EE312 Chapter 8
Two-Period model:
The consumption-savings decision

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1 Overview

- Two-period model: Consumers, ~~Firms~~ and Government

- Macroeconomics studies how key economic variable evolve over time
- The simplest way to think about intertemporal decision is in a two-period model
  - The first period is current period (or today)
  - The second period represents the future (or tomorrow)
- The consumer makes intertemporal choice between current consumption and future consumption.

  - Saving (lending) and dissaving (borrowing).
  - The real interest rate is the relative price of future consumption in terms of current consumption.

- Key trade-off: consuming today or consuming tomorrow or the consumption-saving decision
- Decisions to be affected by changes in the real interest rate and in current and future incomes.

2 Consumer

- Assume the consumer receives exogenous income.
- To keep this simple, we will omit labour/leisure choice.

2.1 The consumer's budget constraint:

- Since consumers live for 2 period, we need to consider their budget constraints in both current and future periods
- Let c = current consumptions

  s = current saving

  y = current income

  t = current lump-sum taxes
- We will consider budget constraints for each period as following

2.1.1 Current budget constrant

- The current disposable income (y - t) equals consumption plus saving (c + s).

$$c + s = y - t \tag{1}$$

- Assuming bonds directly traded in the credit market and one single real interest rate (r) for borrowers and lenders.
- A bond is a promise to pay $1+r$ units of consumption goods tomorrow in exchange for 1 unit of consumption goods today.

- r is the real interest rate on bonds.
- Consumer can exchange one unit of current consumption for (1+r) units of future consumption in the credit market.

  $\Leftrightarrow$ Consumer can exchange (1+r) unit of future consumption for 1 unit of current consumption in the credit market.

  $\Rightarrow$ Consumer can exchange 1 unit of future consumption for ..... unit of current consumption in the credit market.
- The relative price of future consumption in terms of current consumption is $\frac{1}{1+r}$.
- If $s > 0$, the consumer is saving. The consumer is a lender in the credit market.
- If $s < 0$, the consumer is dis-saving. The consumer is a borrower in the credit market.

2.1.2 Future budget constraint

- If $s < 0$, the consumer pays the interest and principal on loan.

$$c' = y' - t' + (1 + r)s \tag{2}$$

- Consumption must be equal to disposable income in the future period $(y' - t')$ plus gross return on savings $(1 + r)s$.

2.1.3 Lifetime budget constraint

- If we combine current and future budget constraint into one, we have:

  Use $c' = y' - t' + (1 + r)s$ and $c + s = y - t$

$$c' = y' - t' + (1 + r)s$$
$$s = \frac{c' - y' + t'}{1 + r}$$

Since $c + s = y - t$, then

$$c + \frac{c' - y' + t'}{1+r} = y - t$$

$$c + \frac{c'}{1+r} = y + \frac{y'}{1+r} - t - \frac{t'}{1+r}$$

PV of lifetime consumption = PV of lifetime income - PV of lifetime taxes

$$c + \frac{c'}{1+r} = \left(y + \frac{y'}{1+r}\right) - \left(t + \frac{t'}{1+r}\right) \tag{3}$$

- The lifetime disposable income is the same as lifetime wealth (we).

$$we \equiv y + \frac{y'}{1+r} - t - \frac{t'}{1+r}$$

$$\Rightarrow c + \frac{c'}{1+r} = we$$

$$c' = -(1+r)c + we \cdot (1+r) \tag{4}$$

where

$we(1+r)$= what could be consumed in the future period if the consumer saved all of his or her current disposable income and consume lifetime wealth (after earning the real interest rate r on savings) in the future period.

$(1+r)c$ = future value of current consumption

- We can plot future consumption c' against current consumption c Life time budget constraint : $c' = \underbrace{-(1+r)}_{\text{slope}} c + \underbrace{we(1+r)}_{\text{intercept}}$

where

E = endowment point where s = 0;

$c = y - t$;

$c' = y' - t'$.

Above E, the consumer is a lender, but below E, a borrower.

2.2 The consumer's preference

- A consumption bundle (in this model) is a combination of current and future consumptions.
- Properties of consumer preference:
  - More is preferred to less.
  - Diversity in the consumption bundle is preferred (consumption smoothing).
  - Current and future consumptions are normal goods.
- The consumer's indifference curves

The ICs are downward-sloped and convex.

Slope = $-MRS_{c,c'}$ = the marginal rate of substitution of c' for c

$MRS_{c,c'}$ is falling as c increases

2.3 Consumer optimization

2.3.1 Optimization bundle

- The consumer choses c, c' to maximize utility subject to the life time budget constraint.
- Consumer choses a consumption bundle on the BC. The indifference curve is tangent to the life time budget constraint.
- The optimization condition:

$$\frac{MU_c}{MU_{c'}} = MRS_{c,c'} = (1+r) \tag{5}$$

- The consumer is optimizing where the marginal rate of substitution of c for c' equals the relative price of c in terms of c'.

  The willingness to trade c for c' equals the market rate of trading c for c'.
- The optimized consumption bundle is $(c, c') = (c^*, c'^*)$
- The consumer is a lender:

At A,$(c, c') = (c^*, c'^*)$ and $c^* < (y - t)$ and $s = y - t - c^* = BD > 0$.

- The consumer is a borrower.

At A, $(c, c') = (c^*, c'^*)$ and $c^* > (y - t)$ and $s = y - t - c* = BD < 0$

2.3.2 An increase in current income

- An increase in current income results in an increase in lifetime wealth. "A pure income effect"
- The budget line shifts *horizontally* to the right

$$we_1 = y_1 + \frac{y'}{1+r} - t - \frac{t'}{1+r}$$
$$we_2 = y_2 + \frac{y'}{1+r} - t - \frac{t'}{1+r}$$
$$\Delta we = we_2 - we_1 = y_2 - y_1$$

- Both current and future consumptions increase (normal goods).
  - The increase in c is smaller than the increase in y.
  - Saving increases; hence, c^* increases.
  - The consumer prefers diversity in the **consumption bundle** – consumption smoothing.

$$\Delta s = \Delta y - \Delta t - \Delta c$$

and because $\Delta t = 0$, and $\Delta y > \Delta c > 0$,

$$\Delta s > 0$$

- Increase in current y for a lender.

  - Both c and c' increase (A to B).
  - $\Delta c = c_1 c_2 < \Delta Y = AD$
  - $\Delta s > 0$
  - $s = y - t - c^*$
  - $\Delta s = \Delta y - \Delta t - \Delta c = BD > 0$

Consumption Smoothing

- An increase in current income causes an increase in consumption in both periods and an increase in savings.
- This behavior arises because of the consumer's desire to **smooth consumption over time.**
- The theory predicts aggregate consumption is less volatile than aggregate output.
- Theory is qualitatively consistent with data
  - Aggregate consumption is less volatile than GDP.
  - Consumption of **nondurables and services** is even less volatile.
  - Consumption of **durables** is more volatile.
    Durable consumption is more like investment.
    Returns of service flow from durable goods.
- While consumption is less volatile than GDP, it is still not smooth enough to be in line with the theory.

2.3.3 An increase in future income

- An increase in future income results in an increase in lifetime wealth.
- The budget line shifts *vertically* to the top.

$$we_1 = y + \frac{y_1'}{1+r} - t - \frac{t'}{1+r}$$
$$we_2 = y + \frac{y_2'}{1+r} - t - \frac{t'}{1+r}$$
$$\Delta we = we_2 - we_1 = \frac{y_2' - y_1'}{1+r}$$

- Future consumption increases, but by a lesser amount than the increase in future income.

  $\Rightarrow$ Saving decreases; current consumption increases.

- The increase in future income is smoothened into increases in both future and current consumption.

  $$\Delta s = \Delta y - \Delta t - \Delta c$$

  and because $\Delta t = 0$, and $\Delta y = 0$,

  $$\Delta c > 0 \text{ and } \Delta s < 0$$

  – Both c and c' increase (A to B).

- $\Delta c' = c'_1 c'_2$
- $\Delta c' < \Delta y' = AD$
- $\Delta s < 0$

2.3.4 Temporary and permanent increases in income

- Consumer's response will be different when they face temporary or permanent changes in income
- A permanent increase in income has a larger positive effect on lifetime wealth and current income than a temporary increase.

  Milton Friedman's **'permanent income hypothesis'**: the level of current consumption depends on the level of permanent income.
- The consumer will tend to save most of the temporary increase in income.
- Temporary changes in income yield small changes in permanent income, hence will have small effects on current consumption.
- Permanent increase: increase both y and y'

Temporary versus Permanent Δy

- A temporary increase in y = HL: the budget line shifts from AB to ED.
  - The consumption bundle rises from H to J.
  - Current consumption rises less than current income; saving increases – consumption smoothing
- A permanent increase in $y = y_2 - y_1 = y_2'$
  - y_1': the budget line shifts from AB to GF.
  - $y_2 - y_1 = HL = y_2' - y_1' = LM$.
  - The consumption bundle rises from H to K – Larger effect on current consumption: c_1c_3

2.3.5 Effect of a tax cut

- The effect of the government's tax cut on consumption depends on whether the cut is temporary or permanent.
- If **temporary**, the increase in consumption will be small; most of the increased income is saved.
- If **permanent**, the increase in consumption will be large.

2.3.6 An increase in the real interest rate and the budget line

- Recall $\frac{1}{1+r}$ is the relative price of future consumption in terms of current consumption.
- The slope of the budget line is $-(1+r)$.
- A change in the real interest rate causes the budget line to rotate.
- Changes in intertemporal decision between current and future consumptions.
- Analysis of the substitution effect and income effect.

$$we = y + \frac{y'}{1+r} - t - \frac{t'}{1+r}$$
$$we(1+r) = y(1+r) + y' - t(1+r) - t'$$

- The budget line rotates upwards as the real interest rate increases.
  - 'we' decreases (horizontal intercept).
  - '$we(1+r)$' increases if r increases (vertical intercept).
  - The endowment point (E) remains the same (no change in the initial endowment of $y-t, y'-t'$).
  - E is the pivot point.

An increase in the real interest rates:

The increase in the real interest rate rotates the budget line upwards with the same E.

The dual effects of a higher r

- An increase in the real interest rate (r) causes a change in the relative price of current and future consumptions.
- Future consumption becomes cheaper; current consumption becomes more expensive.
- Higher return on savings; less sacrifice in current consumption is needed for given future consumption.
- Lender: cheaper future consumption.
- Borrower: more expensive current consumption

Effect of higher r on the lender

- Future consumption becomes cheaper in terms of current consumption.
- **The substitution effect**: more future consumption for less current consumption.
- **The income effect**: given savings yield more future income – higher current and future consumptions.
- Higher future consumption; unclear current consumption and savings, given y and y'

Stronger substitution effect

AD = substitution effect; lower c.
DB = income effect; higher c.
$AD > DB$; lower c at c_3, assuming a lender.

An increase in r for a borrower

- Current consumption becomes more expensive in terms of future consumption.
  - **The substitution effect**: more future consumption and less current consumption.
  - **The income effect**: loans for future consumption become more expensive – reduced current and future consumptions.
- Current consumption decreases while saving increases; unclear future consumption.

AD = substitution effect; lower c' and higher c
DB = income effect; lower c and c'
Net effect: lower c; unclear c' .

Conclusions on effects of Δr

- A higher real interest rate (r) has an intertemporal substitution effect: Future consumption is substituted for current consumption – saving increases.

- Positive income effect for lenders but negative income effect for borrowers.
- No certainty that current consumption will fall if the real interest rate rises.

Variable	Current consumption	Saving
Income (increase) - Temporary Current c Future c - Permanent		
Tax cut (more cut) - Temporary - Permanent		
Interest rate (increase) - Borrower - Saver		

3 Government and Competitive Equilibrium

3.1 Government

- Government in this model lives for two periods and has similar set-up for its budget constraints. Define the following:

  G = current government purchase of goods.

  G'= future government purchase of goods.

  T = current taxes collected by the government.

  N = number of consumers, each paying the current tax of t; so $T = Nt$.

  T'= future taxes; and $T' = Nt'$.

  Government borrows by issuing bonds (B) at the real interest rate of r.

- Government budget constraint: Government spending is financed by taxes and bond issue in each period.
- Government's current budget constraint: $G = T + B$ or $G - T = B$.

  - $B > 0$ government is a borrower;
  - $B < 0$ government is a lender.

- Government's future budget constraint: $G' + (1 + r)B = T'$
- Government present-value budget constraint: The present value of government purchases must equal the present value of taxes.

$$
\begin{aligned}
G &= T + B \\
G' + (1 + r)B &= T' \\
\text{Total govt outlays in the future} &= \text{Future taxes} \\
B &= \frac{T' - G'}{1 + r} \\
G + \frac{G'}{1 + r} &= T + \frac{T'}{1 + r}
\end{aligned}
$$

3.2 Competitive equilibrium

3.2.1 Definition

- Consumers and government interact in the credit market: Trading of future consumption goods for current consumption goods through the interest rate.
- **A competitive equilibrium** is a set of endogenous quantities c, c', s, B and price r given exogenous quantities y, y', G, G', T, T', N that satisfy the following equilibrium conditions
  - Each consumer optimizes current and future consumptions and saving, given r
  - Government budget constraint holds
  - The credit market clears.

3.2.2 Credit Market Clearing

- Private savings (S^p) equals government borrowing (B) or $S^p = B$

- National saving is the sum of private savings and government savings: $S = S^p + S^g$

$$\begin{aligned} S &= S^p + S^g \\ S^p &= B \\ S^g &= -B \\ S &= B - B = 0 \end{aligned}$$

3.2.3 Income-Expenditure Identity

- The credit-market clearing implies that the income-expenditure identity holds (Walraus' law).

Resource constraint:

$$Y = C + G$$

Consumer:

$$S^p = Y - C - T$$

Government:

$$\begin{aligned} B &= G - T \\ \Rightarrow Y - C - T &= G - T \\ Y &= C + G \end{aligned}$$

3.2.4 Equilibrium representation

Demand-Supply approach

- Suppose a given budget deficit equal to $G-T = B_1$ which represents the demand for credit
- r^* is the equilibrium interest rate where r_1: excess supply and r_0: excess demand

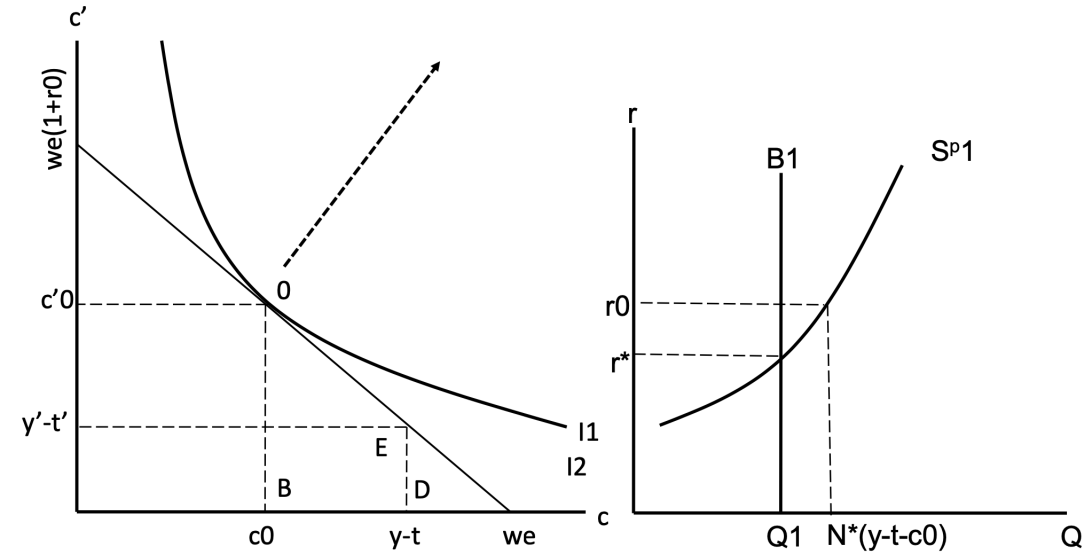
Edgeworth box approach

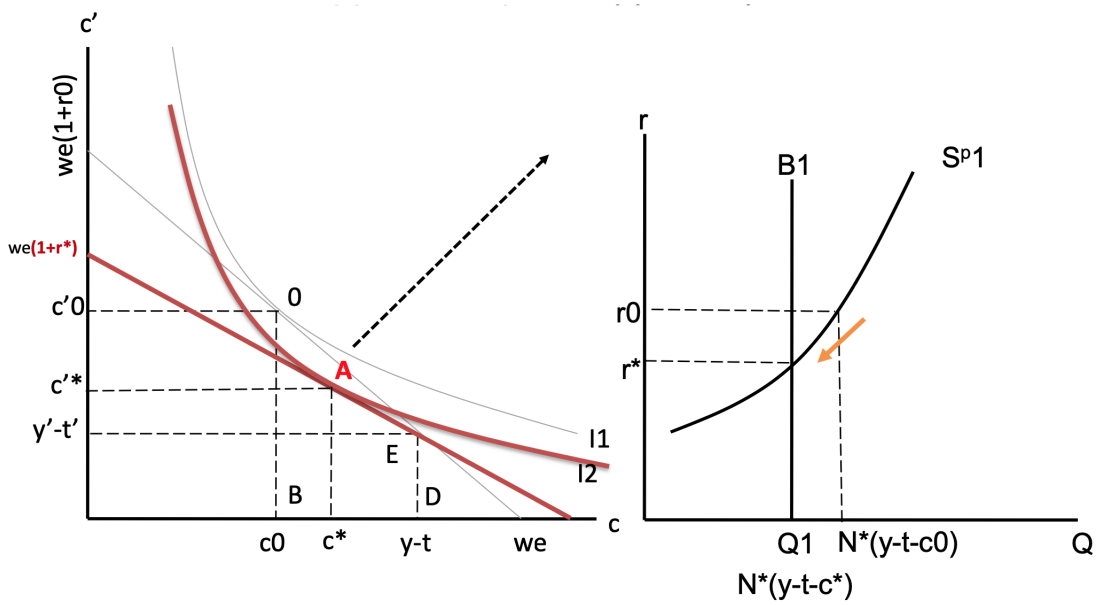
- Suppose that $N \cdot (y - t - c^*) = G - T$
- With r^*, consumers are maximizing utility at point-A.
- With r^*, credit market is cleared; net private saving is equal to net government borrowing ($N \cdot BD$ = G - T = B_1)

Disequilibrium and adjustment

- Given $r_0 > r^*$, does point-0 represent the equilibrium when $N \cdot (y - t - c_0) > B_1$.

  = Interest rate will fall to r^* and point-A is the new equilibrium





- How does an increase in current income affect (equilibrium) real interest rate?

- How does an increase in future income affect (equilibrium) real interest rate?

- How does an increase in permanent income affect (equilibrium) real interest rate?

4 The Ricardian Equivalence Theorem

4.1 Introduction

- A change in current taxes with an equal and opposite change in the present value of future taxes has no effect on the real interest rate and the consumption of individual consumers.
  - Assume equilibrium in the credit market, given r.
  - Current and future government spending are held constant.
  - Consumers' life-time budget constraint and government's present-value budget constraint.

4.2 Algebraic formulation

- Start with government's budget constraint:

$$G + \frac{G'}{1+r} = Nt + \frac{Nt'}{1+r}$$
$$t + \frac{t'}{1+r} = \frac{1}{N}\Big(G + \frac{G'}{1+r}\Big)$$

From consumer's

$$c + \frac{c'}{1+r} = \Big(y + \frac{y'}{1+r}\Big) - \Big(t + \frac{t'}{1+r}\Big)$$
$$\Rightarrow c + \frac{c'}{1+r} = \Big(y + \frac{y'}{1+r}\Big) - \frac{1}{N}\Big(G + \frac{G'}{1+r}\Big)$$

- Consumer and government's budget constraints are simplified into:

$$t + \frac{t'}{1+r} = \frac{1}{N}\Big(G + \frac{G'}{1+r}\Big) \tag{6}$$
$$c + \frac{c'}{1+r} = \Big(y + \frac{y'}{1+r}\Big) - \frac{1}{N}\Big(G + \frac{G'}{1+r}\Big) \tag{7}$$

- The change in current taxes (Δt) is matched by $-(1+r)\Delta t$ so that equation (6) holds.
  - Equation 7 remains unchanged, given r (as y, y', G, G' and N are the same).

- The credit market clears so as the resource constraint $Y = C+G$
- No welfare change for consumers.

4.3 A current tax cut for a borrower

- A current tax cut equals a future tax increase.
- Lifetime wealth and consumption bundle (A) are the same.
- Only the endowment point changes from E_1 to E_2.

4.4 Unchanged credit market

- Private saving and government borrowing increase by the same amount.
- Equilibrium r is the same.

4.5 A tax cut is not a free lunch

- Current tax cut gives all consumers higher current disposable income.
- But consumers must bear higher future taxes by the same amount.
- No welfare gain for consumers

4.6 Ricardian equivalence assumptions: when does it hold?

- A tax change affects every consumer by the same amount so that the present-value tax burden is unchanged.
  - If some consumers receive higher tax cuts, then their lifetime wealth and consumption choices (and the real interest rate) change.
  - Future tax burdens may be shared unequally.
- Government can redistribute wealth among income classes through tax policy
- Government debt is paid off during the lifetimes of current consumers.
  - But the government can postpone debt payment (and future taxes) to next generations.
  - The old receive tax cuts and higher disposable incomes; the young pay higher future taxes.
- The government can redistribute wealth between generations
  - The effect of the social security programs
- The tax is lump-sum.
  - All taxes cause distortions in the relative prices and consumption choices.
  - Welfare loss is greater than tax revenues.
- **Perfect credit markets:** consumers can borrow and lend as much at the same interest rate.

- But consumers have limits on borrowing.
- The borrowing rate is higher than the lending rate.
- Government borrows at a lower rate.
- Credit-constrained consumers benefit from a tax cut.