

## **Assignment 9**

**Due: 24/11/2020**

From the data set `Assignment 9.dta`:

### **Requirements:**

1. Perform unit root test of series  $y$  and  $x$ .
2. Perform cointegration test of series  $y$  and  $x$  using set up of (i) linear trend; (ii) restricted trend; (iii) unrestricted constant; (iv) restricted constant; and (v) no trend, with one lag term.
3. Perform cointegration test of series  $y$  and  $x$  using set up of linear trend with (i) one lag term; (ii) two lag terms; and (iii) three lag terms.
4. From (2) & (3), determine the most appropriated set up and make the conclusion of the test whether  $y$  and  $x$  are cointegrated series? If yes, how many cointegrating equations.
5. Estimated VECM models of  $y$  and  $x$  using (i) one lag term; (ii) two lag terms; and (iii) three lag terms. Determine the optimal lags. Specify cointegrating equation and speed of adjustment parameters.

1. `. dfuller y, trend lags(1) regress`

Augmented Dickey-Fuller test for unit root                      Number of obs    =            498

|      | Test<br>Statistic | Interpolated Dickey-Fuller |                      |                       |
|------|-------------------|----------------------------|----------------------|-----------------------|
|      |                   | 1% Critical<br>Value       | 5% Critical<br>Value | 10% Critical<br>Value |
| Z(t) | 1.000             | -3.980                     | -3.420               | -3.130                |

MacKinnon approximate p-value for Z(t) = 1.0000

| D.y    | Coef.    | Std. Err. | t     | P> t  | [95% Conf. Interval] |          |
|--------|----------|-----------|-------|-------|----------------------|----------|
| Y      |          |           |       |       |                      |          |
| L1.    | .0001178 | .0001179  | 1.00  | 0.318 | -.0001137            | .0003494 |
| LD.    | .6997015 | .0248993  | 28.10 | 0.000 | .6507799             | .7486231 |
| _trend | 2.897751 | 1.159296  | 2.50  | 0.013 | .619992              | 5.175511 |
| _cons  | 1811.233 | 147.0426  | 12.32 | 0.000 | 1522.327             | 2100.139 |

The result shows p-value = 1 (reject null hypothesis of unit root)  
 $\therefore$  The trend exists.

`. dfuller x, trend lags(1) regress`

Augmented Dickey-Fuller test for unit root                      Number of obs    =            498

|      | Test<br>Statistic | Interpolated Dickey-Fuller |                      |                       |
|------|-------------------|----------------------------|----------------------|-----------------------|
|      |                   | 1% Critical<br>Value       | 5% Critical<br>Value | 10% Critical<br>Value |
| Z(t) | 0.601             | -3.980                     | -3.420               | -3.130                |

MacKinnon approximate p-value for Z(t) = 0.9970

| D.x    | Coef.    | Std. Err. | t     | P> t  | [95% Conf. Interval] |          |
|--------|----------|-----------|-------|-------|----------------------|----------|
| x      |          |           |       |       |                      |          |
| L1.    | .0001061 | .0001764  | 0.60  | 0.548 | -.0002405            | .0004526 |
| LD.    | .46018   | .0349881  | 13.15 | 0.000 | .3914361             | .5289239 |
| _trend | 4.166909 | 1.14105   | 3.65  | 0.000 | 1.924999             | 6.408818 |
| _cons  | 2128.626 | 140.0551  | 15.20 | 0.000 | 1853.449             | 2403.803 |

The result shows p-value = 0.9970 (reject null hypothesis of unit root)  
 $\therefore$  The trend exists.

## 2. Linear trend

```
. vecrank y x, trend(t) lag(1/1) max
```

| Johansen tests for cointegration |       |            |                 |           |                                    |
|----------------------------------|-------|------------|-----------------|-----------|------------------------------------|
| Trend: trend                     |       |            | Number of obs = |           | 499                                |
| Sample: 2 - 500                  |       |            | Lags =          |           | 1                                  |
|                                  |       |            |                 |           | 5%                                 |
| maximum                          |       |            |                 | trace     | critical                           |
| rank                             | parms | LL         | eigenvalue      | statistic | value                              |
| 0                                | 4     | -7387.4577 | .               | 790.3357  | > 18.17 → Reject $H_0: r=0$        |
| 1                                | 7     | -6992.3441 | 0.79477         | 0.1085*   | < 3.74 → Fail to reject $H_0: r=1$ |
| 2                                | 8     | -6992.2899 | 0.00022         |           |                                    |

| maximum |       |            |            |               | 5%                                 |
|---------|-------|------------|------------|---------------|------------------------------------|
| rank    | parms | LL         | eigenvalue | max statistic | critical value                     |
| 0       | 4     | -7387.4577 | .          | 790.2272      | > 16.87 → Reject $H_0: r=0$        |
| 1       | 7     | -6992.3441 | 0.79477    | 0.1085        | < 3.74 → Fail to reject $H_0: r=1$ |
| 2       | 8     | -6992.2899 | 0.00022    |               |                                    |

Both Johanson trace test and Max Eigenvalues test suggest to use rank = 1

## Restricted trend

```
. vecrank y x, trend(rt) lag(1/1) max
```

| Johansen tests for cointegration |       |            |                 |           |                             |
|----------------------------------|-------|------------|-----------------|-----------|-----------------------------|
| Trend: rtrend                    |       |            | Number of obs = |           | 499                         |
| Sample: 2 - 500                  |       |            | Lags =          |           | 1                           |
|                                  |       |            |                 |           | 5%                          |
| maximum                          |       |            |                 | trace     | critical                    |
| rank                             | parms | LL         | eigenvalue      | statistic | value                       |
| 0                                | 2     | -8050.4781 | .               | 2116.3764 | > 25.32 → Reject $H_0: r=0$ |
| 1                                | 6     | -7075.2453 | 0.97993         | 165.9107  | > 12.25 → Reject $H_0: r=1$ |
| 2                                | 8     | -6992.2899 | 0.28286         |           |                             |

| maximum |       |            |            |               | 5%                          |
|---------|-------|------------|------------|---------------|-----------------------------|
| rank    | parms | LL         | eigenvalue | max statistic | critical value              |
| 0       | 2     | -8050.4781 | .          | 1950.4657     | > 18.96 → Reject $H_0: r=0$ |
| 1       | 6     | -7075.2453 | 0.97993    | 165.9107      | > 12.52 → Reject $H_0: r=1$ |
| 2       | 8     | -6992.2899 | 0.28286    |               |                             |

Both Johanson trace test and Max Eigenvalues test suggest to use rank = 2

## Unrestricted constant

```
. vecrank y x, trend(c) lag(1/1) max
```

### Johansen tests for cointegration

Trend: constant      Number of obs =      499  
Sample: 2 - 500      Lags =      1

| maximum |       |            |            | trace     | 5% critical |                     |
|---------|-------|------------|------------|-----------|-------------|---------------------|
| rank    | parms | LL         | eigenvalue | statistic | value       |                     |
| 0       | 2     | -8050.4781 | .          | 2086.8946 | > 15.41     | → Reject $H_0: r=0$ |
| 1       | 5     | -7083.8611 | 0.97923    | 153.6607  | > 3.76      | → Reject $H_0: r=1$ |
| 2       | 6     | -7007.0308 | 0.26504    |           |             |                     |

| maximum |       |            |            | max       | 5% critical |                     |
|---------|-------|------------|------------|-----------|-------------|---------------------|
| rank    | parms | LL         | eigenvalue | statistic | value       |                     |
| 0       | 2     | -8050.4781 | .          | 1933.2339 | > 14.07     | → Reject $H_0: r=0$ |
| 1       | 5     | -7083.8611 | 0.97923    | 153.6607  | > 3.76      | → Reject $H_0: r=1$ |
| 2       | 6     | -7007.0308 | 0.26504    |           |             |                     |

Both Johanson trace test and Max Eigenvalues test suggest to use rank = 2

## Restricted constant

```
. vecrank y x, trend(rc) lag(1/1) max
```

### Johansen tests for cointegration

Trend: rconstant      Number of obs =      499  
Sample: 2 - 500      Lags =      1

| maximum |       |            |            | trace     | 5% critical |                     |
|---------|-------|------------|------------|-----------|-------------|---------------------|
| rank    | parms | LL         | eigenvalue | statistic | value       |                     |
| 0       | 0     | -8811.3759 | .          | 3593.1922 | > 19.96     | → Reject $H_0: r=0$ |
| 1       | 4     | -7096.9792 | 0.99896    | 164.3988  | > 9.42      | → Reject $H_0: r=1$ |
| 2       | 6     | -7014.7798 | 0.28069    |           |             |                     |

| maximum |       |            |            | max       | 5% critical |                     |
|---------|-------|------------|------------|-----------|-------------|---------------------|
| rank    | parms | LL         | eigenvalue | statistic | value       |                     |
| 0       | 0     | -8811.3759 | .          | 3428.7934 | > 15.67     | → Reject $H_0: r=0$ |
| 1       | 4     | -7096.9792 | 0.99896    | 164.3988  | > 9.24      | → Reject $H_0: r=1$ |
| 2       | 6     | -7014.7798 | 0.28069    |           |             |                     |

Both Johanson trace test and Max Eigenvalues test suggest to use rank = 2

No trend

```
. vecrank y x, trend(n) lag(1/1) max
```

Johansen tests for cointegration

```
Trend: none                      Number of obs =    499
Sample:  2 - 500                  Lags =            1
```

| maximum |       |            |            | trace     | 5% critical |                     |
|---------|-------|------------|------------|-----------|-------------|---------------------|
| rank    | parms | LL         | eigenvalue | statistic | value       |                     |
| 0       | 0     | -8811.3759 | .          | 3204.1194 | > 12.53     | → Reject $H_0: r=0$ |
| 1       | 3     | -7211.7592 | 0.99836    | 4.8860    | > 3.84      | → Reject $H_0: r=1$ |
| 2       | 4     | -7209.3162 | 0.00974    |           |             |                     |

|         |       |            |            | 5%        |                             |
|---------|-------|------------|------------|-----------|-----------------------------|
| maximum |       |            |            | max       | critical                    |
| rank    | parms | LL         | eigenvalue | statistic | value                       |
| 0       | 0     | -8811.3759 |            | 3199.2334 | > 11.44 → Reject $H_0: r=0$ |
| 1       | 3     | -7211.7592 | 0.99836    | 4.8860    | > 3.84 → Reject $H_0: r=1$  |
| 2       | 4     | -7209.3162 | 0.00974    |           |                             |

Both Johanson trace test and Max Eigenvalues test suggest to use rank = 2

### 3. One lag term

```
. vecrank y x, trend(t) lag(1/1) max
```

Johansen tests for cointegration

Trend: trend

Number of obs = 499

Sample: 2 - 500

Lags = 1

|         |       |            |            |           | 5%                                                   |
|---------|-------|------------|------------|-----------|------------------------------------------------------|
| maximum |       |            |            | trace     | critical                                             |
| rank    | parms | LL         | eigenvalue | statistic | value                                                |
| 0       | 4     | -7387.4577 | .          | 790.3357  | $> 18.17 \rightarrow \text{Reject } H_0: r=0$        |
| 1       | 7     | -6992.3441 | 0.79477    | 0.1085*   | $< 3.74 \rightarrow \text{Fail to reject } H_0: r=1$ |
| 2       | 8     | -6992.2899 | 0.00022    |           |                                                      |

| rank | parms | LL         | eigenvalue | max statistic | critical value |
|------|-------|------------|------------|---------------|----------------|
| 0    | 4     | -7387.4577 | .          | 790.2272      | 16.87          |
| 1    | 7     | -6992.3441 | 0.79477    | 0.1085        | 3.74           |
| 2    | 8     | -6992.2899 | 0.00022    |               |                |

$\rightarrow$  Reject  $H_0: r=0$   
 $<$  3.74  $\rightarrow$  Fail to reject  $H_0: r=1$

Both Johanson trace test and Max Eigenvalues test suggest to use rank = 1

Two lag terms

```
. vecrank y x, trend(n) lag(1/1) max
```

Johansen tests for cointegration

Trend: none

Number of obs = 499

Sample: 2 - 500

Lags = 1

|         |       |            |            |           | 5%                          |
|---------|-------|------------|------------|-----------|-----------------------------|
| maximum |       |            |            | trace     | critical                    |
| rank    | parms | LL         | eigenvalue | statistic | value                       |
| 0       | 0     | -8811.3759 | .          | 3204.1194 | > 12.53 → Reject $H_0: r=0$ |
| 1       | 3     | -7211.7592 | 0.99836    | 4.8860    | > 3.84 → Reject $H_0: r=1$  |
| 2       | 4     | -7209.3162 | 0.00974    |           |                             |

|         |       |            |            | 5%        |                             |
|---------|-------|------------|------------|-----------|-----------------------------|
| maximum |       |            |            | max       | critical                    |
| rank    | parms | LL         | eigenvalue | statistic | value                       |
| 0       | 0     | -8811.3759 | .          | 3199.2334 | > 11.44 → Reject $H_0: r=0$ |
| 1       | 3     | -7211.7592 | 0.99836    | 4.8860    | > 3.84 → Reject $H_0: r=1$  |
| 2       | 4     | -7209.3162 | 0.00974    |           |                             |

Both Johanson trace test and Max Eigenvalues test suggest to use rank = 2

## Three lag terms

```
. vecrank y x, trend(t) lag(3/3) max
```

### Johansen tests for cointegration

Trend: trend      Number of obs =      497  
Sample: 4 - 500      Lags =      3

|         |       |            |            | 5%        |                                    |
|---------|-------|------------|------------|-----------|------------------------------------|
| maximum |       |            | trace      | critical  |                                    |
| rank    | parms | LL         | eigenvalue | statistic | value                              |
| 0       | 12    | -6756.658  | .          | 140.0434  | > 18.17 → Reject $H_0: r=0$        |
| 1       | 15    | -6688.113  | 0.24106    | 2.9534*   | < 3.74 → Fail to reject $H_0: r=1$ |
| 2       | 16    | -6686.6363 | 0.00592    |           |                                    |

|         |       |            |            | 5%        |                                    |
|---------|-------|------------|------------|-----------|------------------------------------|
| maximum |       |            | max        | critical  |                                    |
| rank    | parms | LL         | eigenvalue | statistic | value                              |
| 0       | 12    | -6756.658  | .          | 137.0900  | > 16.87 → Reject $H_0: r=0$        |
| 1       | 15    | -6688.113  | 0.24106    | 2.9534    | < 3.74 → Fail to reject $H_0: r=1$ |
| 2       | 16    | -6686.6363 | 0.00592    |           |                                    |

Both Johanson trace test and Max Eigenvalues test suggest to use rank = 1

4. By performing cointegration test, there are at least one cointegrating equation.  
∴ Y and X are Cointegrated series.

5. One lag term

```
. vec y x, lag(1/1)
```

Vector error-correction model

Sample: 2 - 500

Number of obs = 499

AIC = 28.41227

Log likelihood = -7083.861

HQIC = 28.42883

Det(Sigma\_ml) = 7.34e+09

SBIC = 28.45448

| Equation | Parms | RMSE    | R-sq   | chi2     | P>chi2 |
|----------|-------|---------|--------|----------|--------|
| D_y      | 2     | 331.479 | 0.9988 | 399343.6 | 0.0000 |
| D_x      | 2     | 430.726 | 0.9953 | 105050.2 | 0.0000 |

|     |       | Coef.     | Std. Err. | z       | P> z  | [95% Conf. Interval] |           |
|-----|-------|-----------|-----------|---------|-------|----------------------|-----------|
| D_y | _cel  |           |           |         |       |                      |           |
|     | L1.   | -1.303404 | .0095397  | -136.63 | 0.000 | -1.322102            | -1.284707 |
|     | _cons | -107.843  | 69.40373  | -1.55   | 0.120 | -243.8718            | 28.18583  |
| D_x | _cel  |           |           |         |       |                      |           |
|     | L1.   | -.8364345 | .0123959  | -67.48  | 0.000 | -.8607301            | -.812139  |
|     | _cons | 168.0502  | 90.1839   | 1.86    | 0.062 | -8.706965            | 344.8074  |

Cointegrating equations

| Equation | Parms | chi2     | P>chi2 |
|----------|-------|----------|--------|
| _cel     | 1     | 1.71e+10 | 0.0000 |

Identification: beta is exactly identified

Johansen normalization restriction imposed

| beta |       | Coef.     | Std. Err. | z        | P> z  | [95% Conf. Interval] |           |
|------|-------|-----------|-----------|----------|-------|----------------------|-----------|
| _cel | y     | 1         | -         | -        | -     | -                    | -         |
|      | x     | -1.500508 | .0000115  | -1.3e+05 | 0.000 | -1.500531            | -1.500486 |
|      | _cons | -1678.204 | -         | -        | -     | -                    | -         |
|      |       |           |           |          |       |                      |           |

## Two lag terms

. vec y x, lag(2/2)

Vector error-correction model

Sample: 3 - 500  
 Number of obs = 498  
 AIC = 27.18032  
 Log likelihood = -6758.899  
 HQIC = 27.21018  
 Det(Sigma\_ml) = 2.11e+09  
 SBIC = 27.25641

| Equation | Parms | RMSE    | R-sq   | chi2     | P>chi2 |
|----------|-------|---------|--------|----------|--------|
| D_y      | 4     | 213.415 | 0.9995 | 964110.3 | 0.0000 |
| D_x      | 4     | 263.876 | 0.9982 | 280728.2 | 0.0000 |

|     |       | Coef.    | Std. Err. | z     | P> z  | [95% Conf. Interval] |
|-----|-------|----------|-----------|-------|-------|----------------------|
| D_y | _ce1  |          |           |       |       |                      |
|     | L1.   | -.427199 | .0483639  | -8.83 | 0.000 | -.5219905 -.3324075  |
|     | y     |          |           |       |       |                      |
|     | LD.   | .4022605 | .0224005  | 17.96 | 0.000 | .3583563 .4461646    |
|     | x     |          |           |       |       |                      |
| D_x | LD.   | .4814843 | .0598769  | 8.04  | 0.000 | .3641276 .5988409    |
|     | _cons | 171.4547 | 44.03061  | 3.89  | 0.000 | 85.15624 257.7531    |
|     | _ce1  |          |           |       |       |                      |
|     | L1.   | .2903827 | .0597995  | 4.86  | 0.000 | .1731779 .4075875    |
|     | y     |          |           |       |       |                      |
| D_x | LD.   | .5734396 | .0276971  | 20.70 | 0.000 | .5191544 .6277249    |
|     | x     |          |           |       |       |                      |
|     | LD.   | .3676105 | .0740347  | 4.97  | 0.000 | .2225051 .5127159    |
|     | _cons | 252.237  | 54.44157  | 4.63  | 0.000 | 145.5335 358.9405    |

Cointegrating equations

| Equation | Parms | chi2     | P>chi2 |
|----------|-------|----------|--------|
| _ce1     | 1     | 8.09e+08 | 0.0000 |

Identification: beta is exactly identified

Johansen normalization restriction imposed

| beta |       | Coef.     | Std. Err. | z        | P> z  | [95% Conf. Interval] |
|------|-------|-----------|-----------|----------|-------|----------------------|
| _ce1 | y     | 1         | .         | .        | .     | .                    |
|      | x     | -1.500176 | .0000527  | -2.8e+04 | 0.000 | -1.50028 -1.500073   |
|      | _cons | -577.9622 | .         | .        | .     | .                    |

Cointegrating Equation :  $Y_t = 577.9622 + 1.500176 X_t$

### Three lag terms

. vec y x, lag(3/3)

Vector error-correction model

Sample: 4 - 500

Number of obs = 497

AIC = 27.1245

Log likelihood = -6727.439

HQIC = 27.16771

Det(Sigma\_ml) = 1.96e+09

SBIC = 27.23459

| Equation | Parms | RMSE    | R-sq   | chi2     | P>chi2 |
|----------|-------|---------|--------|----------|--------|
| D_y      | 6     | 211.673 | 0.9995 | 980003.4 | 0.0000 |
| D_x      | 6     | 253.098 | 0.9984 | 305162.2 | 0.0000 |

↳ Lowest SBIC  
∴ Optimal lags = 3

|     |       | Coef.             | Std. Err. | z     | P> z  | [95% Conf. Interval] |
|-----|-------|-------------------|-----------|-------|-------|----------------------|
| D_y | _ce1  |                   |           |       |       |                      |
|     | L1.   | <u>-0.3193909</u> | .058155   | -5.49 | 0.000 | -.4333726 -.2054092  |
|     |       |                   |           |       |       |                      |
|     | Y     |                   |           |       |       |                      |
|     | LD.   | .3553209          | .0559202  | 6.35  | 0.000 | .2457194 .4649225    |
|     | L2D.  | .0174858          | .0312998  | 0.56  | 0.576 | -.0438607 .0788323   |
|     |       |                   |           |       |       |                      |
|     | x     |                   |           |       |       |                      |
|     | LD.   | .6042668          | .0763396  | 7.92  | 0.000 | .4546439 .7538896    |
|     | L2D.  | .0521968          | .0634683  | 0.82  | 0.411 | -.0721988 .1765925   |
|     | _cons | 181.5838          | 45.38519  | 4.00  | 0.000 | 92.63049 270.5372    |
| D_x | _ce1  |                   |           |       |       |                      |
|     | L1.   | .4434064          | .0695359  | 6.38  | 0.000 | .3071185 .5796943    |
|     |       |                   |           |       |       |                      |
|     | Y     |                   |           |       |       |                      |
|     | LD.   | .4713427          | .0668638  | 7.05  | 0.000 | .3402922 .6023932    |
|     | L2D.  | .012763           | .0374252  | 0.34  | 0.733 | -.060589 .0861151    |
|     |       |                   |           |       |       |                      |
|     | x     |                   |           |       |       |                      |
|     | LD.   | .5220109          | .0912792  | 5.72  | 0.000 | .3431069 .700915     |
|     | L2D.  | .0911602          | .0758891  | 1.20  | 0.230 | -.0575797 .2399001   |
|     | _cons | 130.797           | 54.26707  | 2.41  | 0.016 | 24.43548 237.1585    |

↳ Speed of adjustment =  $\frac{1}{|-0.3193909|} = 3.13 \approx 3$  period

Cointegrating equations

| Equation | Parms | chi2     | P>chi2 |
|----------|-------|----------|--------|
| _ce1     | 1     | 6.64e+08 | 0.0000 |

Identification: beta is exactly identified

Johansen normalization restriction imposed

| beta |       | Coef.     | Std. Err. | z        | P> z  | [95% Conf. Interval] |
|------|-------|-----------|-----------|----------|-------|----------------------|
| _ce1 |       |           |           |          |       |                      |
|      | y     | 1         | .         | .        | .     | .                    |
|      | x     | -1.500049 | .0000582  | -2.6e+04 | 0.000 | -1.500163 -1.499935  |
|      | _cons | -92.13439 | .         | .        | .     | .                    |

Cointegrating Equation :  $Y_t = 92.13439 + 1.500049X_t$