DEALING WITH UNCERTAINTY: EXPECTED VALUES, SENSITIVITY ANALYSIS, AND THE VALUE OF INFORMATION

EE465/EE463 Project Evaluation

Semester 2/2014

Based on Chapter 7 of *Cost-Benefit Analysis: Concepts and Practice*, Boardman et al., 2011.

Topics

- Expected value analysis
- Sensitivity analysis
- Information and quasi-option value

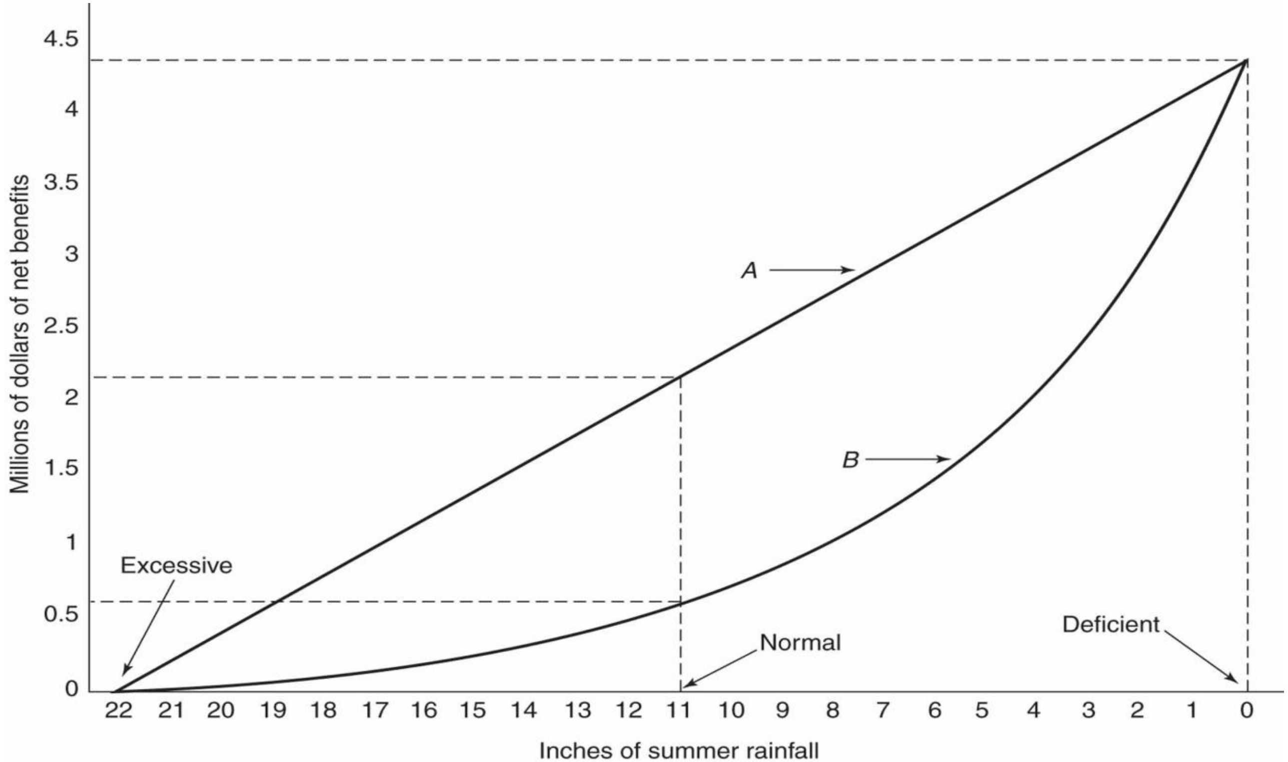
EXPECTED VALUE ANALYSIS

Expected Value Analysis

- Expected value analysis consists of modeling uncertainty as contingencies with specific probabilities of occurrence.
- Modeling uncertainty as risks begins with the specification of a set of contingencies that are *exhaustive* and *mutually exclusive*.
- Contingencies – possible events, outcomes ,or states of the world such that one and only one of the relevant set of possibilities will actually occur.
  - Two considerations:

1. the contingencies capture the full range of likely variation in net benefits of the policies
2. The contingencies (scenarios) represent the possible outcomes between the extremes.

Representatives of Contingencies



- Suppose there are 2 contingencies: “excessive” and “deficient”. Assume all the rainfall amounts between 0 and 22 are equally likely.
  - E[NB] = 0.5*0 + 0.5*4.4 = 2.2
- If NB follows line B <u>and</u> there are 3 contingencies:

  E[NB] = (1/3*0)+(1/3*0.6)+(1/3*4.4) = 1.6

Example: Rainfall and irrigation with 3 outcomes

Now suppose that there are three contingencies (low, normal, high), and the corresponding probabilities are:

- prob(low) = ¼ ; NB(low) = 4.5
- prob(normal) = ½ ; NB(normal) = 0.6
- prob(high) = ¼ ; NB(high) = 0.0

➔ *E[NB] = 0.25*4.5 + 0.50*0.6 + 0.25*0.0 = 1.425*

Note: Probabilities may be based solely on historically observed frequencies; on subjective assessments by clients, analysts, or other experts based on a variety of information and theory, or both.

Calculating the Expected Value of Net Benefits

Let p_i = probability of event *i* occurring ($0 \le p_i \le 1$)

B_i = benefit if event *i* occurs

C_i = cost if event *i* occurs

The expected net benefits are given by:

$$E[NB] = \sum_{i=1}^{n} p_i(B_i - C_i)$$

- Analysts often model risk problems as "*game against nature*" – assumes that nature will randomly, and nonstrategically, select a particular state of the world.
- A game against nature in *normal form* has the followings:
  - *State of nature* and the *probabilities of occurrences, actions*
  - *Payoffs*

A Game against Nature: Expected Values of Asteroid Defense Alternatives

State of Nature	*Exposure to a Collision with an Asteroid Larger Than One Kilometer in Diameter*	*Exposure to a Collision with an Asteroid between 20 Meters and 1 Kilometer in Diameter*	*No Exposure to Collision with an Asteroid Larger Than 20 Meters in Diameter*	
Probabilities of states of nature (over next century)	.001	.004	.995	
Actions (alternatives)	*Payoffs (net costs in billions of 2000 dollars)*			*Expected Value*
Forward-based asteroid defense	5,060	1,060	60	69
Near-Earth asteroid defense	10,020	2,020	20	**38**
No asteroid defense	30,000	6,000	0	54

Choose near-Earth asteroid defense: Expected net cost = $38 billion.

Example - Expected value of payoffs (present value of net costs) for forward-based asteroid defense is:

(0.001*$5,060m) + (0.004*$1,060m) + (0.995*$60m) = $59m

Decision Trees and Expected Net Benefits

- Basic expected value analysis can be extended to situations where costs and benefits accrue over several years, as long as the risks in each year are independent of the actions in the previous year.
  - Calculate the PV of expected net benefits for each year and calculate the NPV of this stream of benefits.
- However, when either the net benefits accruing under contingencies or the probabilities of the contingencies depend on the previous contingencies, then **decision analysis** is needed.
- Decision analysis can be thought of as a *sequential, or extended-form game* against nature.

Decision Analysis

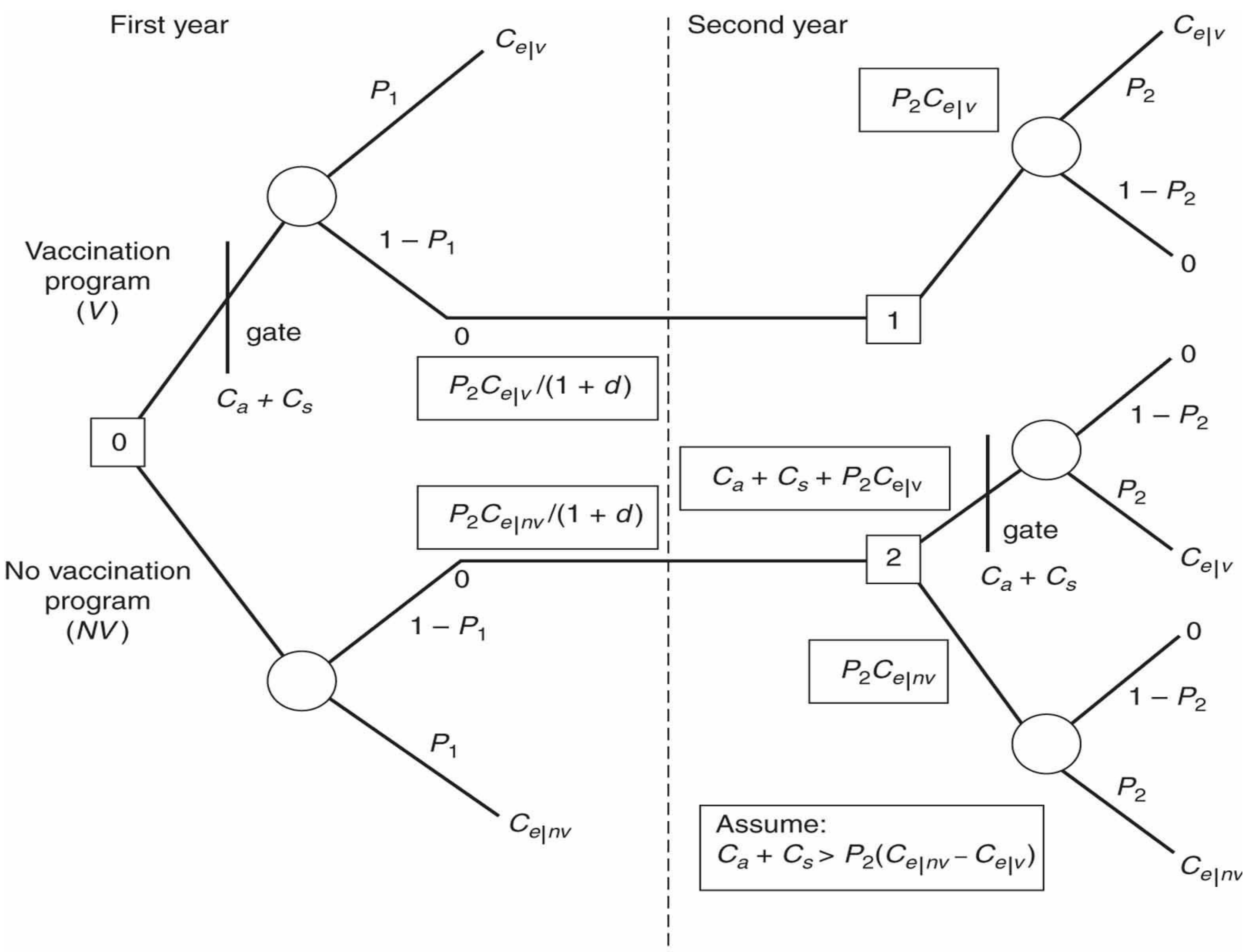
Decision analysis has two stages:

- First, specify the logical structure of the decision problem in terms of sequences of decisions and realizations of contingencies using a diagram (called a *decision tree*) that links an initial decision (*trunk*) to final outcomes (*branches*).
- Second, use *backward induction* (from final outcomes to the initial decision), calculate expected values of net benefits across contingencies and eliminate dominated branches (ones with lower expected values of net benefits).

Example: CBA of a Vaccination Program

- Consider a vaccination program against a type of influenza (which may infect a population over the next 2 years).
  - Costs – immunization expenditures (C_a) and possible adverse side effects (C_s)
  - Benefits – the adverse health effects that are avoided if an epidemic occurs.
- Decisions:
  1. Implement vaccination program
  2. No vaccination program
- Let P be the probability that the influenza infects the population.
- Costs from adverse health effects when epidemic occur:
  - $C_{e|v}$ – costs when epidemic occurs given that the vaccination program has been implemented.
  - $C_{e|nv}$ – costs when epidemic occurs given that the vaccination program has not been implemented.

Decision Tree for Vaccination Program



Solving the Decision Tree

Numerical Example

- Let $P_1 = 0.2$, $P_2 = 0.2$, $d = 0.05$, $C_{e|v} = 0.5C_{e|nv}$, $C_a = 0.1C_{e|nv}$, and $C_s = 0.01C_{e|nv}$

SENSITIVITY ANALYSIS

Sensitivity Analysis

There are several key ideas to sensitivity analysis:

- We face uncertainty about the predicted impacts and the values assigned to them.
- Most plausible estimates comprise the *base case*.
- The purpose of sensitivity analysis is to show how sensitive predicted net benefits are to changes in assumptions.
  - If the sign of net benefits doesn't change after considering the range of assumptions, then the analysis is robust and we can have greater confidence in it.
- Looking at all combinations of assumptions is infeasible.

3 Main Approaches in Sensitivity Analysis

1. Partial sensitivity analysis:

- Asks: how do net benefits change as *one* assumption varies?
- Should be used for the most important or uncertain assumptions.

2. Best/worst case analysis:

- Can be used to find worst-case and best-case scenarios (subset of assumptions).

3. Monte Carlo sensitivity analysis:

- Creates a distribution of net benefits from drawing key assumptions from a probability distribution, with variance and mean drawn from information on the risk of the project.

Vaccination Example Revisited

Assume the followings:

- 2 groups of population: high risk and low risk (from influenza)
- Only a fraction of each group can be recruited to receive vaccine.
- Some fraction of the vaccinated will suffer from possible side-effects and will convert to high-risk status; costs included in C_s.
- Those who contract the flu must be confined to bed rest for a number of days. → loss = # of hours lost x wage rate
- The value of each life save is $3 million.
- 2 benefits from vaccine: self-immunity and herd immunity

➔ $C_{e|v} < C_{e|nv}$

Base-Case Values for Vaccination Program CBA

Parameter	*Value [Range]*	*Comments*
County Population (N)	380,000	Total population in the county
Fraction High Risk (r)	.06 [.04, .08]	One-half population over age 64
Low-Risk Vaccination Rate (v_l)	.05 [.03, .07]	Fraction of low-risk persons vaccinated
High-Risk Vaccination Rate (v_h)	.60 [.40, .80]	Fraction of high-risk persons vaccinated
Adverse Reaction Rate (α)	.03 [.01, .05]	Fraction vaccinated who become high risk
Low-Risk Mortality Rate (m_l)	.00005 [.000025, .000075]	Mortality rate for low-risk infected
High-Risk Mortality Rate (m_h)	.001 [.0005, .002]	Mortality rate for high-risk infected
Herd Immunity Effect (θ)	1.0 [.5, 1.0]	Fraction of effectively vaccinated who contribute to herd immunity effect
Vaccine Effectiveness Rate (e)	.75 [.65, .85]	Fraction of vaccinated who develop immunity
Hours Lost (t)	24 [18, 30]	Average number of work hours lost to illness

Base-Case Values for Vaccination Program (Cont'd)

Infection Rate (i)	.25 [.20, .30]	Infection rate without vaccine
First-Year Epidemic Probability (p_1)	.40	Chance of epidemic in current year
Second-Year Epidemic Probability (p_2)	.20	Chance of epidemic next year
Vaccine Dose Price (q)	$9/dose	Price per dose of vaccine
Overhead Cost (o)	$120,000	Costs not dependent on number vaccinated
Opportunity Cost of Time (w)	$12/hour	Average wage rate (including benefits) in the county
Value of Life (L)	$3,000,000	Assumed value of life
Discount Rate (d)	.05	Real discount rate
Number High-Risk Vaccinations (V_h)	13,680	High-risk persons vaccinated: $v_h rN$
Number Low-Risk Vaccinations (V_l)	17,860	Low-risk persons vaccinated: $v_l(1 - r)N$
Fraction Vaccinated (v)	.083	Fraction of total population vaccinated: $rv_h + v_l(1 - r)$

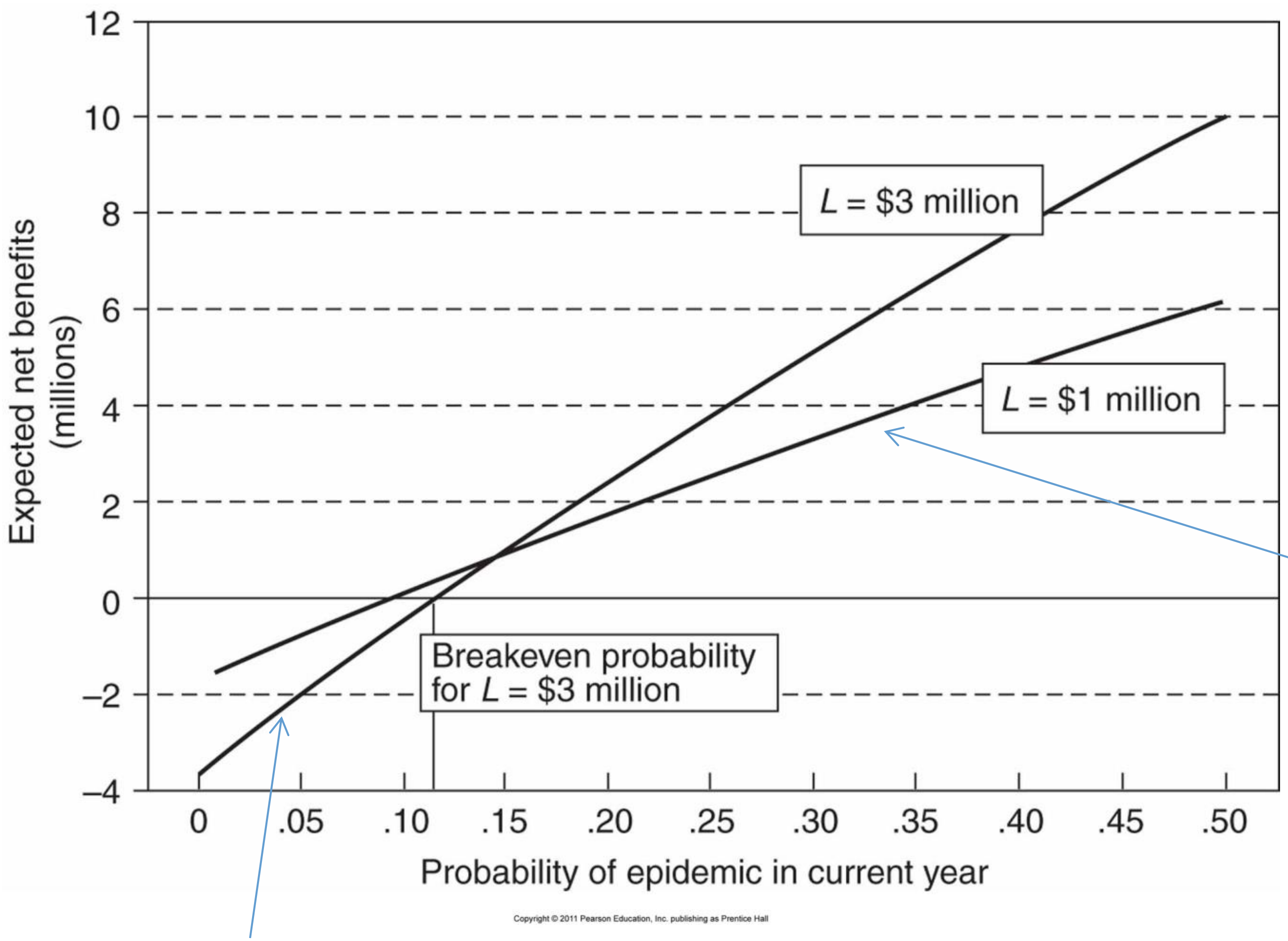
Formulas for Calculating the Net Benefits of Vaccination Program

Variable	Value (millions of dollars)	Formula
C_a	0.404	$o + (V_h + V_l)q$
C_s	3.111	$\alpha(V_h + V_l)(wt + m_h L)$
$C_{e\|nv}$	57.855	$i[rN(wt + m_h L) + (1 - r)N(wt + m_l L)]$
$C_{e\|v}$	36.014	$(i - \theta ve)\{(rN - eV_h)(wt + m_h L) + [(1 - r)N - eV_l](wt + m_l L)\}$
EC_v	22.036	$C_a + C_s + p_1 C_{e\|v} + (1 - p_1) p_2 C_{e\|v}/(1 + d)$
EC_{nv}	29.754	$p_1 C_{e\|nv} + (1 - p_1) p_2 C_{e\|nv}/(1 + d)$
$E[NB]$	7.718	$EC_{nv} - EC_v$

I. Partial Sensitivity Analysis

- The value of a parameter where net benefits switch sign is called the *breakeven* value.
- The idea is to choose an important assumption in the model and find the "breakeven" value for the parameter of interest.
  - Ask: At what value of "x" would the project be worthwhile?
- A thorough investigation of sensitivity ideally considers the impact of changes in each of the important assumptions.
- *Practical problem: how can you determine what is important (and therefore crucial for sensitivity analysis) before doing the sensitivity analysis!*
  - Need to perform the sensitivity analysis before identifying important assumptions ("chicken and egg" problem)

Expected Net Benefits of Vaccination

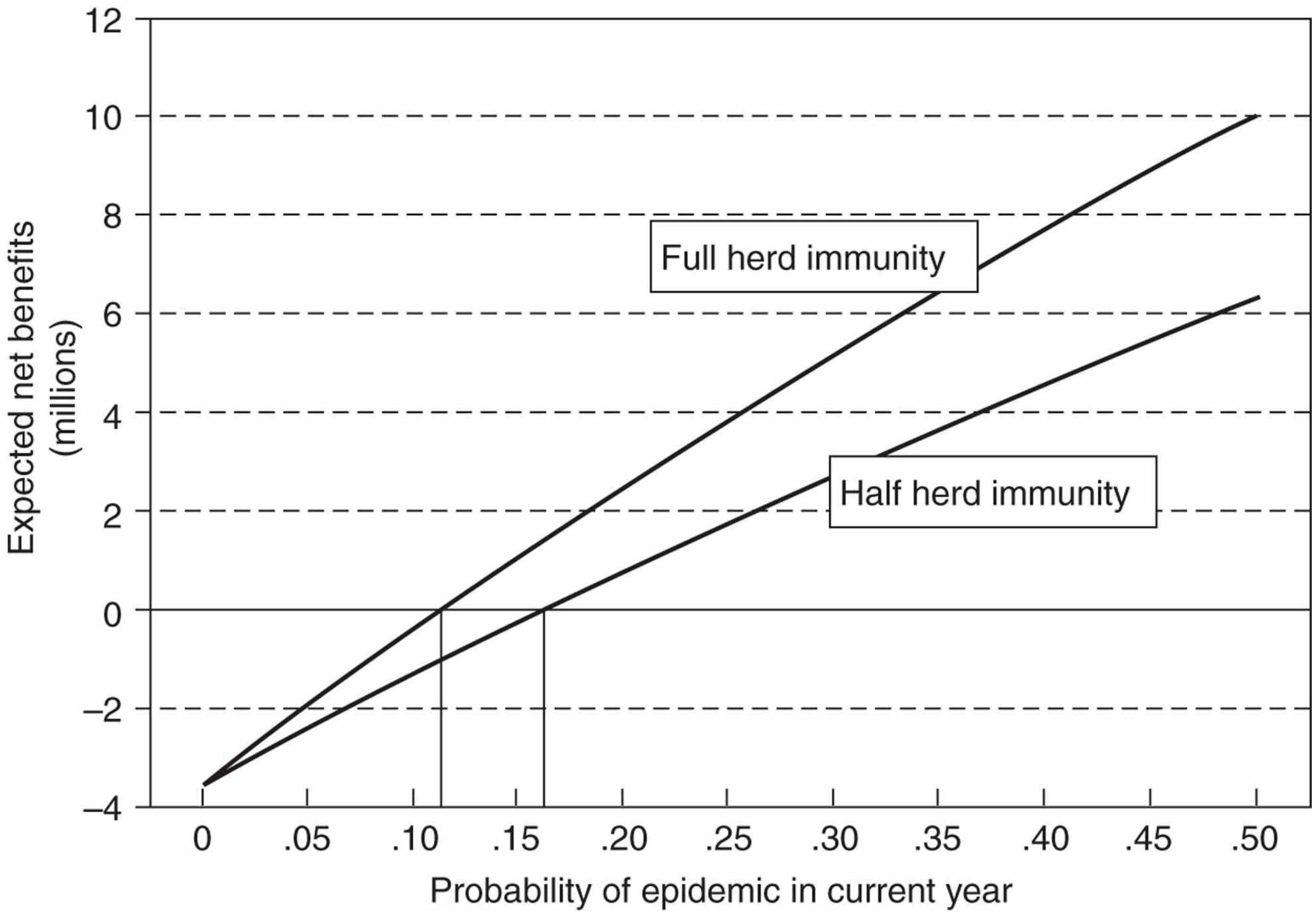


Change the assumptions on the value of life saved

Changing the assumption for P1 (range from 0 to 0.5).

Expected Net Benefits of Vaccination

(change the assumption on the size of the herd immunity effect)



II. Best and Worst Case Analysis

- Base-Case Analysis: Assign the most *plausible* numerical values to unknown parameters to produce an estimate of net benefits that is thought to be most representative.
- Worst-Case Analysis: Assign the least favorable of the plausible range of values to the parameters; use the most conservative assumptions.
  - Most valuable when E[NB] is positive.
  - Acknowledges that the decision makers may be risk-averse.
- Best Case: Assign the most favorable of the plausible range of values to the parameters.
  - Most valuable when E[NB] is negative.

III. Monte Carlo Sensitivity Analysis

- Partial and best/worst case sensitivity analyses have two limitations.
  1. They may not take account of all of the available information about the assumed values of parameters.
  2. These techniques do not directly provide information about the variance of the statistical distribution of the realized net benefits.
- The essence of Monte Carlo analysis is playing games of chance many times to elicit a distribution of outcomes.
  - It plays an important role in the investigation of statistical estimators whose properties cannot be adequately determined through mathematical techniques alone.

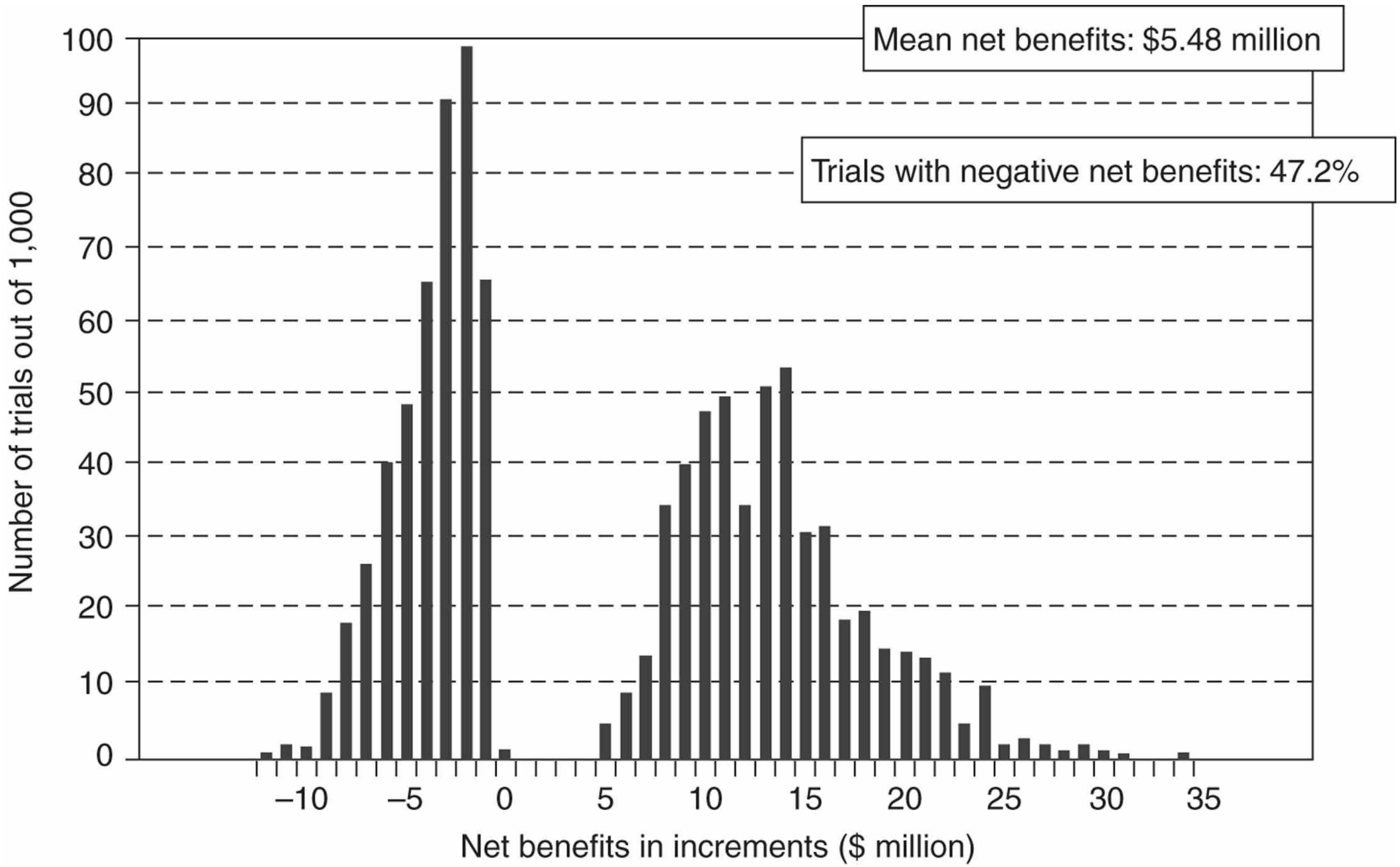
Basis Steps of Monte Carlo Analysis (MCA)

1. Specify probability distributions for all of the important uncertain quantitative assumptions .
   - If all values are equally likely, use a uniform distribution.
   - If a value near the expected value is more plausible, use a normal distribution.

2. Execute a trial by taking a random draw from the distribution for each parameter to arrive at a specific value for computing realized net benefits.

3. Repeat the trial many times to generate a distribution of NB.
   - An approximation of the probability distribution of net benefits can be obtained by creating a histogram.

Histogram of Realized Net Benefits



INFORMATION AND QUASI-OPTION VALUE

The Value of Information

- The value of information in the context of a game against nature answers the question: By how much would the information increase the expected value of playing the game?
  - Compare the expected net benefits of the optimal choice in the game without information with the expected net benefits resulting from the optimal choice in the game with information.
  - The value of the information is the difference between the net benefits.
- The value of information derives from the fact that it leads to different optimal decisions (i.e., if the end decision doesn't change, the value doesn't provide any value).

Reformulated Games against Nature:

Value of Device for Detecting Large Asteroids

State of Nature	*Game One* *p* = .001 — Exposure to a Collision with an Asteroid Larger Than 1 Kilometer in Diameter	*Game Two* *p* = .999 — Exposure to a Collision with an Asteroid between 20 Meters and 1 Kilometer in Diameter	*Game Two* *p* = .999 — No Exposure to Collision with an Asteroid Larger Than 20 Meters in Diameter
Probabilities of states of nature (over next century)	1	.004004	.995996

Game 1 | Game 2

Actions (alternatives)	Game 1: Payoffs (net costs in billions of 2000 dollars)	Game 1: Expected Value	Game 2: Payoffs (net costs in billions of 2000 dollars)		Game 2: Expected Value
Forward-based asteroid defense	5,060	**5,060**	1,060	60	64.01
Near-Earth asteroid defense	10,020	10,020	2,020	20	28.01
No asteroid defense	30,000	30,000	6,000	0	**24.02**

- Game 1: Choose 'forward-based asteroid defense' (ENC = 5,060 $bn)
- Game 2: Choose 'no asteroid defense' (ENC = 24.02 $bn)
- Expected net cost of decision with detection device:

  (0.001*5060) + (0.999*24.02) = $29.06 bn

➔ Value of information provided by detection device: $38 bn - $29.06 bn= $8.94 bn

Quasi-Option Value

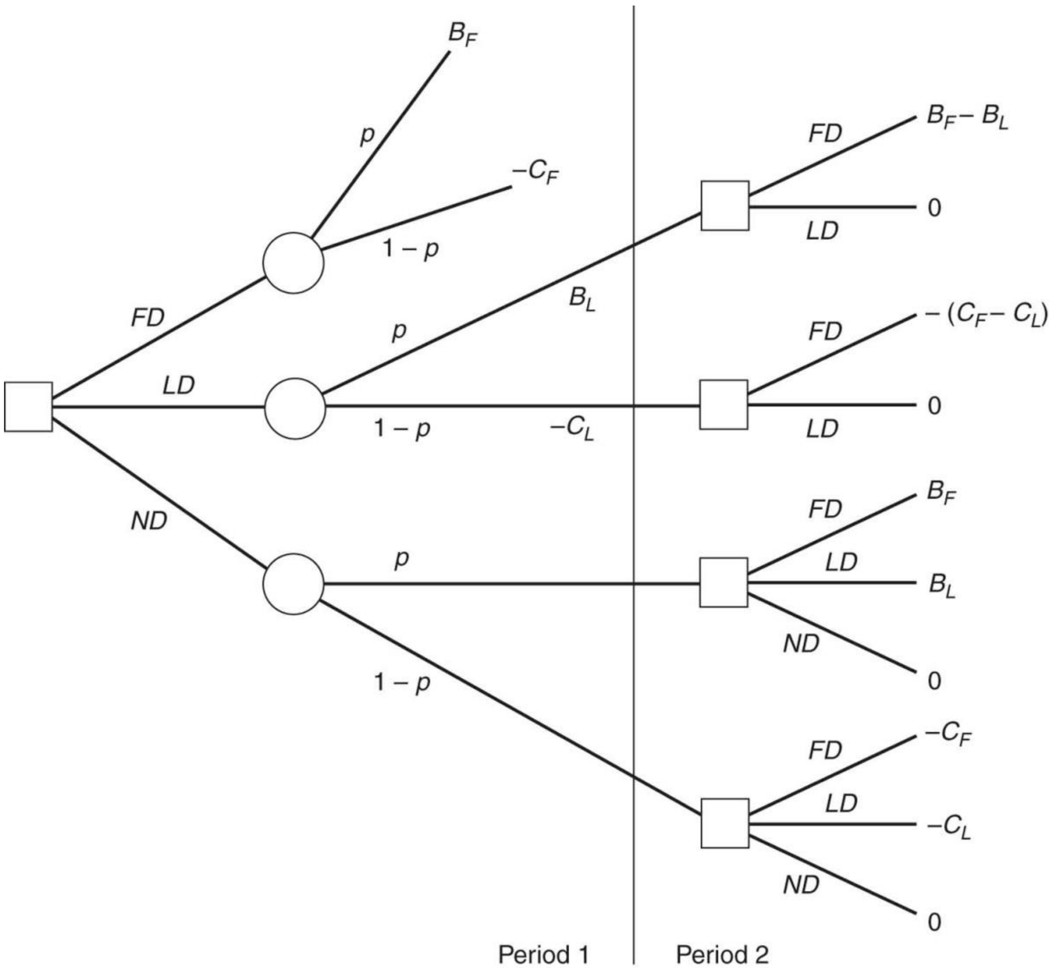
- Quasi-option value is the expected value of information gained by delaying an *irreversible decision*.
  - It can be quantified by formulating a multi-period decision problem that allows for the revelation of information about the value of options in later periods.
- 2 types of learning:
  1. Exogenous learning: learning is revealed no matter what option is taken. After the first period we discover with a certainty which of the two contingencies will occur.
  2. Endogenous learning: information is generated only through the activity (whatever the program is) itself.

Benefits and Costs of Alternative Development Policies Assuming No Learning

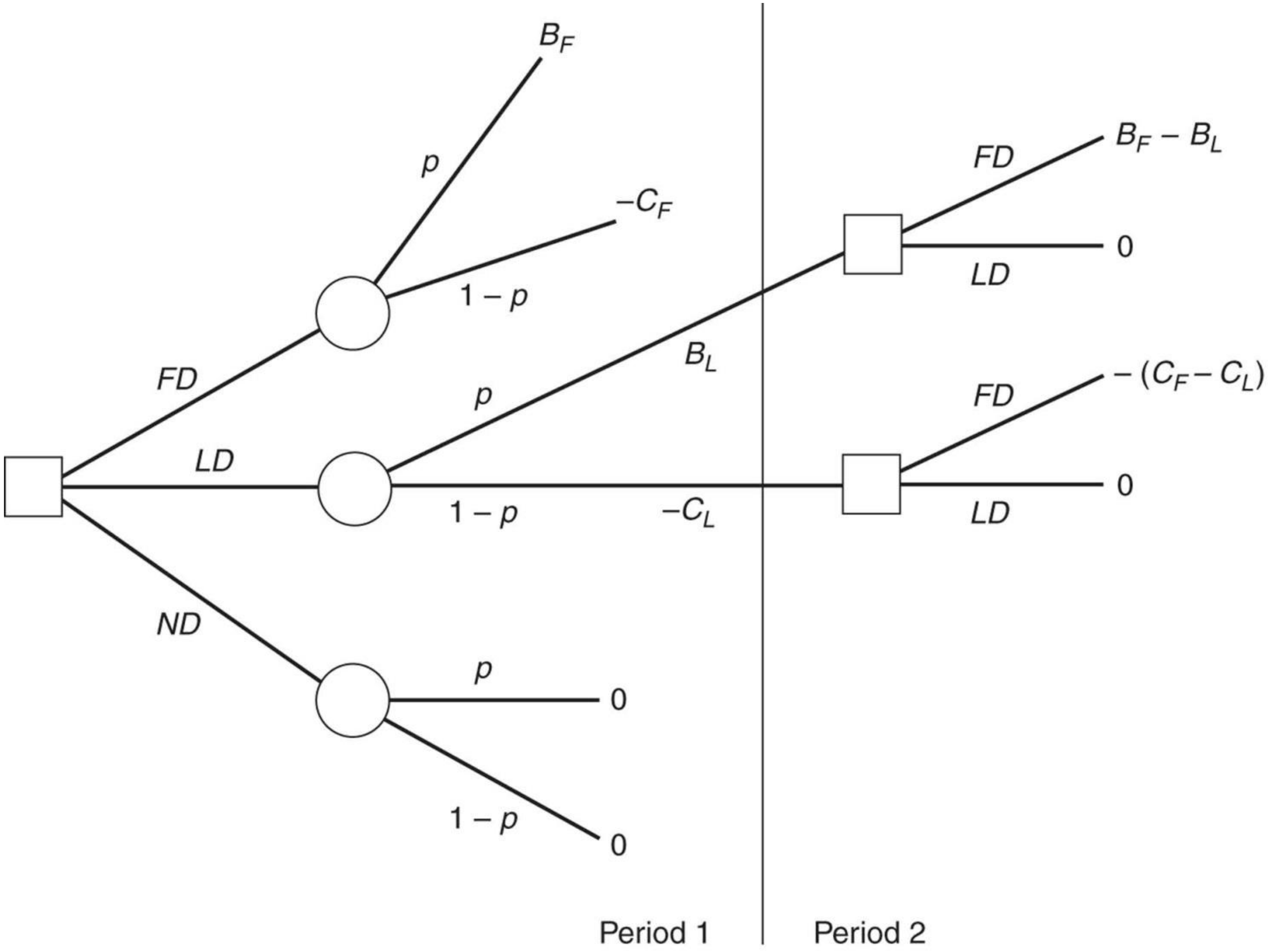
	Preservation Contingencies	
	Low Value	***High Value***
Full development (FD)	B_F	$-C_F$
Limited development (LD)	B_L	$-C_L$
No development (ND)	0	0
Probability of contingency	p	$1-p$

Expected value of full development:	$E[FD] = pB_F - (1-p)C_F$
Expected value of limited development:	$E[LD] = pB_L - (1-p)C_L$
Expected value of no development:	$E[ND] = 0$
Adopt full development if:	$pB_F - (1-p)C_F > pB_L - (1-p)C_L$ and $pB_F - (1-p)C_F > 0$

Exogenous Learning



Endogenous Learning



Expected Values for Decision Problems: Quasi-Option Values (QOV) Measured Relative to No Learning Case

	No Learning	Exogenous Learning	Endogenous Learning
$E[FD]$	$pB_F - (1-p)C_F$	$pB_F - (1-p)C_F$ $QOV = 0$	$pB_F - (1-p)C_F$ $QOV = 0$
$E[LD]$	$pB_L - (1-p)C_L$	$p[B_L + (B_F - B_L)/(1+d)]$ $-(1-p)C_L$ $QOV = p(B_F - B_L)/(1+d)$	$p[B_L + (B_F - B_L)/(1+d)]$ $-(1-p)C_L$ $QOV = p(B_F - B_L)/(1+d)$
$E[ND]$	0	$pB_F/(1+d)$ $QOV = pB_F/(1+d)$	0 $QOV = 0$

Numerical Illustration of Quasi-Option Value ($ million)

Assumptions:				
$B_F = 100$	$C_F = 80$			
$B_L = 50$	$C_L = 40$			
$p = .5$	$d = .08$	**No Learning**	**Exogenous Learning**	**Endogenous Learning**
$E[FD]$		10.00	10.00	10.00
$E[LD]$		5.00	28.15	28.15
$E[ND]$		0.00	46.30	0.00