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EE325 Section 1 HW 2 Due Thursday February 20th (23:00 hr.),2020

Use 4 decimal places for numerical answers

1. In Table 1.a. X_i is total microeconomics exam point (total points are 100) and Y_i is GPA of each student.

$\overline{point} = \bar{x} = \frac{1}{8}\sum_{i=1}^{8} X_i = (\frac{1}{8}\times 63) + (\frac{1}{8}\times 72) + \ldots + (\frac{1}{8}\times 90)$
$= 77.625$

$\overline{GPA} = \bar{y} = 3.2125$

Table 1.a

Student	Y_i (GPA)	X_i (point)	1) $(X_i - \bar{X})$	2) $(Y_i - \bar{Y})$	1×2	$(X_i - \bar{X})^2$
1	2.8	63	-14.625	-0.4125	6.03	213.89
2	3.4	72	-5.625	0.1875	-1.05	31.64
3	3	78	0.375	-0.2125	-0.08	0.14
4	3.5	81	3.375	0.2875	0.97	11.39
5	3.6	87	9.375	0.3875	3.63	87.89
6	3.0	75	-2.625	-0.2125	0.56	6.89
7	2.7	75	-2.625	-0.5125	1.35	6.89
8	3.7	90	12.375	0.4875	6.03	153.14
Av.	3.2125	77.625		Sum:	17.44	511.87

1.1 Now consider the two-variable $Y_i = \beta_0 + \beta_1 X_i + u_i$, $u_i \sim NIID(0, \sigma^2)$. Use OLS to find the estimator of β_0 and β_1. (Note: *NIID* = Normally, Identically, and Independently Distributed).

$$\hat{Y} = \hat{\beta}_0 + \hat{\beta}_1 X_i$$

$$\widehat{GPA} = \hat{\beta}_0 + \hat{\beta}_1(\text{point})$$

$$\hat{\beta}_1 = \frac{\sum_{i=1}^{8}(Y_i - \bar{Y})(X_i - \bar{X})}{\sum_{i=1}^{8}(X_i - \bar{X})^2} = \frac{17.44}{511.87} \approx 0.034066 \ \#$$

$$\hat{\beta}_0 = \bar{Y} - \hat{\beta}_1\bar{X} = 3.2125 - (0.034066)(77.625) \approx 0.568132 \ \#$$

$$\widehat{GPA} = 0.5655 + 0.0341(\text{point})$$

1.2 For each observation i, find $\widehat{Y}_i$ and $\widehat{u}_i$. Show that $\sum_{i=0}^{N} \hat{u}_i = 0$.

The regression equation is $\hat{Y}_i = 0.5681 + 0.034 X_i$

when $i=1$; $\hat{Y}_1 = 0.5681 + 0.034(63) = 2.714$
$i=2$; $\hat{Y}_2 = 0.5681 + 0.034(72) = 3.021$
$i=3$; $\hat{Y}_3 = 0.5681 + 0.034(78) = 3.225$
$i=4$; $\hat{Y}_4 = 0.5681 + 0.034(81) = 3.327$
$i=5$; $\hat{Y}_5 = 0.5681 + 0.034(87) = 3.532$
$i=6$; $\hat{Y}_6 = 0.5681 + 0.034(75) = 3.123$
$i=7$; $\hat{Y}_7 = 0.5681 + 0.034(75) = 3.123$
$i=8$; $\hat{Y}_8 = 0.5681 + 0.034(90) = 3.634$

From $\hat{u}_i = Y_i - \hat{Y}_i$

when $i=1$; $\hat{u}_1 = 2.8 - 2.714 = 0.086$
$i=2$; $\hat{u}_2 = 3.4 - 3.021 = 0.379$
$i=3$; $\hat{u}_3 = 3.0 - 3.225 = -0.225$
$i=4$; $\hat{u}_4 = 3.5 - 3.327 = 0.173$
$i=5$; $\hat{u}_5 = 3.6 - 3.532 = 0.068$
$i=6$; $\hat{u}_6 = 3.0 - 3.123 = -0.123$
$i=7$; $\hat{u}_7 = 2.7 - 3.123 = -0.423$
$i=8$; $\hat{u}_8 = 3.7 - 3.634 = 0.066$

$$\sum_{i=1}^{n} \hat{u}_i = \sum_{i=1}^{n} Y_i - \hat{Y}_i = 0.086 + 0.379 + (-0.225) + 0.173 + 0.068 + (-0.123) + (-0.423) + 0.066 = 0 \ \#$$

1.3 Find $var(\widehat{u_i}), var(\widehat{\beta_0}), var(\widehat{\beta_1})$

$$\sigma^2 = Var(\hat{u}_i) = \frac{\sum_{i=1}^{n} \hat{u}_i^2}{n-2} = \frac{0.4347}{\cancel{8-2}\ 6} = 0.0725 \ \#$$

$$Var(\hat{\beta}_0) = \frac{\sum_{i=1}^{n} X_i^2}{n\sum_{i=1}^{n} x_i^2}\hat{\sigma}^2 = \frac{48717(0.0725)}{8(511.875)} = 0.8625 \ \#$$

$$Var(\hat{\beta}_1) = \frac{\hat{\sigma}^2}{\sum_{i=1}^{n} x_i^2} = \frac{0.0725}{511.875} = 0.00014163 \ \#$$

2. Data is listed in the table

X_i	Y_i	1) $(X_i-\bar{X})$	2) $(Y_i-\bar{Y})$	394 1×2	440 $(X_i-\bar{X})^2$: Sum
10	0	-10	-9.1	91	100
12	2	-8	-7.1	56.8	64
14	5	-6	-4.1	24.6	36
16	6	-4	-3.1	12.4	16
18	7	-2	-2.1	4.2	4
22	10	2	0.9	1.8	4
24	10	4	0.9	3.6	16
26	15	6	5.9	35.4	36
28	16	8	6.9	55.2	64
30	20	10	10.9	109	100

$$\bar{X} = \frac{10+12+\ldots+30}{10} = 20 \quad ; \quad n=10$$

$$\bar{Y} = \frac{0+2+\ldots+20}{10} = 9.1$$

2.1 From the simple regression model $Y_i = \beta_0 + \beta_1 X_i + u_i$, $u_i \sim NIID(0,\sigma^2)$. Find estimators of β_0 and β_1 from the OLS method and interpret the meaning.

$$\hat{\beta}_1 = \frac{\sum_{i=1}^{10}(Y_i-\bar{Y})(X_i-\bar{X})}{\sum_{i=1}^{10}(X_i-\bar{X})^2} = \frac{\sum_{i=1}^{n} x_i y_i}{\sum_{i=1}^{n} x_i^2} = \frac{394}{440} = 0.8955 \ \#$$

$$\hat{\beta}_0 = \bar{Y} - \hat{\beta}_1\bar{X} = 9.1 - (0.8955)(20) = -8.81 \ \#$$

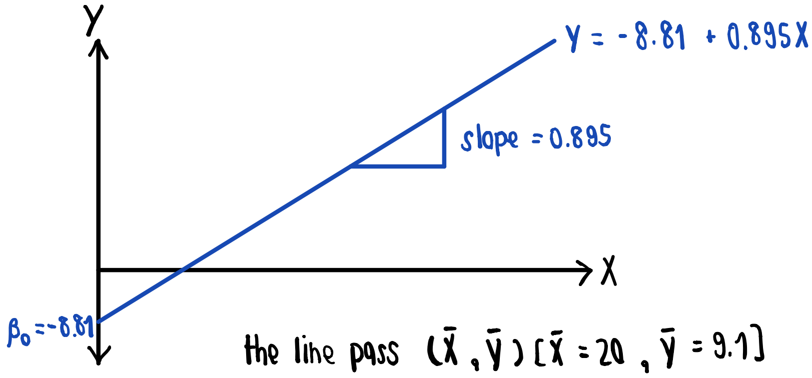
2.2 Find the value of $\widehat{Y}_i$ and $\widehat{u}_i$. Show that $\sum_{i=0}^{N} \widehat{u}_i = 0$.

$$\sigma^2 = Var(\hat{u}_i) = \frac{\sum \hat{u}_i^2}{n-2} = \frac{14.091}{10-2} = 1.761375 \ \#$$

$$Var(\hat{\beta}_0) = \frac{\sigma^2}{\sum (X_i - \bar{X})^2} = \frac{1.761375}{440} = 0.004003 \ \#$$

$$Var(\hat{\beta}_1) = \frac{\sum X_i^2}{n \sum (X_i - \bar{X})^2}(\sigma^2) = \frac{4440(1.761375)}{10(440)} = 1.7774 \ \#$$

2.3 Plot graph and draw regression line. Does the line pass $(\bar{X}, \bar{Y})$?



the line pass $(\bar{X}, \bar{Y})$ $[\bar{X} = 20 , \bar{Y} = 9.1]$

sub $\bar{X}$ in regression function

$$Y = -8.81 + 0.895X$$
$$= -8.81 + 0.895(20)$$
$$\bar{Y} = 9.1$$

$\therefore$ The line pass $(\bar{X}, \bar{Y})$ #

2.4 If X_i =16, what is the predicted Y?

$$\hat{Y}_i = \beta_0 + \beta_1 X_i = -8.8091 + 0.8955(16) = 5.5189 \ \#$$

2.5 Find $var(\widehat{u}_i)$, $var(\widehat{\beta_0})$, $var(\widehat{\beta_1})$

$$Var(\hat{u}_i) = \frac{\sum_{i=1}^{n} \hat{u}_i^2}{n-2} = 1.7614$$

$$Var(\hat{\beta}_0) = \frac{\sum_{i=1}^{n} X_i^2 \hat{\sigma}^2}{n \sum_{i=1}^{n} X_i^2} = 1.7774$$

$$Var(\hat{\beta}_1) = \frac{\hat{\sigma}^2}{\sum_{i=1}^{n} X_i^2} = 0.0040$$

3. Consider the below regression function:

$$Y_i = \beta_1 X_i + u_i \,,$$

Where $u_i \sim NIID(0, \sigma^2)$. Find an OLS estimator of β_1. Then, provide a proof that this is an unbiased estimator. Please state the SLR assumption(s) when used.

To prove that $\hat{\beta}_1$ is unbiased under assumption SLR 1-4

From $\hat{\beta}_1 = \dfrac{\Sigma (y_i - \bar{y})(x_i - \bar{x})}{\Sigma (x_i - \bar{x})^2}$

Let $k_i = x_i - \bar{x}$

$$k_i = \frac{x_i}{\Sigma_i x_i^2} = \frac{x_i - \bar{x}}{\Sigma_i (x_i - \bar{x})^2}$$

Now, write as ;

$$\hat{\beta}_1 = \sum_{i=1}^{n} (y_i - \bar{y}) k_i$$

$$= \sum_{i=1}^{n} (\beta_0 + \beta_1 x_i + u_i - \beta_0 - \beta_1 \bar{x}) k_i$$

$$= \Sigma_i \beta_1 (x_i - \bar{x}) k_i + \Sigma_i u_i k_i$$

$$= \beta_1 \Sigma_i x_i k_i + \Sigma_i u_i k_i$$

$$= \beta_1 \Sigma_i x_i \left(\frac{x_i}{\Sigma_i x_i^2} \right) + \Sigma_i u_i k_i$$

$$= \beta_1 \frac{\Sigma_i x_i^2}{\Sigma_i x_i^2} + \Sigma_i u_i k_i$$

$E(\hat{\beta}_1) = E[\beta_1 + \Sigma_i u_i k_i] = \beta_1 + E[\Sigma_i u_i k_i]$ SLR4

SLR 4: $E(u_i \mid x_i) = 0$ This assumption takes the value of x as given (or fixed)

So, we can treat x_i as a constant

$$E(\hat{\beta}_1) = \beta_1 + \Sigma_i k_i E(u_i)$$

$$E(\hat{\beta}_1) = \beta_1$$

$\therefore$ β_1 is unbiased estimator #