var\left(\frac{1}{3}x_1\right) + var\left(\frac{1}{3}x_2\right) + var\left(\frac{1}{3}x_3\right) - cov$$

$$= \frac{1}{9}\sigma + \frac{1}{9}\sigma + \frac{1}{9}\sigma - \frac{1}{4}\sigma^2$$

$$= \frac{1}{3}\sigma - \frac{1}{4}\sigma^2$$

cov = 0

6. Given X_1, X_2, X_3, X_4 are independent identically distributed random variables from population with mean u and variance σ^2. $\bar{X}$ is estimator used to estimate mean value. $\bar{X} = \frac{1}{4}(X_1 + X_2 + X_3 + X_4)$

a. Find $E(\bar{X})$ and $\mathrm{var}(\bar{X})$ in term of μ and σ

$$E(\bar{x}) = E\left(\frac{1}{4}\sum_{i=1}^{4} x_i\right)$$

$$= E\left[\left(\frac{1}{\cancel{4}}\right)(\cancel{4}x_i)\right]$$

$$= E(x_i) = \mu$$

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$$var(\bar{x}) = var\frac{1}{4}\left(\sum_{i=1}^{4} x_i\right)$$

$$= var\left[\left(\frac{1}{\cancel{4}}\right)\left(\cancel{4}x_i\right)\right]$$

$$= var(x_i) = \sigma$$

b. Given $\tilde{X} = \frac{1}{8}X_1 + \frac{1}{4}X_2 + \frac{1}{8}X_3 + \frac{1}{2}X_4$ is another estimator of μ. Show that $\tilde{X}$ is an unbiased estimator of u check var $\neq \sigma$; unbiase

$$E(\tilde{x}) = E\left(\frac{1}{8}x_1 + \frac{1}{4}x_2 + \frac{1}{8}x_3 + \frac{1}{2}x_4\right)$$

$$= E\left(\frac{1}{8}x_1\right) + E\left(\frac{1}{4}x_2\right) + E\left(\frac{1}{8}x_3\right) + E\left(\frac{1}{2}x_4\right)$$

$$= \frac{1}{8}E(x_1) + \frac{1}{4}E(x_2) + \frac{1}{8}E(x_3) + \frac{1}{2}E(x_4)$$

$$= \frac{1}{8}\mu + \frac{1}{4}\mu + \frac{1}{8}\mu + \frac{1}{2}\mu$$

$$= \mu$$

$\tilde{X}$ is an unbiased estimator of μ

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c. Between $\bar{X}$ and $\tilde{X}$, which one is the better estimator for μ? Why?

$$var(\tilde{x}) = var\left(\frac{1}{8}x_1 + \frac{1}{4}x_2 + \frac{1}{8}x_3 + \frac{1}{2}x_4\right)$$

$$= var\left(\frac{1}{8}x_1\right) + var\left(\frac{1}{4}x_2\right) + var\left(\frac{1}{8}x_3\right) + var\left(\frac{1}{2}x_4\right)$$

$$= \frac{1}{64}\sigma + \frac{1}{16}\sigma + \frac{1}{64}\sigma + \frac{1}{4}\sigma$$

$$= \frac{38}{64}\sigma$$

$\bar{X}$ is the better estimator for μ