

UNCERTAINTY AND CONSUMER BEHAVIOR (PINDYCK, CH. 5)

- DESCRIBING RISK
- PREFERENCES TOWARD RISK
- REDUCING RISK
- THE DEMAND FOR RISKY ASSETS

• PROBABILITY: LIKE LIHOOD THAT "A GIVEN OUTCOME" WILL OCCUR.

• EXPECTED VALUE: PROBABILITY WEIGHTED AVERAGE OF "PAYOFFS" ASSOCIATED W/ ALL POSSIBLE OUTCOMES.

• PAYOFFS: VALUE ASSOCIATED W/ A POSSIBLE OUTCOME.

EX: W/ 2 POSSIBLE OUTCOMES,

$$THE\ EXPECTED\ VALUE\ (E(X)) = P_{r_1} \cdot X_1 + P_{r_2} \cdot X_2$$

W/ n POSSIBLE OUTCOMES,

$$E(X) = P_{r_1} \cdot X_1 + P_{r_2} \cdot X_2 + \dots + P_{r_n} \cdot X_n$$

• VARIABILITY: DEGREE OF WHICH POSSIBLE OUTCOMES OF "UNCERTAIN EVENT" DIFFER.

	OUTCOME 1		OUTCOME 2		EXPECTED INCOME
	PROB.	INCOME (€)	PROB.	INCOME (€)	
<u>JOB 1</u> (SALE COMMISSION)	0.5	2000	0.5	1000	$0.5 \cdot 2000 + 0.5 \cdot 1000 = 1500\ €$
<u>JOB 2</u> (FIXED SALARY)	0.99	1510	0.01	510	$0.99 \cdot 1510 + 0.01 \cdot 510 = 1500\ €$

THE TWO JOBS SHARE THE SAME EXPECTED INCOME: 1500 €

STANDARD DEVIATION (σ)	$X_i - E(X)$		$X_i - E(X)$	
	OUTCOME 1	DEVIATION	OUTCOME 2	DEVIATION
JOB 1	2000	+500	1000	-500
JOB 2	1510	+10	510	-990

TO CALCULATE VARIANCE (= THE AVG. OF THE SQUARED DEVIATION)

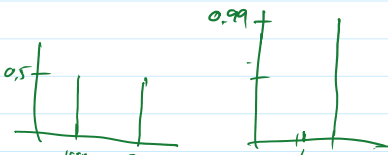
$$\begin{aligned}
 \text{JOB 1: } \sigma^2 &= 0.5 (1500)^2 + 0.5 (-500)^2 \\
 &= 250,000 \\
 \sigma &= \sqrt{250,000} = 500 \text{ (STANDARD DEVIATION)}
 \end{aligned}$$

$$\begin{aligned}
 \text{JOB 2: } \sigma^2 &= 0.99 (10)^2 + 0.01 (-990)^2 \\
 &= 0.99 (100) + 0.01 (980,100) \\
 &= 9900
 \end{aligned}$$

$$\sigma = \sqrt{9900} = 99.50 \text{ (STANDARD DEVIATION)}$$

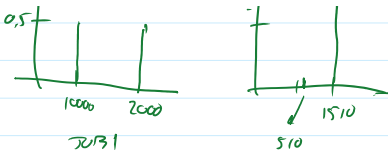
CONCLUSION

EVEN THOUGH THE EXPECTED INCOME FROM JOB 1 AND JOB 2 ARE NO DIFFERENT, JOB 2 IS MUCH LESS

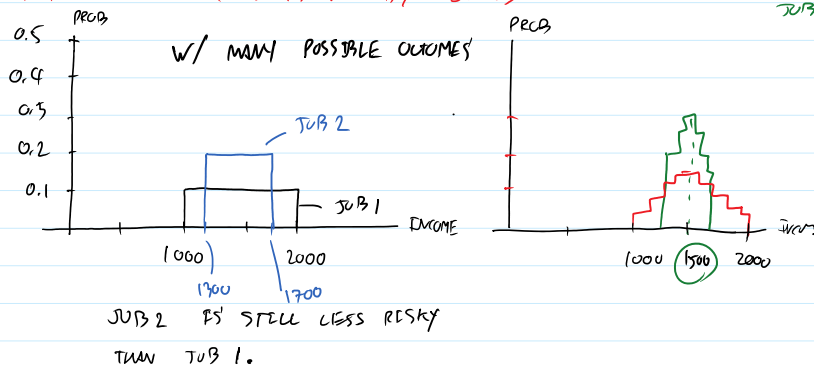


CONCLUSION

$\sigma = \sqrt{9900} = 99.50$ (STANDARD DEVIATION)
 EVENTHOUGH THE EXPECTED INCOME FROM JOB 1 AND JOB 2 ARE NO DIFFERENT, JOB 2 IS MUCH LESS RISKY THAN JOB 1 (MEASURED BY S.D.)



MODIFIED SITUATION



DECISION MAKING

LET'S MODIFY OUR EXAMPLE AGAIN ...

	OUTCOME 1	DEVIATION SQUARED	OUTCOME 2	DEVIATION SQUARED	EXPECTED INCOME	S.D
JOB 1	2100	250,000	1100	250,000	1600	500
JOB 2	1510	$(1510 - 1500)^2 = 10^2 = 100$	510	$(510 - 1500)^2 = (-990)^2 = 980,100$	1500	99.5

IMPLICATION

- IF THIS GUY WANTS TO MAXIMIZE THE EXPECTED INCOME, HE MAY TAKE JOB 1.
- HOWEVER, HIS FRIEND MAY DO IT DIFFERENTLY, DEPENDING ON "ATTITUDE TOWARD RISK"

DIAGRAM)

IF SO, THIS PERSON, OF COURSE, WILL NOT WANT TO TAKE THE RISKY CHOICE.

NOTE: FOR RISK-AVERSE AGENT, LOSSES ARE MORE IMPORTANT THAN GAINS (IN TERMS OF THE CHANGE IN UTILITY)

INCOME UTILITY

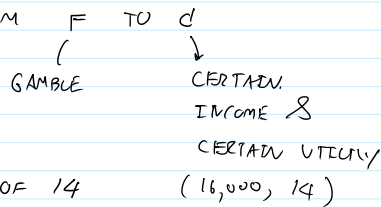
$$(20,000 \rightarrow 30,000) \rightarrow (16 \rightarrow 18) \Rightarrow \Delta U = +2$$

$$(20,000 \rightarrow 10,000) \rightarrow (16 \rightarrow 10) \Rightarrow \Delta U = -6$$

FACT 4: A RISK-AVERSE AGENT MAY WANT TO PAY A RISK PREMIUM: THE MAXIMUM AMOUNT OF MONEY THAT HE/SHE IS WILLING TO PAY TO AVOID TAKING A RISK.

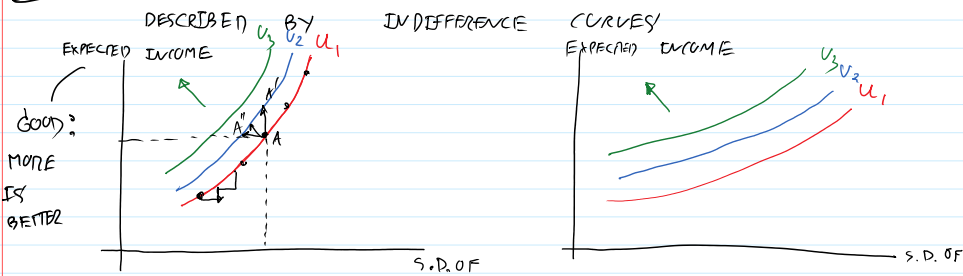
POINT OF DEPARTURE: SUPPOSE THIS GUY IS NOW AT POINT F, i.e., HE IS NOW TAKING A RISKY JOB. (PERHAPS A DICTATOR FORCES HIM TO BE HERE)

IN THE PICTURE, HE IS WILLING TO PAY THE RISK PREMIUM (SIZE = $CF = 4000 \text{€}$) TO BRING HIM FROM F TO d



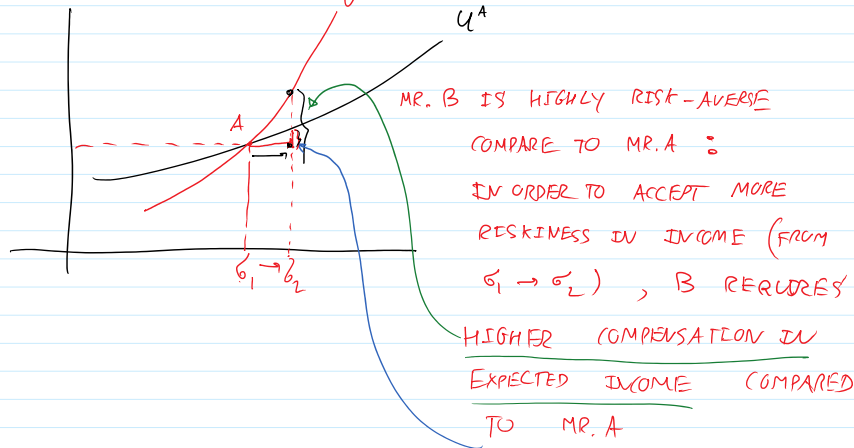
NOTE: OBSERVE THAT F AND d GIVE THE SAME "EXPECTED UTILITY" OF 14

FACT 5: RISK AVERSE AGENT'S PREFERENCES CAN BE DESCRIBED BY INDIFFERENCE CURVES



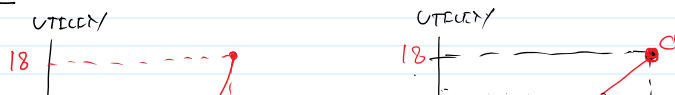
INCOME MEASURED (VARIABILITY OF INCOME)

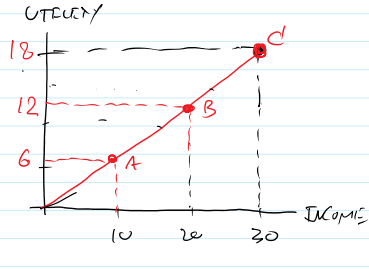
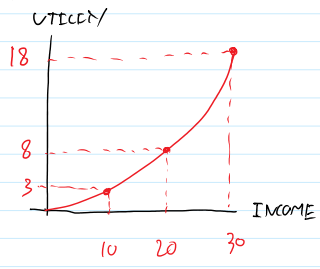
BAD: LESS IS PREFERRED TO MORE U^B U^A



MR. B IS HIGHLY RISK-AVERSE COMPARE TO MR. A: IN ORDER TO ACCEPT MORE RISKINESS IN INCOME (FROM $\sigma_1 \rightarrow \sigma_2$), B REQUIRES HIGHER COMPENSATION IN EXPECTED INCOME COMPARED TO MR. A

FACT # 6: ABOUT RISK-LOVING AGENT AND RISK-NEUTRAL AGENT





RISK AVERSE

$$U(x) = x^a$$

WHERE $a < 1$

Ex: $U(x) = x^{\frac{1}{2}}$

SUPPOSE $x_1 = 100$
 $x_2 = 400$
 $Pr(x_1) = 0.5$
 $Pr(x_2) = 0.5$

RISK NEUTRAL

$$U(x) = 2x$$

GENERAL FORM: $U(x) = Ax^a$

WHERE $a = 1$

SUPPOSE $x_1 = 100$
 $x_2 = 400$
 $Pr(x_1) = 0.5$
 $Pr(x_2) = 0.5$

RISK LOVING

$$U(x) = Ax^a$$

WHERE $a > 1$

SUPPOSE $x_1 = 100$
 $x_2 = 400$
 $Pr(x_1) = 0.5$
 $Pr(x_2) = 0.5$

$U(x) = x^2$

$E(x) = 0.5(100) + 0.5(400)$
 $= 50 + 200$
 $= 250$
 EXPECTED INCOME

$E(x) = 250$
 $E[U(x)] = \frac{1}{2}U(100) + \frac{1}{2}U(400)$
 $= \frac{1}{2}(2 \cdot 100) + \frac{1}{2}(2 \cdot 400)$
 $= 500$

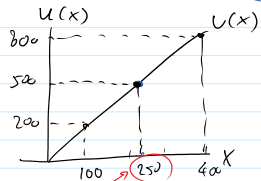
$E(x) = 250$
 $E[U(x)] = \frac{1}{2}U(100) + \frac{1}{2}U(400)$
 $= \frac{1}{2}(100)^2 + \frac{1}{2}(400)^2$
 $= 85,000$

SUPPOSE $U(100) = 10$
 $U(400) = 20$

$U(250) = 2 \cdot 250 = 500$

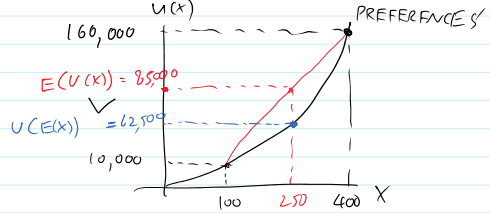
$U(250) = (250)^2 = 62,500$

$E[U(x)] = \frac{1}{2}U(100) + \frac{1}{2}U(400)$
 $= \frac{1}{2} \cdot 10 + \frac{1}{2} \cdot 20$
 $= 15$
 EXPECTED UTILITY OF INCOME

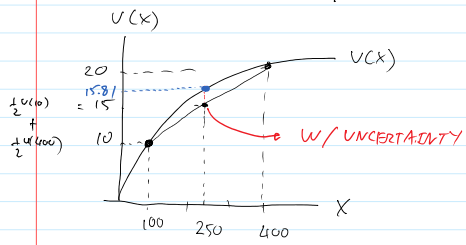


HERE $U[E(x)] < E[U(x)]$
 $62,500 < 85,000$

SO, $U(x) = x^2$ DESCRIBES RISK-LOVING PREFERENCES



HERE $U[E(x)] = E[U(x)]$
 THIS GUY IS INDIFFERENT BETWEEN CERTAIN EVENT AND UNCERTAIN EVENT, GIVEN THE SAME EXPECTED INCOME.



$0.5(100) + 0.5(400)$ PROB OF GETTING 250 BHT IN THIS SCENARIO IS
 AT THE SAME EXPECTED INCOME (250),
 $= 250 \cdot 1$ EQUAL TO 1

$$U[E(X)] > E[U(X)]$$

UTILITY FROM
EXPECTED INCOME
OF 250

15.81

↓
EXPECTED UTILITY
OF INCOME

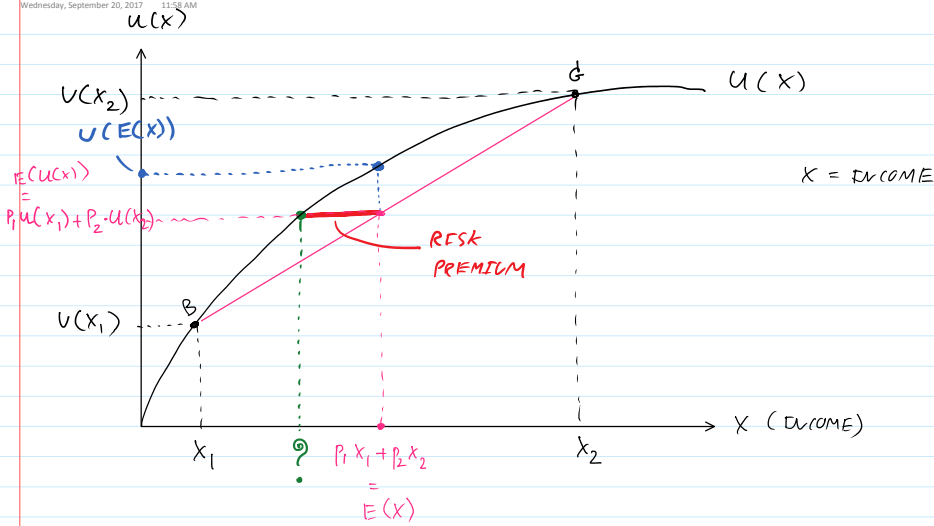
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W/O RISK

W/ RISK

SO, THIS UTILITY FUNCTION: $U(X) = X^{\frac{1}{2}}$
DESCRIBES RISK-AVERSE BEHAVIOR

NOTICE THAT: MU OF INCOME IS
DIMINISHING.



HOW TO AVOID OR REMOVE RISK?

SINCE UNCERTAINTY AND RISK IS PERVASIVE,
 A RISK-AVERSE AGENT WILL TRY TO REMOVE OR AVOID RISK
 BY...

- ① DIVERSIFICATION: DO NOT PUT ALL YOUR EGGS IN ONE BASKET.
- ② BUYING INSURANCE
- ③ INVESTING TO OBTAIN INFORMATION

① DIVERSIFICATION: PRACTICE OF REDUCING RISK BY "ALLOCATING"

RESOURCES TO A VARIETY OF ACTIVITIES
 WHOSE OUTCOMES ARE NOT RELATED.
 (STATE 1) (STATE 2)
 Prob(H) = 0.5 Prob(C) = 0.5

(Ex)

	HOT WEATHER	COLD WEATHER	E(π)	S.D. σ(π)
AIR CON	30,000	12,000	21,000	$\sqrt{\frac{1}{2}(30,000 - 21,000)^2 + \frac{1}{2}(12,000 - 21,000)^2} = 9,000$
HEATER	12,000	30,000	21,000	
SELL BOTH	$\frac{1}{2} \cdot 30,000 + \frac{1}{2} \cdot 12,000 = 21,000$ $\frac{1}{2} \cdot 12,000 + \frac{1}{2} \cdot 30,000 = 21,000$		21,000	0

COEFFICIENT OF VARIATION
 $C.V. = \frac{S.D. OF \pi}{E(\pi)} = \frac{9,000}{21,000} = \frac{3}{7}$

WHEN HE SELLS BOTH BY ALLOCATING HIS TIME EQUALLY
 HIS PROFIT WILL BE 21,000 CERTAINLY, REGARDLESS
 OF WEATHER!

WHEN WEATHER IS HOT, HE GETS 15,000 FROM SELLING AIRCON
 AND GET 6,000 FROM SELLING HEATER.
 WHEN WEATHER IS COLD, HE GETS 6,000 FROM SELLING AIRCON
 AND GETS 15,000 FROM SELLING HEATER.

THEREFORE, RISK DIVERSIFICATION IN THIS CASE "ELIMINATES"
 ALL RISK!

IT WORKS WELL WHEN OUTCOMES ARE PERFECTLY
NEGATIVELY CORRELATED.

② BUYING INSURANCE

- EX
- YOUR MACBOOK'S VALUE = 50,000 BAHIT
 - COST OF BUYING APPLE CARE = 1000 BAHIT
 - ANY DAMAGE WILL REDUCE THE VALUE OF YOUR MAC BY 10,000 BAHIT.
 - PROB (DAMAGE) = 0.1 OR 10%

	GET DAMAGED	NO DAMAGE	$E(x)$	S.D.	$C.V. = \frac{S.D.}{E(x)}$
DON'T BUY	40,000	50,000	$0.1(40,000) + 0.9(50,000) = 49,000$	$\sqrt{0.1(-)^2 + 0.9(-)^2} = 3,000$	$\frac{3,000}{49,000}$
BUY	$50,000 - 1,000 - 10,000 + 10,000 = 49,000$	$50,000 - 1,000 = 49,000$	49,000	○	○

BUYING INSURANCE HELPS REMOVING ALL RISK!

③ BUYING INFORMATION TO REDUCE / AVOID RISK.

D-I-Y

THE DEMAND FOR RISKY ASSETS

ASSETS : SOMETHING THAT PROVIDES A FLOW OF MONEY OR SERVICES TO ITS OWNER.

RISKY ASSET : ASSET THAT PROVIDES AN UNCERTAIN FLOW OF MONEY OR SERVICES TO ITS OWNER.

RISK-FREE ASSET : ASSET THAT PROVIDES A FLOW OF MONEY OR SERVICES THAT IS KNOWN W/ CERTAINTY.

EXPECTED RETURN : RETURN THAT AN ASSET SHOULD EARN ON AVERAGE.

ACTUAL RETURN : RETURN THAT AN ASSET EARNS.

BASIC CONCEPT ABOUT A RANDOM VARIABLE

A RANDOM VARIABLE (RV), w TAKES VALUE w_1, w_2, \dots, w_s WITH PROBABILITIES $\pi_1, \pi_2, \pi_3, \dots, \pi_s$

NOTE: $\sum_{i=1}^s \pi_i = 1$

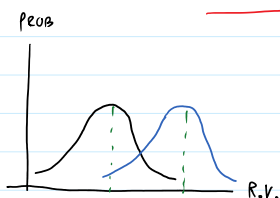
\Rightarrow MEAN (EXPECTED VALUE) OF THE DISTRIBUTION IS ... AVERAGE VALUE OF THE R.V.

$$E(w) = \mu_w = \pi_1 w_1 + \pi_2 w_2 + \dots + \pi_s w_s = \sum_{i=1}^s \pi_i w_i$$

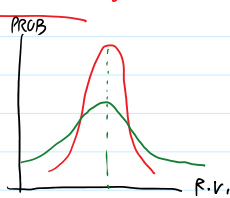
\Rightarrow THE DISTRIBUTION'S VARIANCE IS THE R.V.'S AVERAGE SQUARED DEVIATION FROM THE MEAN.

$$S.D. = \sigma_w = \sqrt{\sum_{i=1}^s (w_i - \mu_w)^2 \cdot \pi_i}$$

IT MEASURES THE R.V.'S VARIABILITY.



TWO DISTRIBUTIONS W/
THE SAME VARIANCE
AND DIFFERENT MEANS



THE TWO DISTRIBUTIONS W/
THE SAME MEAN
BUT DIFFERENT VARIANCES.

PREFERENCE OVER RISKY ASSETS

FACT#1 AN INVESTOR'S UTILITY FUNCTION IS DESCRIBED BY

$$U(\text{EXPECTED RETURN}, \text{RISK})$$

FACT#2 THE HIGHER "MEAN (OR EXPECTED) RETURN" IS PREFERRED, GIVEN A LEVEL OF RISK.

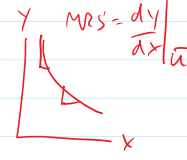
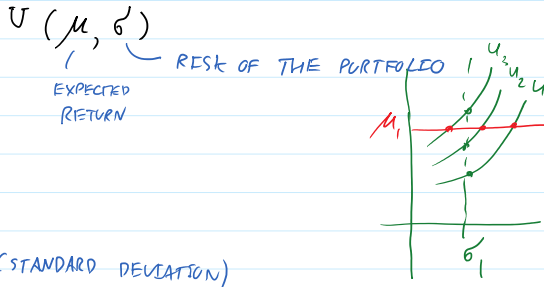
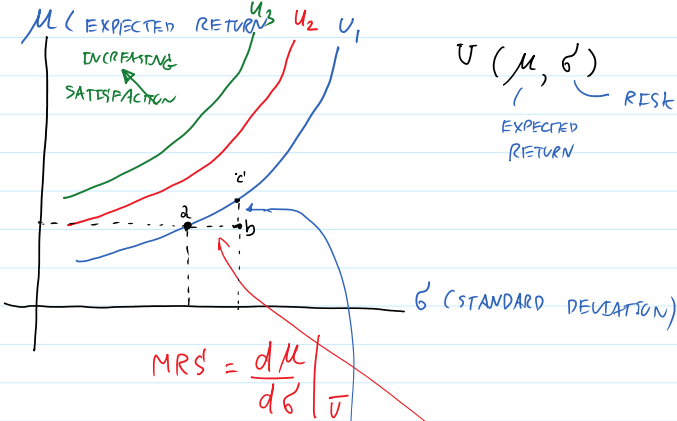
FACT#3 LESS VARIATION IN RETURN IS PREFERRED, (LESS RISK)

SO RISK IS CONSIDERED AS "BAD": LESS IS PREFERRED TO MORE.

$$U(\text{EXPECTED RETURN}) \quad U_2 \quad U_1$$

(LESS RISK)

SO RISK IS CONSIDERED AS "BAD": LESS IS PREFERRED TO MORE.



PROOF: HOW IS THE MRS COMPUTED?

$$U = f(\mu, \sigma)$$

$$dU = \frac{dU}{d\mu} d\mu + \frac{dU}{d\sigma} d\sigma = 0 \quad (\text{TOTAL DERIVATIVE})$$

TRAVELLING FROM b → c

TRAVELLING FROM a → b

$$\frac{dU}{d\mu} d\mu = - \frac{dU}{d\sigma} d\sigma$$

$$\frac{d\mu}{d\sigma} = - \frac{\frac{dU}{d\sigma}}{\frac{dU}{d\mu}}$$

MARGINAL UTILITY OF σ

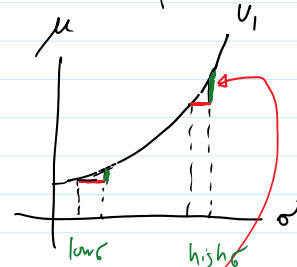
MARGINAL UTILITY OF μ

w/ good x & y

$$MRS = - \frac{dy}{dx} \Big|_U$$

$$MRS = - \frac{MU_x}{MU_y} = - \frac{\frac{dU}{dx}}{\frac{dU}{dy}}$$

$$MRS = \frac{d\mu}{d\sigma} = - \frac{MU_\sigma}{MU_\mu}$$



MRS IN INCREASING σ B/C' ...

AT LOW LEVEL OF σ COMPARED TO A HIGH LEVEL OF σ , THE INVESTOR WOULD REQUIRE MORE EXPECTED RETURN (μ) TO BE COMPENSATED WHEN HIS PORTFOLIO IS W/ HIGH RISK ALREADY.

BUDGET CONSTRAINT FOR RISKY ASSETS

CONSIDER TWO ASSETS

- RISK-FREE ASSET
- STOCK (\approx RISKY ASSET)

- RISK-FREE ASSET'S RETURN = r_f
- RISKY STOCK'S RATE OF RETURN = m_s IF STATE S OCCURS.

SO RISKY STOCK'S MEAN RATE-OF-RETURN IS

$$r_m = \sum_{s=1}^S m_s \pi_s$$

- A BUNDLE CONTAINING SOME OF THE RISKY STOCK AND SOME OF RISK-FREE ASSET IS SO CALLED "PORTFOLIO."

SUPPOSE b = FRACTION OF WEALTH USED TO BUY THE RISKY STOCK.

$1-b$ = FRACTION OF WEALTH USED TO BUY NON-RISKY ASSET.

W/ A GIVEN VALUE OF b , THE AVERAGE RATE-OF-RETURN OF THE PORTFOLIO :

$$r_b = b \cdot r_m + (1-b) \cdot r_f$$

IF $b=0$, $r_b = r_f$ (WHEN PUTTING ALL MONEY INTO RISK-FREE ASSET)

IF $b=1$, $r_b = r_m$ (WHEN PUTTING ALL MONEY INTO RISKY ASSET)

OBSERVE THAT THE PORTFOLIO'S EXPECTED RATE-OF-RETURN (r_b)

RISES AS b RISES. (= INVESTING MORE ON STOCK INCREASES HIS EXPECTED RATE OF RETURN)

$$\sigma_b^2 = \sum_{s=1}^S \left\{ [b \cdot m_s + (1-b)r_f] - \underbrace{[b r_m + (1-b)r_f]}_{r_b} \right\} \cdot \pi_s \quad \sigma^2 = E[x - \bar{x}]^2$$

$$\sigma_b^2 = \sum_{s=1}^S \left\{ b m_s + (1-b)r_f - [b r_m + (1-b)r_f] \right\}^2 \cdot \pi_s$$

$$= \sum_{s=1}^S [b m_s + \cancel{(1-b)r_f} - b r_m - \cancel{(1-b)r_f}]^2 \cdot \pi_s$$

$$= \sum_{s=1}^S (b m_s - b r_m)^2 \cdot \pi_s$$

$$= \sum_{s=1}^S (m_s - r_m)^2 \cdot \pi_s$$

$$\sigma_b^2 = b^2 \cdot \sigma_m^2$$

SO

$$\sigma_b = b \cdot \sigma_m$$

VARIABILITY OF THE RATE-OF-RETURN ON STOCK,

RISKINESS

OF THE PORTFOLIO

IF $b=0$, $\sigma_b = 0$.

IF $b=1$, $\sigma_b = 1 \cdot \sigma_m = \sigma_m \rightarrow$ RISKINESS OF THE PORTFOLIO

IS DETERMINED BY

RISKINESS OF THE STOCK

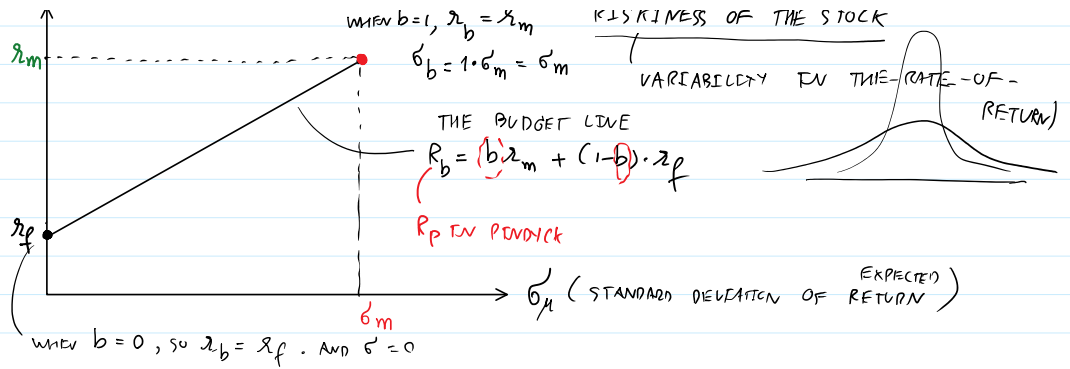
μ (EXPECTED RETURN)

r_m

WHEN $b=1$, $r_b = r_m$

$\sigma_b = 1 \cdot \sigma_m = \sigma_m$

VARIABILITY IN THE RATE-OF-



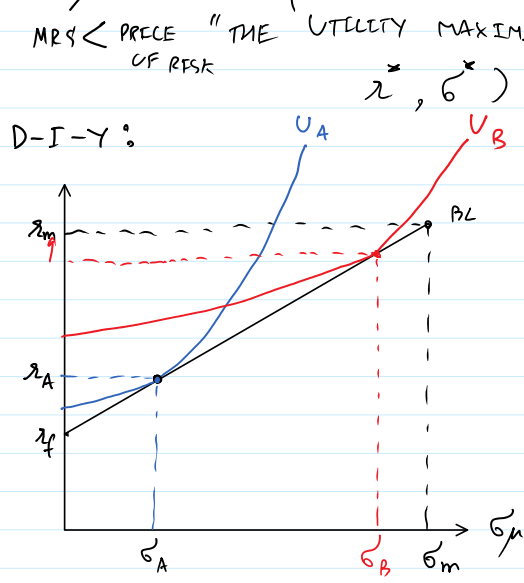
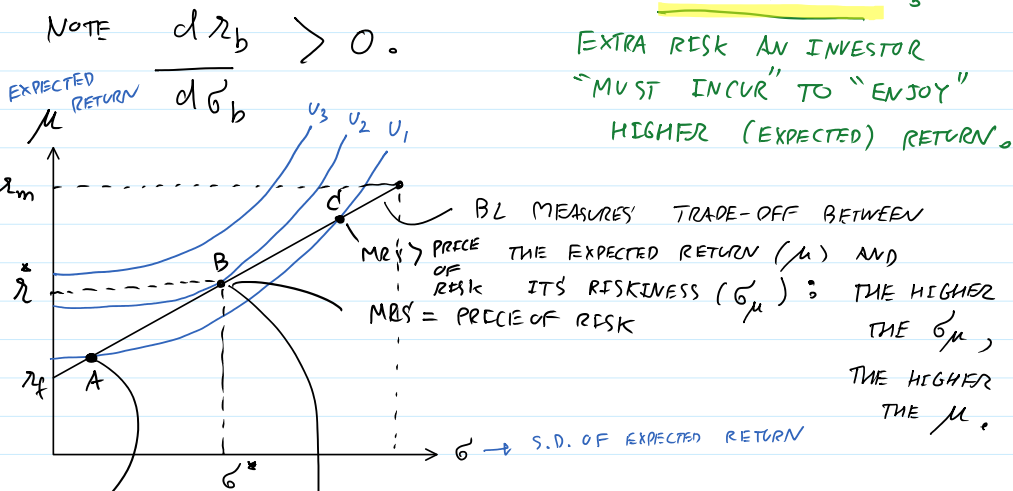
FROM $r_b = b r_m + (1-b) r_f$

$$= r_f + b(r_m - r_f)$$

$$= r_f + \frac{\sigma_b}{\sigma_m} (r_m - r_f) \quad \left[\text{RECALL THAT } \sigma_b = b \sigma_m \right]$$

SO $\frac{dr_b}{d\sigma_b} = \frac{(r_m - r_f)}{\sigma_m}$

→ SLOPE OF THE BUDGET LINE WHICH REFLECTS "PRICE OF RISK": EXTRA RISK AN INVESTOR "MUST INCUR" TO "ENJOY" HIGHER (EXPECTED) RETURN.



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