1. a) Find the projection matrix $\underline{P}_c$ onto the column space of $\underline{A}$

$$\underline{A} = \begin{bmatrix} 3 & 3 & 6 \\ 1 & 1 & 2 \end{bmatrix}$$

b) Find the 3 by 3 projection matrix $\underline{P}_R$ onto the row space of $\underline{A}$. What is the closest vector in the row space to the vector $\underline{b} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$?

c) Multiply $\underline{P}_C \underline{A}$ and then $\underline{P}_c \underline{A} \underline{P}_R$. Explain your answer?

d) Find the basis for the subspace of all vectors orthogonal to the row space of $\underline{A}$.

2. Find the least squares solution to $\underline{A}\underline{x} = \underline{b}$ where

$$\underline{A} = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix} \quad \text{and} \quad \underline{b} = \begin{bmatrix} 1 \\ 3 \\ 8 \\ 2 \end{bmatrix}$$

3. If $\underline{A}$ has eigenvalues 0 and 1, corresponding to the eigenvectors $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$ and $\begin{bmatrix} 2 \\ -1 \end{bmatrix}$,

a) how can you tell in advance that $\underline{A}$ is symmetric? What are its trace and determinant?

b) What is $\underline{A}$? Compute $A^5 \begin{bmatrix} 1 \\ 1 \end{bmatrix}$.

4. Multinational companies in the US, Asia, and Europe have assets of $12 trillion. At the start, $6 trillion are in the US, $6 trillion in Europe. Each year half the US money stays home, ¼ each goes to Asia and Europe. For Asia and Europe, half stays home and half is sent to the US.

a) Find the eigenvalues and eigenvectors of this matrix *A*.
b) What is the limiting distribution of the $12 trillion as the world ends ($k \rightarrow \alpha$)?