Exercise: Solving Inequality

1 Part I

1. Let a, b, and c be real numbers. Show that if $a \geq b$ and $c < 0$, then $a^2 + b^2 + 5a - 5b - 4c > 0$.

2. Let x, y be real numbers. Suppose that $x < 0$ and $y > 3$. Determine whether each of the following inequalities is true or not. Explain your answer.

   (a) $3x - xy > 0$.

   (b) $\frac{3}{x} - \frac{y}{x} + x^2 + y^3 > 18$

3. Find the solution set for each of following inequalities.

   (a)
   $$\frac{2x}{1-x} \geq \frac{1-x}{2x}$$

   (b)
   $$\frac{16x^4 - 81}{6x^2 + x - 12} < 0$$

   (c)
   $$\frac{x^4 - 2x^2 - 8}{2x + 1} \geq 0$$

   (d)
   $$\frac{x^4 + x^3 + 2x^2 + 2x}{x - 1} \leq 0$$

   (e)
   $$\left(\frac{x}{x-3} - 2\right)\left(\frac{e^x}{\cos(x) + 2}\right) \geq 0$$

4. Let x and y be real numbers with $|x| < \frac{1}{2}$. Show that $|xy - x| < \frac{|y|+1}{2}$.

5. Let x be a real number with $|x| \leq 3$. Determine if the following inequality is true or not.
   $$|x^2 - 4| \geq 5|x + 2|$$
   Explain your answer.

6. Let x and y be real numbers with $x > 1$ and $y < -2$. Determine if the following inequality is true or not.
   $$1 + \frac{|y|}{x} > \frac{4 - y}{2|x|}$$
   Explain your answer.

7. Find the solution set for each of following inequalities.

(a) $1 < |x-1| \leq 6$

(b)
$$\frac{(|x|+5)(x-1)}{|x-2|+x^2+x+1} \geq 0$$

(c)
$$2x + \frac{1}{|x|} \geq 1$$

(d)
$$\frac{|2-x|}{x^3+3x^2+11x+18} \leq 0$$

(e)
$$\frac{x^2+1-|x-1|}{5-|x+3|} \leq 0$$

2 Part II

1. Let a and b be real numbers with $0 < b < a < 1$. Determine whether each of the following inequalities is true or not. Explain your answer.

   (a) $\frac{(b+1)^3}{|a|} < \frac{(a+1)^3}{b}$

   (b) $\frac{b}{a} < \frac{a^3-1}{b^3-1}$

2. Let x and y be real numbers with $y > x > 1$. Show that
$$y(y-2) > |x-1|^2 - 1.$$

3. Find the solution set for each of following inequalities.

   (a)
$$\frac{x^2-1}{x} < \frac{x+1}{2x}$$

   (b)
$$2 \leq \left|\frac{x-1}{x}\right| \leq 7$$

   (c)
$$\frac{x^2+|x|+1}{x^7+x^5+x^2+1} \leq 0$$

   (d)
$$\frac{|x+3|-2}{5} + \frac{1}{|x-1|+1} \leq 1$$

   (e)
$$\frac{|x-1|-x^2-1}{5-|x+3|} \geq 0$$

3 Part III

1. Let a_1, a_2, a_3, and a_4 be some constant real numbers such that

$$a_1 < a_2 < a_3 < a_4.$$

Determine the solution set for each of the following inequalities in terms of a_1, a_2, a_3, and a_4.

(a) $(x-a_1)(x-a_2)(x-a_3)(x-a_4) \geq 0$
(b) $(x-a_1)^2(x-a_2)(x-a_3)^2(x-a_4)^3 \geq 0$

2. Let a and b be real numbers with $|b| < |a| < 1$ and $a > 0$.

(a) Show that $a-1 \ < \ 0 \ < \ b+a \ < \ 2a \ < \ 1+a.$
(b) Show that

$$\frac{2a^2-2a}{b+a} > \frac{a^2-1}{2a}.$$

3. Find the solution set for each of following inequalities.

(a)
$$\frac{1}{x} < 8$$

(b)
$$\frac{9-x}{x-2} \geq \frac{2-x}{x-9}$$

(c)
$$\frac{|2x+5|-2}{3|x|-1} \leq 1$$

(d)
$$\frac{|x^2-x+1|}{|3x+1|} > 1$$

(e)
$$\frac{|x^2-2|+2}{x^2-3|x|+2} < 0$$

4. A math class counts the midterm exam scores as 1/3 of the grade and the final exam scores as 2/3 of the grade. Paul scored 48% on the midterm.

To get an A, Paul must have the total score between 90% and 100% inclusive;
to get a B, Paul must have the total score between 80% and 89% inclusive;
to get a C, Paul must have the total score between 70% and 79% inclusive.

Suppose that all scores with decimal digits get rounded up to the closest higher integers.

(a) What range of scores (in %) on the final exam would make Paul get a C?

(b) What is the highest grade that Paul could get for this class?

5. The weekly demand (the number bought by consumers) for the a product is given by the formula

$$d = 9000 - 60p$$

where p is the price each in dollars.

(a)What is the demand when the price is 30 each?

(b) In what price range will the demand be above 6000?

6. A store's revenue $R(x)$ (in Baht) on the sale of x cupcakes is determined by the formula $R(x) = 50x - x^2$. The cost $C(x)$(in Baht) for producing x cupcakes is given by the formula $C(x) = 2x + 400$. For what values of x is the store's profit positive? Note: profit = revenue - cost.

4 Additional Problems

1. Find the solution set for each of the following inequities.

   (a) $|x-1| < 7$
   (b) $3 < |x+2| < 4$
   (c) $|2x+3| \leq 2$
   (d) $|3x-1| \geq 1$
   (e) $\frac{|x+2|}{2} > |x|$
   (f) $|x+1| - |2x-1| < 0$
   (g) $|2-x| - x \leq 0$
   (h) $|x+1| > 3 - |x|$
   (i) $x^2 - 2 \geq \frac{1}{2}|x-1|$
   (j) $|x+3| \geq 2 - x$
   (k) $|2x^2 + x - 1| \geq 2$
   (l) $\frac{|x|}{2} + 3 > x^2$
   (m) $|x| > \frac{2}{|x+1|}$
   (n) $\frac{x^2}{x+1} < |x|$
   (o) $\frac{4}{|x|} \leq \frac{1}{|x|}$
   (p) $|x^2+1| < |x+1|$
   (q) $(x^2+1)(|x+2| - |x|) \geq 0$
   (r) $\frac{|1-x|}{x^3+2x^2+5x+4} \leq 0$

2. Find the solution set for each of the following inequities.

   (a) $|x-1| < 7$
   (b) $3 < |x+2| < 4$
   (c) $|2x+3| \leq 2$
   (d) $|3x-1| \geq 1$
   (e) $\frac{|x+2|}{2} > |x|$
   (f) $|x+1| - |2x-1| < 0$
   (g) $|2-x| - x \leq 0$
   (h) $|x+1| > 3 - |x|$
   (i) $x^2 - 2 \geq \frac{1}{2}|x-1|$
   (j) $|x+3| \geq 2 - x$
   (k) $|2x^2 + x - 1| \geq 2$
   (l) $\frac{|x|}{2} + 3 > x^2$
   (m) $|x| > \frac{2}{|x+1|}$

(n) $\frac{x^2}{x+1} < |x|$

(o) $\frac{4}{|x|} \leq \frac{1}{|x|}$

(p) $|x^2 + 1| < |x + 1|$

(q) $(x^2 + 1)(|x + 2| - |x|) \geq 0$

(r) $\frac{|1-x|}{x^3+2x^2+5x+4} \leq 0$