Answer Keys: Chapter3 Differential calculus in Economic Theory

1. A) $\frac{dy}{dx} = 0$

   b) $\frac{dy}{dx} = 6x^2$

   c) $\frac{dy}{dx} = \frac{1}{2}\ x^{\frac{-1}{2}}$

   d) $\frac{dy}{dx} = 4x^{-3}$

   e) $\frac{dy}{dx} = 16x^3 - 6x^2 + 2x - 3$

   f) $\frac{dy}{dx} = 36x^2 + 8x - 6$

   g) $\frac{dy}{dx} = \frac{-5x^2+5}{(1+x^2)^2}$

   h) $\frac{dy}{dx} = 1 + 3\ln|x|$

2. a) $f(x) = 16x^5 - 2x^3 + 4x^2 - 3x - 32$

   $f'(x) = 80x^4 - 6x^2 + 8x - 3$

   $f''(x) = 320x^3 - 12x + 8$

   $f'''(x) = 960x^2 - 12$

   b) $f(x) = (3x+2)^3$

   $f'(x) = 3(3x+2)^2(3) = 9(3x+2)^2$

   $f''(x) = 18(3x+2)(3) = 162x+108$

   $f'''(x) = 162$

   c) $f(x) = (6x-1)(2x-3)^2$

   $f'(x) = (6x-1)2(2x-3)(2)+(2x-3)^2(6)$

   $f''(x) = 144x-152$

   $f'''(x) = 144$

3. a) $f_x=6x-2y$ $f_{xx}=6$

   $f_y= -2x+8y$ $f_{yy}=8$

   b) $f_u= 4u+4+5v$ $f_{uu}=4$

   $f_v=5u+10$ $f_{vv}=0$

4. a) $dy = (8x+5)\,dx$

   b) $dy = (28x^3 - 6x^2 + 5)dx$

c) $dy = \frac{1}{(x+5)\ln 10}dx$

d) $dy = (-3x^2-6x)dx$

e) $dy = (4x - 1)dx$

5. a) $\frac{dy}{dx} = 6(4x-1)4 = 96x - 24$

   b) $\frac{dy}{dx} = 3(5 - x^2)^2(-2x) = -6x(5 - x^2)^2$

   c) $\frac{dy}{dx} = 2a(bx^2+cx)(2bx+c)$

   d) $\frac{dy}{dx} = \frac{-Fx}{Fy} = \frac{-(3x^2-4yx+3y^2)}{-2x^2+6xy}$

   e) $\frac{dy}{dx} = \frac{-Fx}{Fy} = \frac{-(2xy^2+e^x)}{2x^2y-e^y}$

6. a) $f'(1)=18$ $f'(2)=18$

   b) $f'(1)=10$ $f'(2)=\frac{5}{4}$

   c) $f'(1)=\frac{9}{4}$ $f'(2)=9$

   d) $f'(1)=2$ $f'(2)= 2^{\frac{-1}{3}} = 0.79$

7. $z=x^2-8xy-y^2= 9t^2-8(3t)(1 - t) - (1 - t)^2 = 32t^2 - 22t - 1$

   $\frac{dz}{dt} = 64t - 22$

8. $x= t+\frac{1}{t}$, $\frac{dx}{dt}= 1+\ln|t|$

   $y=t - \frac{1}{t}$, $\frac{dy}{dt} = 1 - \ln|t|$

   $z= (t+\frac{1}{t})^2 + (t+\frac{1}{t})(t - \frac{1}{t}) + (t - \frac{1}{t})^2$, then $\frac{dz}{dt} = 6t - 2t^{-3}$

9. $g(x) = \frac{x}{1+x}$

   $g'(x) = \frac{(1+x)1-x}{(1+x)^2}$

   $g''(x) = -2(1+x)^{-3}$

   $g'''(x) = 6(1+x)^{-4}$

10. a) $TR= PQ=18Q - 0.2Q^2$

    $AR=18 - 0.2Q$

    $MR=18 - 0.4Q$

    b) $TR=10Q$

AR=MR=10

11.a) $MC=0.06Q^2 – 6Q+12$

$AC= 0.02Q^2 – 3Q+12 + \frac{300}{Q}$

b) $MC= \frac{1}{3} Q^{\frac{-2}{3}} - 1.2Q +30$

$AC= Q^{\frac{-2}{3}} - 0.6Q +30+ \frac{120}{Q}$

12.a) $MU_1= Q_1^{-0.5} Q_2^{0.5}$

$MU_2= Q_2^{-0.5} Q_1^{0.5}$

b) $MU_1= 12Q_1+2Q_2$

$MU_2= 2Q_1+8Q_2$

13. From $c=500+0.75Y^d$

MPC= 0.75 and MPS =1 – MPC=1-0.75= 0.25

14. $VC= Q^3-5Q^2+12Q$

$\frac{dVC}{dQ} = 3Q^2 – 10Q +12$ = MVC (Marginal Variable cost) which is the change in variable cost due to one unit change in quantity.

15. $Q_d=10-0.5P$

$\frac{dQ}{dP} = - 0.5$

a) $E_p^d = \frac{dQ}{dP}\left(\frac{P}{Q}\right) = -0.5\left(\frac{2}{9}\right) = \frac{-1}{9}$

$|E_p^d| = \frac{1}{9} < 1$ **Inelastic**

b) $E_p^d = \frac{dQ}{dP}\left(\frac{P}{Q}\right) = -0.5\left(\frac{10}{5}\right) = -1$

$|E_p^d| = 1$ **Unitary Elastic**

c) $E_p^d = \frac{dQ}{dP}\left(\frac{P}{Q}\right) = -0.5\left(\frac{18}{1}\right) = -9$

$|E_p^d| > 1$ **Relatively Elastic**

16. $Q_x=100-2P_x+P_y+0.1I$

a) $E_p{}^d = \frac{dQx}{dPx}\left(\frac{Px}{Qx}\right) = -2\left(\frac{10}{192}\right) = \frac{-5}{48}$

b) $E^c = \frac{dQx}{dPy}\left(\frac{Py}{Qx}\right) = 1\left(\frac{12}{192}\right) = \frac{1}{16}$

$|E^c| = \frac{1}{16} < 1$ **Inelastic, then Good x and Good y are substitute goods.**

c) $E^I = \frac{dQx}{dI}\left(\frac{I}{Qx}\right) = 0.1\left(\frac{1000}{192}\right) = \frac{25}{48}$

$0 < |E^I| < 1$ **,then Good x is necessary good.**

17. $TP= Q=10L^{0.5} K^{0.5}$

$MPL= \frac{dTP}{dL} = 5 L^{-0.5}K^{0.5}$

$MPK=\frac{dTP}{dK} = 5 L^{0.5} K^{-0.5}$

18. $S=(12000-900P)A^{0.5} R^{0.5}$ At P=6, R=49, A=8100

a) Elasticity of sale with respect to advertising

$E_{S,A} = \frac{dS}{dA}\left(\frac{A}{S}\right) = 0.5(12000-900P)A^{-0.5}R^{0.5}\left(\frac{A}{S}\right) = 0.5$

b) Elasticity of sale with respect to price

$E_{S,P} = \frac{dS}{dP}\left(\frac{P}{S}\right) = -900A^{0.5}R^{0.5}\left(\frac{P}{S}\right) = \frac{-9}{11}$

c) Elasticity of sale with respect to representative

$E_{S,R} = \frac{dS}{dR}\left(\frac{R}{S}\right) = 0.5(12000-900P)A^{0.5}R^{-0.5}\left(\frac{R}{S}\right) = 0.5$

19. From $Q=1.1 L^{0.6}K^{0.2}$

Output elasticity of L= $\frac{dQ}{dL}\left(\frac{L}{Q}\right) = 0.66L^{-0.4}K^{0.2}\left(\frac{L}{Q}\right) =0.6$

Output elasticity of K $=\frac{dQ}{dK}\left(\frac{K}{Q}\right) = 0.22L^{0.6}K^{-0.8}\left(\frac{K}{Q}\right) = 0.2$

Proof: From $f(L,K) = 1.1 L^{0.6}K^{0.2}$

$f(tL, tK) = 1.1 (tL)^{0.6}(tK)^{0.2}= t^{0.8}\, 1.1 L^{0.6}K^{0.2}$

$=t^{0.8} f(L,K)$

Then, it shows decreasing return to scale.

20. From $Q=a+bP^2+R^{1/2}$

Price elasticity of supply = $E^s{}_p = \frac{dQ}{dP}\left(\frac{P}{Q}\right) = 2bP\left(\frac{P}{Q}\right) = \frac{2bP^2}{Q}$

Rainfall elasticity of supply $=E^s{}_R = \frac{dQ}{dR}\left(\frac{R}{Q}\right) = \frac{1}{2}R^{\frac{-1}{2}}\left(\frac{R}{Q}\right) = \frac{R^{\frac{1}{2}}}{2a+2bP^2+2R^{1/2}}$

21. From $X= Y^{1/2}+P^{-2}$

$E_{X,P} = \frac{dX}{dP}\left(\frac{P}{X}\right) = -2P^{-3}\left(\frac{P}{X}\right) = \frac{-2P^{-2}}{Y^{1/2}+P^{-2}}$

22. $Y= a+b(Y- T_0 - tY)+I_0+dY+G_0+X_0 - M_0 - mY$ ($M_0>0$, $0<m<1$)

$(1-b+bt - d +m)Y = a - bT_0+ I_0 +G_0+X_0 - M_0$

$$Y^* = \frac{a - bT_0 + I_0 + G_0 + X_0 - M_0}{1-b+bt-d+m}$$

$$\frac{dy*}{dI_0} = \frac{1}{1-b+bt-d+m}$$

$$\frac{dy*}{dG_0} = \frac{1}{1-b+bt-d+m}$$

$$\frac{dy*}{dT_0} = \frac{-b}{1-b+bt-d+m}$$

$$\frac{dy*}{dM_0} = \frac{-1}{1-b+bt-d+m}$$

23. $Q_d=Q_s$

$30-2P = 5P-6$

Equilibrium: $P^* = \frac{36}{7}$, $Q^* = \frac{138}{7}$

When the demand is changed to $40 - 2P$

$$\frac{dP*'}{da} = \frac{1}{2+5} = \frac{1}{7}$$

$$\frac{dQ*'}{da} = \frac{5}{2+5} = \frac{5}{7}$$