Assignment 4

DUE DATE: Tuesday 9^{nd}, March 2021.

I pledge to the Honor Code and to obey all rules for taking and performing homework assignments as specified by the course instructor.

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Question 1 (50 points) Your score......................

Given the daily log returns : (R_t) can be explained by the AR(2) model as following:

$$(1 - 1.5B + 0.9B^2)R_t = 0.25 + \varepsilon_t$$

where ε_t is distributed as the Gaussian White Noise with mean $(\mu) = 0$ and variance (σ^2) =0.25
B lag-operator

Question 1.1 (10 points) Your score......................

From the above AR(2) model, Is the model weakly stationary? Write down the reverse characteristic equation and find out the conditions to support your answer.

characteristic eq = $1 - 1.5B + 0.9B^2$

$\therefore$ reverse characteristic eq is $x^2 - 1.5x + 0.9 = 0$

$\lambda_i = a + bi$

$\lambda_1 = 0.75 + 0.581i$

$\lambda_2 = 0.75 - 0.581i$

which $R_1 = \sqrt{a^2 + b^2}$

$R = 0.9487$ #

R is less than 1. So, the model is weakly stationary

Question 1.2 (10 points) | Your score......................

$$(1-1.5B+0.9B^2)R_t = 0.25+\varepsilon_t$$

Calculate the unconditional mean: $E(R_t)$ of R_t and the conditional mean: $E(R_t|F_{t-1})$

$$R_t - 1.5R_{t-1} + 0.9R_{t-2} = 0.25 + \varepsilon_t$$

$$R_t = 0.25 + 1.5R_{t-1} - 0.9R_{t-2} + \varepsilon_t$$

Unconditional mean:

white noise at $\sim(0,\sigma^2)$

$$E(R_t) = 0.25 + 1.5\,E(R_{t-1}) - 0.9\,E(R_{t-2}) + \cancel{E(\varepsilon_t)}$$

* from weak stationary $E(r_t) = E(r_{t-1}) = \ldots = \mu$ ← constant

$$E(R_t) = 0.25 + 1.5\,E(R_t) - 0.9\,E(R_t)$$

$$E(R_t)(1-1.5+0.9) = 0.25$$

$$E(R_t) = \frac{0.25}{0.40} = 0.625 \ \#$$

Conditional mean:

$$E(R_t|F_{t-1}) = 0.25 + 1.5(R_{t-1}) - 0.9R_{t-2} + \cancel{E(\varepsilon_t|F_{t-1})}^{\,0}$$

$$E(R_t|F_{t-1}) = 0.25 + 1.5R_{t-1} - 0.9R_{t-2} \ \#$$

Question 1.3 (10 points) Your score......................

$R_t = 0.25 + 1.5 R_{t-1} - 0.9 R_{t-2} + \varepsilon_t$

Find out the unconditional variance: $Var(R_t)$ of R_t and conditional variance $Var(R_t|F_{t-1})$ of R_t

unconditional Var: $R_t - \mu = 0.25 + 1.5(R_{t-1}-\mu) + (-0.9)(R_{t-2}-\mu) + \varepsilon_t$

$$(R_t-\mu)^2 = (0.25)^2 + (1.5)^2(R_{t-1}-\mu)^2 + (-0.9)^2(R_{t-2}-\mu)^2 + \varepsilon_t^2$$
$$+ 2(0.25)(1.5)(R_{t-1}-\mu)$$
$$+ 2(0.25)(-0.9)(R_{t-2}-\mu)$$
$$+ 2(0.25)(\varepsilon_t)$$
$$+ 2(1.5)(-0.9)(R_{t-1}-\mu)(R_{t-2}-\mu)$$
$$+ 2(1.5)(R_{t-1}-\mu)(\varepsilon_t)$$
$$+ 2(0.9)(R_{t-2}-\mu)(\varepsilon_t)$$

take $E(\cdot)$

from weak stationary $Var(R_t) = Var(R_{t-1}) = \ldots$

$\sigma^2 = 0.25$

$$Var(R_t) = 0.0625 + 2.25\, Var(R_t) + 0.81\, Var(R_t) + Var(\varepsilon_t)$$
$$+ 0.75\, E(\cancel{R_{t-1}-\mu})^{0}$$

$E(R_{t-1}) = \mu$

$$- 0.45\, E(\cancel{R_{t-2}-\mu})^{0}$$
$$+ 0.5\, E(\cancel{\varepsilon_t})\ 0$$
$$- 2.7\, E[(R_{t-1}-\mu)(R_{t-2}-\mu)] \quad \gamma_1$$
$$+ 3\, E[\cancel{(R_{t-1}-\mu)(\varepsilon_t)}]^{0}$$
$$+ 1.8\, E[\cancel{(R_{t-2}-\mu)(\varepsilon_t)}]\ 0$$

$$Var(R_t) = \frac{0.3125 - 2.7\gamma_1}{-2.06} \quad \#$$

$Cov(r_t, r_{t-j})$

$$r_t - \mu = 0.25 + 1.5(R_{t-1} - \mu) - 0.9(R_{t-2} - \mu) + \varepsilon_t$$

$$E[(r_t - \mu)(r_{t-j} - \mu)] = 0.25\,E(\cancel{r_{t-j}} - \mu)^{0} + 1.5\,E[(r_{t-1} - \mu)(r_{t-j} - \mu)]$$
$$- 0.9\,E[(r_{t-2} - \mu)(r_{t-j} - \mu)] + E[(\varepsilon_t)\cancel{(r_{t-j} - \mu)}^{0}]$$

$$\gamma_j = 1.5\,\gamma_{j-1} - 0.9\,\gamma_{j-2}$$

$$\therefore \quad \gamma_1 = 1.5\,\gamma_0 - 0.9\gamma_1$$

$$1.9\,\gamma_1 = 1.5\,Var(r_t)$$

$$\gamma_1 = \frac{1.5}{1.9} Var(r_t)$$

$$\gamma_1 = 0.8\,Var(r_t)$$

$$-2.06\,Var(R_t) = 0.3125 - 2.7(0.8)\,Var(R_t)$$

$$(0.1)Var(R_t) = 0.3125$$

$$Var(R_t) = 3.125 \; \#$$

Conditional var

$$Var(r_t | F_{t-1}) = 2.25 Var(\cancel{r_{t-1}} | F_{t-1}) + 0.81 Var(\cancel{r_{t-2}} | F_{t-1})$$

$$+ Var(\varepsilon_t)$$

$$+ 0 \ldots 0$$

$$= \sigma^2_{\varepsilon t} = 0.25 \#$$

Question 1.4 (10 points) | Your score......................

Calculate the autocorrelation: ρ_l for l=1 and 2 of R_t. Also, write down the autocorrelation: ρ_l when l $\geq$ 2.

from $\gamma_j = 1.5\,\gamma_{j-1} - 0.9\,\gamma_{j-2}$

$$\rho_i = \frac{\gamma_j}{var(r_t)} = \frac{\gamma_j}{\gamma_0} = 1.5\frac{\gamma_{j-1}}{\gamma_0} - 0.9\frac{\gamma_{j-2}}{\gamma_0}$$

$$\rho_j = 1.5\,\rho_{j-1} - 0.9\,\rho_{j-2} \ \#$$

$\rho_1 = 1.5\,\rho_0 - 0.9\,\rho_1$

$\rho_1 = \frac{1.5}{1.9}$

$\rho_1 = 0.8 \ \#$

$\rho_2 = 1.5\,\rho_1 - 0.9\,\rho_0$

$\rho_2 = 1.5(0.8) - 0.9$

$\rho_2 = 0.3 \ \#$

Question 1.5 (10 points)

Your score......................

Given $R_{1000} = 0.01$ $R_{999} = 0.02$ $R_{998} = 0.03$ $\varepsilon_{1000} = -0.01$ $\varepsilon_{999} = -0.02$ $\varepsilon_{998} = -0.03$ Obtain 1-step, 2-setp 95 % interval forecasts for R_t at the forecast origin t = 1000. Also the ∞-step 95 % interval forecasts for R_t. Draw these intervals.

$$R_t = 0.25 + 1.5R_{t-1} - 0.9R_{t-2} + \varepsilon_t$$

1-step ahead:

Point estimator : $\hat{R}_{1000}(1) = 0.25 + 1.5R_{1000} - 0.9R_{999}$

$= 0.25 + 1.5(0.01) - 0.9(0.02)$

$\hat{R}_{1000}(1) = 0.247$

forecasting error : $R_{1001} - \hat{R}_{1000}(1) = \varepsilon_{1001}$

variance : $\text{var}(\varepsilon_{1001} | \cdot) = \sigma_\varepsilon^2$ Constant variance

$= 0.25$

Interval Forecasting : $= 0.247 \pm 1.96\sqrt{0.25}$

$= 0.247 \pm 0.98$

$-0.733 \leq R_{1001} \leq 1.227$ # at 95% C.I.

2 step ahead

Point estimator : $\hat{R}_{1000}(2) = 0.25 + 1.5\,\hat{R}_{1000}(1) - 0.9\,R_{1000}$

$= 0.25 + 1.5(0.249) - 0.9(0.01)$

$= 0.6115$ #

forecasting error : $R_{1002} - \hat{R}_{1000}(2) = 1.5[\varepsilon_{1001}] + \varepsilon_{1002}$

variance : $= (1.5)^2 \sigma_a^2 + \sigma_a^2 + \cancel{2\,\mathrm{Cov}(\varepsilon_{1001}, \varepsilon_{1002})}$

$= 3.25(0.25)$

$= 0.8125$ #

Interval Forecasting : $= 0.249 \pm 1.96\sqrt{0.8125}$

$= 0.247 \pm 1.767$

$-1.52 \le \hat{R}_{1000}(2) \le 2.014$ # at 95% CI.