a. If you require a risk premium of 8%, how much will you be willing to pay for the portfolio?
b. Suppose that the portfolio can be purchased for the amount you found in (*a*). What will be the expected rate of return on the portfolio?
c. Now suppose that you require a risk premium of 12%. What is the price that you will be willing to pay?
d. Comparing your answers to (*a*) and (*c*), what do you conclude about the relationship between the required risk premium on a portfolio and the price at which the portfolio will sell?

5. Consider a portfolio that offers an expected rate of return of 12% and a standard deviation of 18%. T-bills offer a risk-free 7% rate of return. What is the maximum level of risk aversion for which the risky portfolio is still preferred to bills?
6. Draw the indifference curve in the expected return–standard deviation plane corresponding to a utility level of .05 for an investor with a risk aversion coefficient of 3. (*Hint:* Choose several possible standard deviations, ranging from 0 to .25, and find the expected rates of return providing a utility level of .05. Then plot the expected return–standard deviation points so derived.)
7. Now draw the indifference curve corresponding to a utility level of .05 for an investor with risk aversion coefficient $A = 4$. Comparing your answer to Problem 6, what do you conclude?
8. Draw an indifference curve for a risk-neutral investor providing utility level .05.
9. What must be true about the sign of the risk aversion coefficient, A, for a risk lover? Draw the indifference curve for a utility level of .05 for a risk lover.

For Problems 10 through 12: Consider historical data showing that the average annual rate of return on the S&P 500 portfolio over the past 80 years has averaged roughly 8% more than the Treasury bill return and that the S&P 500 standard deviation has been about 20% per year. Assume these values are representative of investors' expectations for future performance and that the current T-bill rate is 5%.

10. Calculate the expected return and variance of portfolios invested in T-bills and the S&P 500 index with weights as follows:

W_{bills}	W_{index}
0	1.0
0.2	0.8
0.4	0.6
0.6	0.4
0.8	0.2
1.0	0

11. Calculate the utility levels of each portfolio of Problem 10 for an investor with $A = 2$. What do you conclude?
12. Repeat Problem 11 for an investor with $A = 3$. What do you conclude?

Use these inputs for Problems 13 through 19: You manage a risky portfolio with expected rate of return of 18% and standard deviation of 28%. The T-bill rate is 8%.

13. Your client chooses to invest 70% of a portfolio in your fund and 30% in a T-bill money market fund. What is the expected value and standard deviation of the rate of return on his portfolio?
14. Suppose that your risky portfolio includes the following investments in the given proportions:

Stock A	25%
Stock B	32%
Stock C	43%

What are the investment proportions of your client's overall portfolio, including the position in T-bills?

15. What is the reward-to-volatility ratio (S) of your risky portfolio? Your client's?
16. Draw the CAL of your portfolio on an expected return–standard deviation diagram. What is the slope of the CAL? Show the position of your client on your fund's CAL.
17. Suppose that your client decides to invest in your portfolio a proportion y of the total investment budget so that the overall portfolio will have an expected rate of return of 16%.
    a. What is the proportion y?
    b. What are your client's investment proportions in your three stocks and the T-bill fund?
    c. What is the standard deviation of the rate of return on your client's portfolio?
18. Suppose that your client prefers to invest in your fund a proportion y that maximizes the expected return on the complete portfolio subject to the constraint that the complete portfolio's standard deviation will not exceed 18%.
    a. What is the investment proportion, y?
    b. What is the expected rate of return on the complete portfolio?
19. Your client's degree of risk aversion is $A = 3.5$.
    a. What proportion, y, of the total investment should be invested in your fund?
    b. What is the expected value and standard deviation of the rate of return on your client's optimized portfolio?
20. Look at the data in Table 6.7 on the average risk premium of the S&P 500 over T-bills, and the standard deviation of that risk premium. Suppose that the S&P 500 is your risky portfolio.
    a. If your risk-aversion coefficient is $A = 4$ and you believe that the entire 1926–2009 period is representative of future expected performance, what fraction of your portfolio should be allocated to T-bills and what fraction to equity?
    b. What if you believe that the 1968–1988 period is representative?
    c. What do you conclude upon comparing your answers to (*a*) and (*b*)?
21. Consider the following information about a risky portfolio that you manage, and a risk-free asset: $E(r_P) = 11\%$, $\sigma_P = 15\%$, $r_f = 5\%$.
    a. Your client wants to invest a proportion of her total investment budget in your risky fund to provide an expected rate of return on her overall or complete portfolio equal to 8%. What proportion should she invest in the risky portfolio, P, and what proportion in the risk-free asset?
    b. What will be the standard deviation of the rate of return on her portfolio?
    c. Another client wants the highest return possible subject to the constraint that you limit his standard deviation to be no more than 12%. Which client is more risk averse?
22. Investment Management Inc. (IMI) uses the capital market line to make asset allocation recommendations. IMI derives the following forecasts:
    - Expected return on the market portfolio: 12%.
    - Standard deviation on the market portfolio: 20%.
    - Risk-free rate: 5%.

    Samuel Johnson seeks IMI's advice for a portfolio asset allocation. Johnson informs IMI that he wants the standard deviation of the portfolio to equal half of the standard deviation for the market portfolio. Using the capital market line, what expected return can IMI provide subject to Johnson's risk constraint?

For Problems 23 through 26: Suppose that the borrowing rate that your client faces is 9%. Assume that the S&P 500 index has an expected return of 13% and standard deviation of 25%, that $r_f = 5\%$, and that your fund has the parameters given in Problem 21.

9. Diversification is based on the allocation of a *fixed* portfolio across several assets, limiting the exposure to any one source of risk. Adding additional risky assets to a portfolio, thereby increasing the total amounts invested, does not reduce dollar risk, even if it makes the rate of return more predictable. This is because that uncertainty is applied to a larger investment base. Nor does investing over longer horizons reduce risk. Increasing the investment horizon is analogous to investing in more assets. It increases total risk. Analogously, the key to the insurance industry is risk sharing—the spreading of risk across many investors, each of whom takes on only a small exposure to any given source of risk. Risk pooling—the assumption of ever-more sources of risk—may increase rate of return predictability, but not the predictability of total dollar returns.

Related Web sites for this chapter are available at **www.mheducation.asia/olc/bodie**

Key Terms

diversification
insurance principle
market risk
systematic risk
nondiversifiable risk
unique risk
firm-specific risk
nonsystematic risk
diversifiable risk
minimum-variance portfolio
portfolio opportunity set
reward-to-volatility ratio
optimal risky portfolio
minimum-variance frontier
efficient frontier of risky assets
input list
separation property
risk pooling
risk sharing

Problem Sets

Basic

1. Which of the following factors reflect *pure* market risk for a given corporation?
   a. Increased short-term interest rates.
   b. Fire in the corporate warehouse.
   c. Increased insurance costs.
   d. Death of the CEO.
   e. Increased labor costs.

2. When adding real estate to an asset allocation program that currently includes only stocks, bonds, and cash, which of the properties of real estate returns affect portfolio *risk?* Explain.
   a. Standard deviation.
   b. Expected return.
   c. Correlation with returns of the other asset classes.

3. Which of the following statements about the minimum variance portfolio of all risky securities are valid? (Assume short sales are allowed.) Explain.
   a. Its variance must be lower than those of all other securities or portfolios.
   b. Its expected return can be lower than the risk-free rate.
   c. It may be the optimal risky portfolio.
   d. It must include all individual securities.

Intermediate

The following data apply to Problems 4 through 10: A pension fund manager is considering three mutual funds. The first is a stock fund, the second is a long-term government and corporate bond fund, and the third is a T-bill money market fund that yields a rate of 8%. The probability distribution of the risky funds is as follows:

	Expected Return	Standard Deviation
Stock fund (*S*)	20%	30%
Bond fund (*B*)	12	15

The correlation between the fund returns is .10.

4. What are the investment proportions in the minimum-variance portfolio of the two risky funds, and what is the expected value and standard deviation of its rate of return?

5. Tabulate and draw the investment opportunity set of the two risky funds. Use investment proportions for the stock fund of zero to 100% in increments of 20%.

6. Draw a tangent from the risk-free rate to the opportunity set. What does your graph show for the expected return and standard deviation of the optimal portfolio?
7. Solve numerically for the proportions of each asset and for the expected return and standard deviation of the optimal risky portfolio.
8. What is the reward-to-volatility ratio of the best feasible CAL?
9. You require that your portfolio yield an expected return of 14%, and that it be efficient, on the best feasible CAL.
   a. What is the standard deviation of your portfolio?
   b. What is the proportion invested in the T-bill fund and each of the two risky funds?
10. If you were to use only the two risky funds, and still require an expected return of 14%, what would be the investment proportions of your portfolio? Compare its standard deviation to that of the optimized portfolio in Problem 9. What do you conclude?
11. Stocks offer an expected rate of return of 18%, with a standard deviation of 22%. Gold offers an expected return of 10% with a standard deviation of 30%.
    a. In light of the apparent inferiority of gold with respect to both mean return and volatility, would anyone hold gold? If so, demonstrate graphically why one would do so.
    b. Given the data above, reanswer (*a*) with the additional assumption that the correlation coefficient between gold and stocks equals 1. Draw a graph illustrating why one would or would not hold gold in one's portfolio. Could this set of assumptions for expected returns, standard deviations, and correlation represent an equilibrium for the security market?
12. Suppose that there are many stocks in the security market and that the characteristics of stocks *A* and *B* are given as follows:

Stock	Expected Return	Standard Deviation
A	10%	5%
B	15	10

Correlation = −1

Suppose that it is possible to borrow at the risk-free rate, r_f. What must be the value of the risk-free rate? (*Hint:* Think about constructing a risk-free portfolio from stocks *A* and *B*.)

13. Assume that expected returns and standard deviations for all securities (including the risk-free rate for borrowing and lending) are known. In this case all investors will have the same optimal risky portfolio. (True or false?)
14. The standard deviation of the portfolio is always equal to the weighted average of the standard deviations of the assets in the portfolio. (True or false?)
15. Suppose you have a project that has a .7 chance of doubling your investment in a year and a .3 chance of halving your investment in a year. What is the standard deviation of the rate of return on this investment?
16. Suppose that you have $1 million and the following two opportunities from which to construct a portfolio:
    a. Risk-free asset earning 12% per year.
    b. Risky asset with expected return of 30% per year and standard deviation of 40%.

    If you construct a portfolio with a standard deviation of 30%, what is its expected rate of return?

The following data are for Problems 17 through 19: The correlation coefficients between pairs of stocks are as follows: Corr(*A*,*B*) = .85; Corr(*A*,*C*) = .60; Corr(*A*,*D*) = .4 5. Each stock has an expected return of 8% and a standard deviation of 20%.

17. If your entire portfolio is now composed of stock *A* and you can add some of only one stock to your portfolio, would you choose (explain your choice):
    a. *B*.
    b. *C*.
    c. *D*.
    d. Need more data.

18. Would the answer to Problem 17 change for more risk-averse or risk-tolerant investors? Explain.
19. Suppose that in addition to investing in one more stock you can invest in T-bills as well. Would you change your answers to Problems 17 and 18 if the T-bill rate is 8%?

The following table of compound annual returns by decade applies to Challenge Problems 20 and 21.

iii. Challenge

	1920s*	1930s	1940s	1950s	1960s	1970s	1980s	1990s
Small-company stocks	−3.72%	7.28%	20.63%	19.01%	13.72%	8.75%	12.46%	13.84%
Large-company stocks	18.36	−1.25	9.11	19.41	7.84	5.90	17.60	18.20
Long-term government	3.98	4.60	3.59	0.25	1.14	6.63	11.50	8.60
Intermediate-term government	3.77	3.91	1.70	1.11	3.41	6.11	12.01	7.74
Treasury bills	3.56	0.30	0.37	1.87	3.89	6.29	9.00	5.02
Inflation	−1.00	−2.04	5.36	2.22	2.52	7.36	5.10	2.93

*Based on the period 1926–1929.

20. Input the data from the table into a spreadsheet. Compute the serial correlation in decade returns for each asset class and for inflation. Also find the correlation between the returns of various asset classes. What do the data indicate?
21. Convert the asset returns by decade presented in the table into real rates. Repeat the analysis of Challenge Problem 20 for the real rates of return.

CFA® PROBLEMS

The following data apply to CFA Problems 1 through 3: Hennessy & Associates manages a $30 million equity portfolio for the multimanager Wilstead Pension Fund. Jason Jones, financial vice president of Wilstead, noted that Hennessy had rather consistently achieved the best record among the Wilstead's six equity managers. Performance of the Hennessy portfolio had been clearly superior to that of the S&P 500 in 4 of the past 5 years. In the one less-favorable year, the shortfall was trivial.

Hennessy is a "bottom-up" manager. The firm largely avoids any attempt to "time the market." It also focuses on selection of individual stocks, rather than the weighting of favored industries.

There is no apparent conformity of style among Wilstead's six equity managers. The five managers, other than Hennessy, manage portfolios aggregating $250 million made up of more than 150 individual issues.

Jones is convinced that Hennessy is able to apply superior skill to stock selection, but the favorable returns are limited by the high degree of diversification in the portfolio. Over the years, the portfolio generally held 40–50 stocks, with about 2%–3% of total funds committed to each issue. The reason Hennessy seemed to do well most years was that the firm was able to identify each year 10 or 12 issues that registered particularly large gains.

On the basis of this overview, Jones outlined the following plan to the Wilstead pension committee:

> Let's tell Hennessy to limit the portfolio to no more than 20 stocks. Hennessy will double the commitments to the stocks that it really favors, and eliminate the remainder. Except for this one new restriction, Hennessy should be free to manage the portfolio exactly as before.

All the members of the pension committee generally supported Jones's proposal because all agreed that Hennessy had seemed to demonstrate superior skill in selecting stocks. Yet the proposal was a considerable departure from previous practice, and several committee members raised questions. Respond to each of the following questions.

1. *a.* Will the limitation to 20 stocks likely increase or decrease the risk of the portfolio? Explain.
   b. Is there any way Hennessy could reduce the number of issues from 40 to 20 without significantly affecting risk? Explain.