(2) $P_s = \pi_s m_s \cdot 1$

$$m_s = \frac{\delta u'(c_s^*)}{u'(c_0^*)} = \delta \frac{c_0^*}{c_s^*}$$

$$\therefore \quad P_s = \delta \pi_s \frac{c_0^*}{c_s^*} \qquad \#$$

(3) $$m_{01} = \frac{\delta u'(c_1)}{u'(c_0)} = \delta \left(\frac{c_1}{c_0}\right)^{\gamma-1}$$

$$\text{FOC} : \quad 1 - \pi_s \delta \left(\frac{c_s}{c_0}\right)^{\gamma-1} R_s = 0$$

$$\Downarrow$$

$$(\gamma-1)\pi_s \delta \left(\frac{c_s}{c_0}\right)^{\gamma-2} R_s \, d\left(\frac{c_s}{c_0}\right) + \pi_s \delta \left(\frac{c_s}{c_0}\right)^{\gamma-1} dR_s = 0$$

$$\mathcal{E} = \frac{R_s}{\frac{c_s}{c_0}} \cdot \frac{d\left(\frac{c_s}{c_0}\right)}{dR_s}$$

$$\frac{d\left(\frac{c_s}{c_0}\right)}{dR_s} = - \frac{\cancel{\pi_s \delta} \left(\frac{c_s}{c_0}\right)^{\cancel{\gamma-1}}}{(\gamma-1)\cancel{\pi_s \delta} \cancel{\left(\frac{c_s}{c_0}\right)^{\gamma-2}} R_s}$$

$$\frac{R_s}{\left(\frac{c_s}{c_0}\right)} \cdot \frac{d\left(\frac{c_s}{c_0}\right)}{dR_s} = -\frac{1}{\gamma-1} = \frac{1}{1-\gamma}$$

$$\#$$

(4)

(a) Let $\mathbb{P} = \begin{bmatrix} \frac{1}{1.05} \\ 6 \end{bmatrix}$, $\mathbb{X} = \begin{bmatrix} 1 & 10 \\ 1 & 5 \end{bmatrix}$

$$[P_1 \quad P_2] = \mathbb{P}'\mathbb{X}^{-1} = \begin{bmatrix} \frac{1}{1.05} & 6 \end{bmatrix} \begin{bmatrix} 1 & 10 \\ 1 & 5 \end{bmatrix}^{-1} = [\,0.2476 \quad 0.7048\,]$$

$\therefore$ risk neutral probabilities are $P_1 R_f = 0.26$ and $P_2 R_f = 0.74$

(b) $\pi_1 = \pi_2 = 0.5$

$$m_1 = \frac{P_1}{\pi_1} = 0.495$$

$$m_2 = \frac{P_2}{\pi_2} = 1.410$$

(6)

(a) $$0 = E[m(R_i - R_f)]$$

$$= E[(a + bR_m)(R_i - R_f)]$$

$$= a\,E[R_1] - aR_f + b\,E[R_m R_i] - bR_f E[R_m]$$

$$= a(E[R_1] - R_f) + b(E[R_m]E[R_i] + \mathrm{cov}[R_m R_i] - R_f E[R_m])$$

$$= (E(R_i) - R_f)(a + bE[R_m]) + b\,\mathrm{cov}(R_m R_i)$$

$$E(R_i) - R_f = -\frac{b\,\mathrm{cov}(R_m R_i)}{(a + bE[R_m])}$$

$$= - \frac{\operatorname{cov}(R_m R_i)}{\sigma_m^2} \cdot \frac{b\,\sigma_m^2}{a + bE(R_m)}$$

$$= -\beta_i \frac{b\,\sigma_m^2}{a + bE(R_m)}$$

$$\therefore \ \gamma = -\frac{b\,\sigma_m^2}{a + bE(R_m)} \quad ※$$

(b) risk-free : $\frac{1}{R_f} = E[a + bR_m] = a + bE(R_m) \rightarrow a = \frac{1}{R_f} - bE(R_m)$

market portfolio : $1 = E[(a+bR_m)R_m]$

$$= aE[R_m] + b\underbrace{E[R_m^2]}_{\sigma_m^2 + [E(R_m)]^2}$$

$$= \left[\frac{1}{R_f} - bE(R_m)\right]E(R_m) + b\left[\sigma_m^2 + [E(R_m)]^2\right]$$

$$= \frac{E(R_m)}{R_f} + b\,\sigma_m^2$$

$$\therefore \ b = \left[1 - \frac{E(R_m)}{R_f}\right]\frac{1}{\sigma_m^2}$$

$$= \frac{R_f - E(R_m)}{R_f\,\sigma_m^2} \quad ※$$