



Practice problem set 8

Semester 1/2016

Integration and its application in economics.

(Solution)

Question 1:

In the manufacture of a product, the marginal cost of producing x units is $C'(x)$ and fixed cost are $C(0)$. Find the total cost function $C(x)$ when:

- a. $C'(x) = 3x + 4, C(0) = 40$.
- b. $C'(x) = ax + b, C(0) = C_0$.

Ans.

- a. $C(x) = \frac{3}{2}x^2 + 4x + 40$
- b. $C(x) = \frac{1}{2}ax^2 + bx + C_0$

Question 2

Let $K(t)$ denote the capital stock of an economy at time t . Then net investment at time t , denoted by $I(t)$, is given by the rate of increase $\frac{dK}{dt}$ of $K(t)$.

- a. If $I(t) = 3t^2 + 2t + 5, t \geq 0$, what is the total increase in the capital stock during the interval from $t = 0$ to $t = 5$?

Ans. $K(5) - K(0) = 175$.

- b. If $K(t_0) = K_0$, find an expression for the total increase in the capital stock from time $t = t_0$ to $t = T$ when the investment function $I(t)$ is as in part (a).

$$\text{Ans. } K(T) - K_0 = (T^3 - t_0^3) + (T^2 - t_0^2) - 5(T - t_0)$$

Question 3:

Given the following demand and supply curves, compute the consumer and producer surplus.

- a. Demand: $P = 200 - 0.2Q$; Supply: $P = 20 + 0.1Q$.

$$\text{Ans. } (Q^*, P^*) = (600, 80).$$

$$\text{Consumer surplus} = 36,000; \text{ Producer surplus} = 18,000.$$

- b. Demand: $P = \frac{6000}{Q+50}$; Supply: $P = Q + 10$.

$$\text{Ans. } (Q^*, P^*) = (50, 60).$$

$$\text{Consumer surplus} = 6,000 \ln 2 - 3000;$$

$$\text{Producer surplus} = 1250.$$

Question 4

Suppose that the profit of a firm as a function of its output x is given by

$$f(x) = 4000 - x - \frac{3000000}{x}, \quad x > 0$$

- a. Find the level of output that maximizes profit. Sketch the graph of f .

$$\text{Ans. } x = 1000\sqrt{3}$$

b. The actual output varies between 1000 and 3000 units. Compute the

average profit $I = \frac{1}{2000} \int_{1000}^{3000} f(x) dx.$

Ans. $I = 2000 - 1500\ln(3) \approx 352$

Question 5:

Evaluate the following integrals by using integrations by substitution:

a. $\int_0^1 x\sqrt{1+x^2} dx$

Ans. Let $u = \sqrt{1+x^2}$. Thus, $\int_0^1 x\sqrt{1+x^2} dx = \int_1^{\sqrt{2}} u^2 du = \frac{1}{3}(2\sqrt{2} - 1)$

b. $\int_1^e \frac{\ln y}{y} dy$

Ans. Let $u = \ln(y)$. Thus, $\int_1^e \frac{\ln y}{y} dy = \frac{1}{2}.$

Question 6:

Evaluate the following integrals by using integrations by parts ($r \neq 0$).

a. $\int_0^T bte^{-rt} dt$

Ans. $\int_0^T bte^{-rt} dt = br^{-2}[1 - (1 + rT)e^{-rT}]$

b. $\int_0^T (a + bt)e^{-rt} dt$

Ans. $\int_0^T (a + bt)e^{-rt} dt = ar^{-1}(1 - e^{-rT}) + br^{-2}[1 - (1 + rT)e^{-rT}]$

Question 7:

- a. Evaluate $\int_0^1 x^p(x^q + x^r + x^s)dx$ where $p, q, r,$ and s are positive numbers.

Ans: $\frac{1}{p+q+1} + \frac{1}{p+r+1} + \frac{1}{p+s+1}$

- b. Let $F(x) = \int_0^x (t^2 + 2)dt$ and $G(x) = \int_0^{x^2} (t^2 + 2)dt$. Find $F'(x)$ and $G'(x)$.

Ans: $F(x) = x^3/3 + 2x$ and $G(x) = x^6/3 + 2x^2$
 $F'(x) = x^2 + 2$ and $G'(x) = 2x^5 + 4x$

Question 8:

Let the demand and supply of goods Q in a perfectly competitive market be the followings;

Demand Function : $P = 25 - Q^2$

Supply Function : $P = 2Q + 1$

- a. Determine the consumer surplus as the equilibrium.
 CS = 42.67 and PS = 16
- b. If the government imposes tax on consumers for \$4 per unit of production, calculate the deadweight loss.
 [DWL = 0.9]

Question 9:

Let $P = 274 - Q^2$ be the demand function in a monopoly market. Suppose further that marginal cost of the monopolist is given by $MC = 4 + 3Q$.

- a. Determine consumer's surplus at the profit-maximizing production level. [Q = 9, P = 193, CS = 486];
- b. Calculate deadweight loss under monopoly.

Q under perfect competition = 15. Thus, deadweight loss is 522.

Question 10:

A company has $MC = 80$ where the demand function is $P = 1400 - 6Q$. At zero production, the company faces loss by \$1,500. Determine the maximum profit by using integral calculus and prove your answer.

[Q = 110 and maximum profit = 73,000]