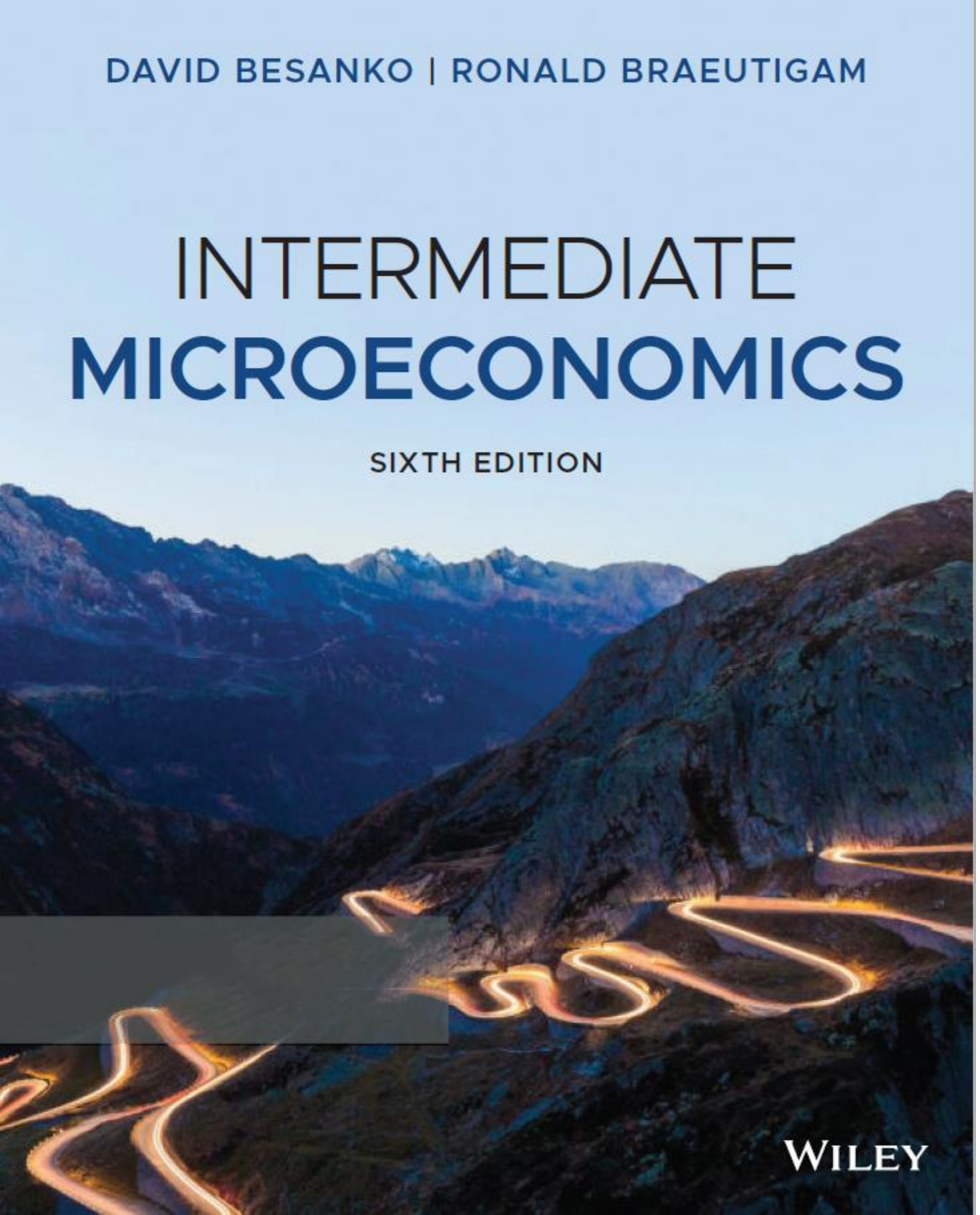


Chapter 8

Cost Curves

Chapter Eight Overview

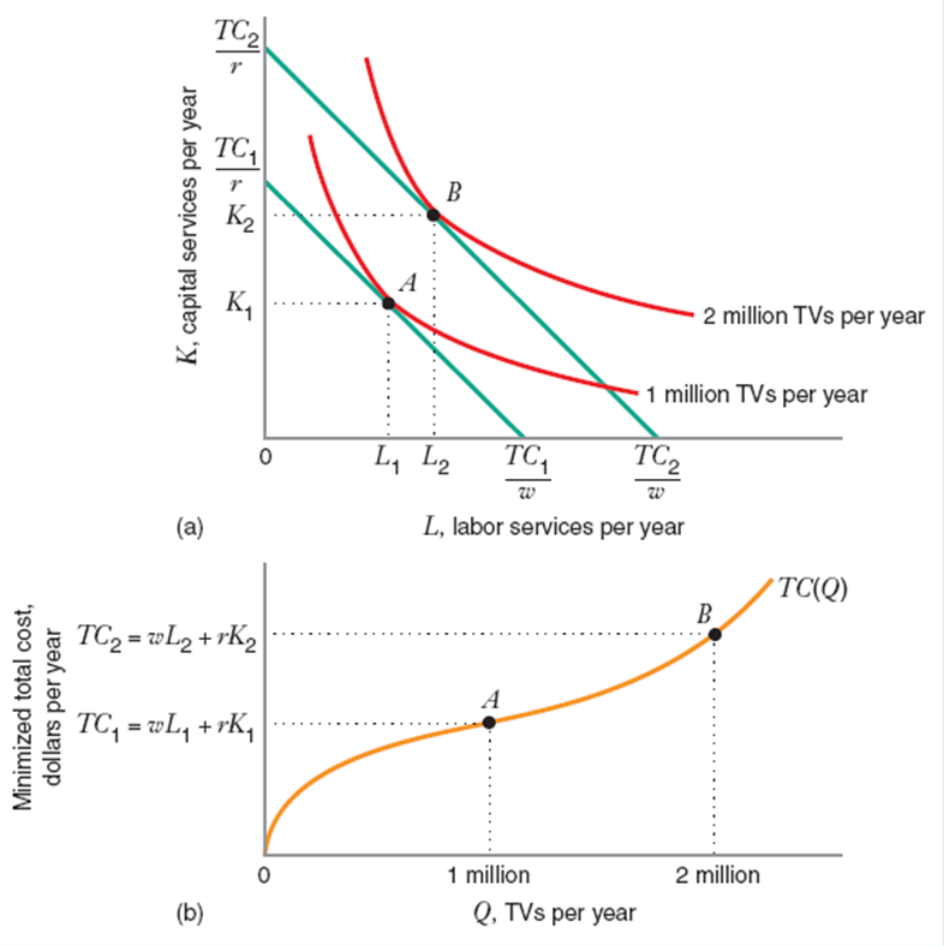
- Long-Run Cost Curves
- Short-Run Cost Curves
- Special Topics in Cost
- Estimating Cost Functions
- Shepard’s Lemma and Duality

Long Run Cost Functions

- The long run total cost function relates minimized total cost to output, Q, and to the factor prices (w and r)
- $TC(Q,w,r) = wL^*(Q,w,r) + rK^*(Q,w,r)$
- Where: L^* and K^* are the long run input demand functions

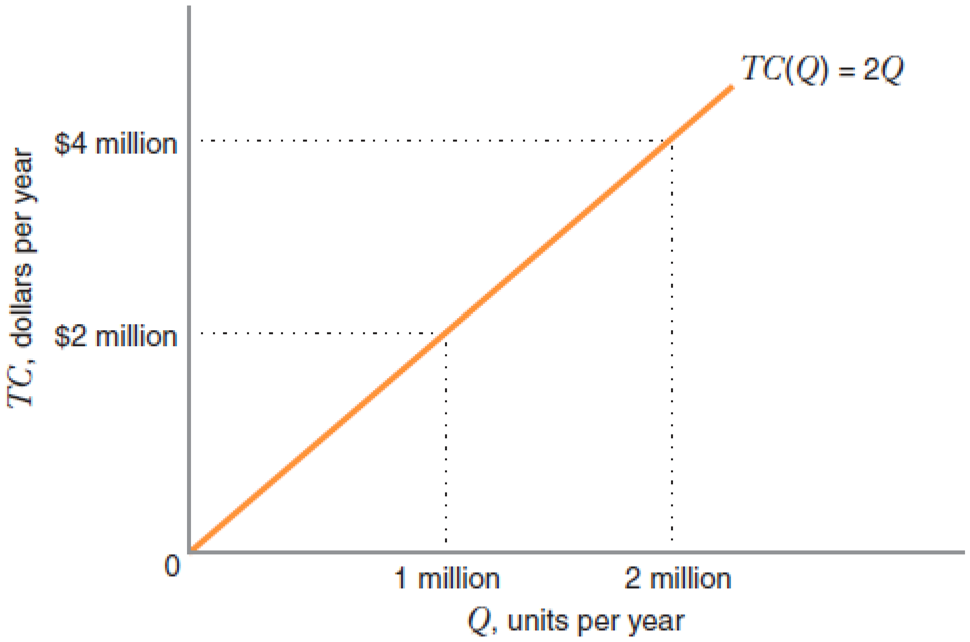
Cost Minimization and the Long-Run Total Cost Curve

- As quantity of output increases from 1 million to 2 million, with input prices(w, r) constant, cost minimizing input combination moves from TC_1 to TC_2 which gives the TC(Q) curve
- Panel (a) shows how the cost-minimizing input combination moves from point A to point B
- Panel (b) shows the long-run total cost curve TC(Q)



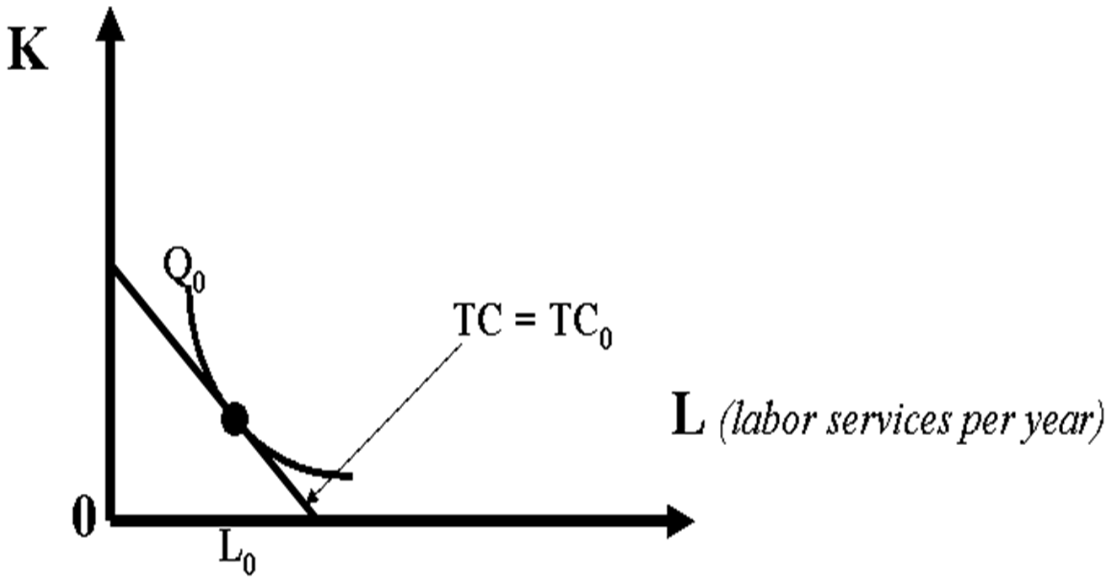
Finding the Long-Run Total Cost Curve from a Production Function

- What is the long run total cost function for production function $Q = 50L^{1/2}K^{1/2}$?
- $L^*(Q,w,r) = (Q/50)(r/w)^{1/2}$
- $K^*(Q,w,r) = (Q/50)(w/r)^{1/2}$
- $TC(Q,w,r) = w[(Q/50)(r/w)^{1/2}]+r[(Q/50)(w/r)^{1/2}]$
- $TC(Q,w,r) = (Q/50)(wr)^{1/2} + (Q/50)(wr)^{1/2}$
- $TC(Q,w,r) = (Q/25)(wr)^{1/2}$
- What is the graph of the total cost curve when w = 25 and r = 100?
- $TC(Q) = 2Q$

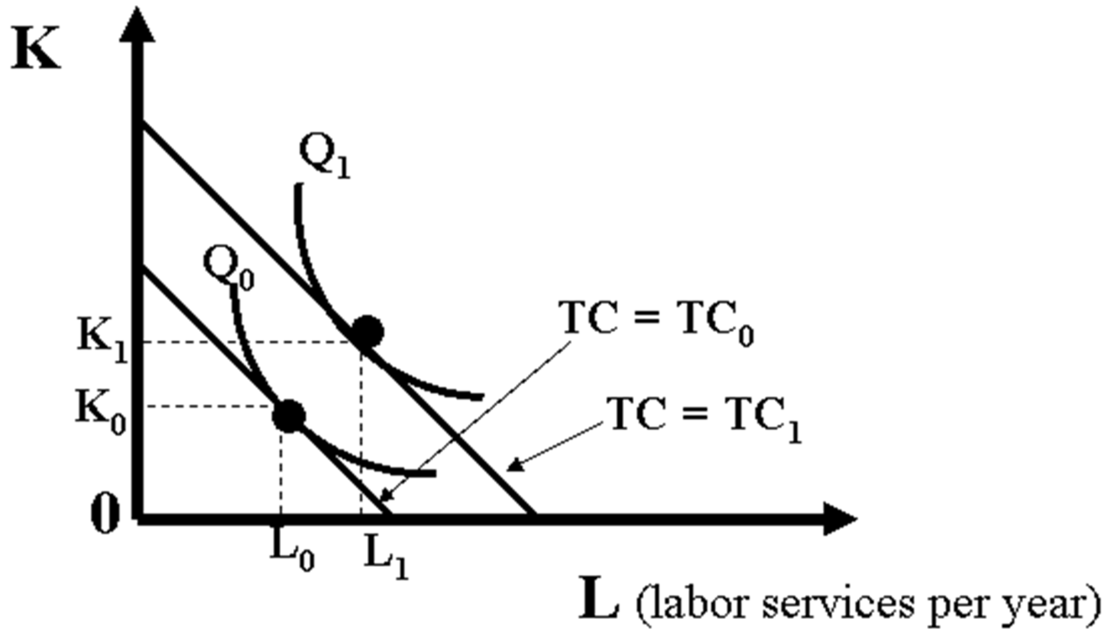


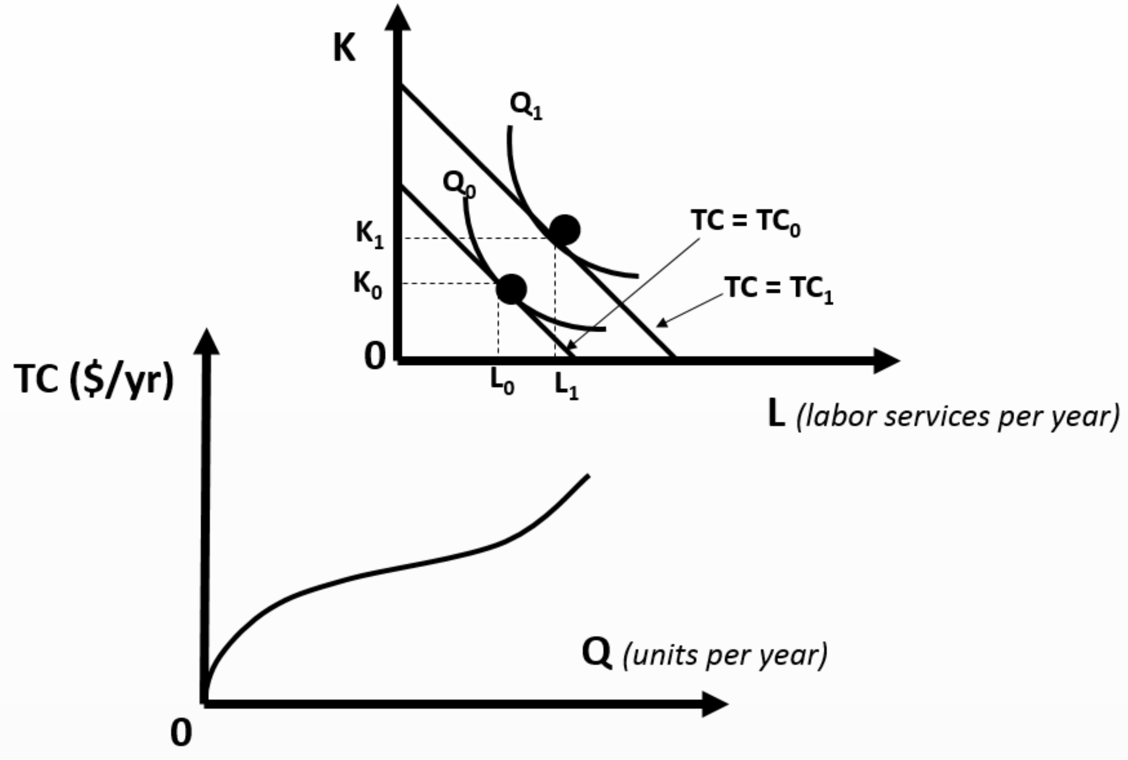
Long Run Total Cost Curve: Tracking Movement

- The long run total cost curve shows minimized total cost as output varies, holding input prices constant
- Graphically, what does the total cost curve look like if Q varies and w and r are fixed?

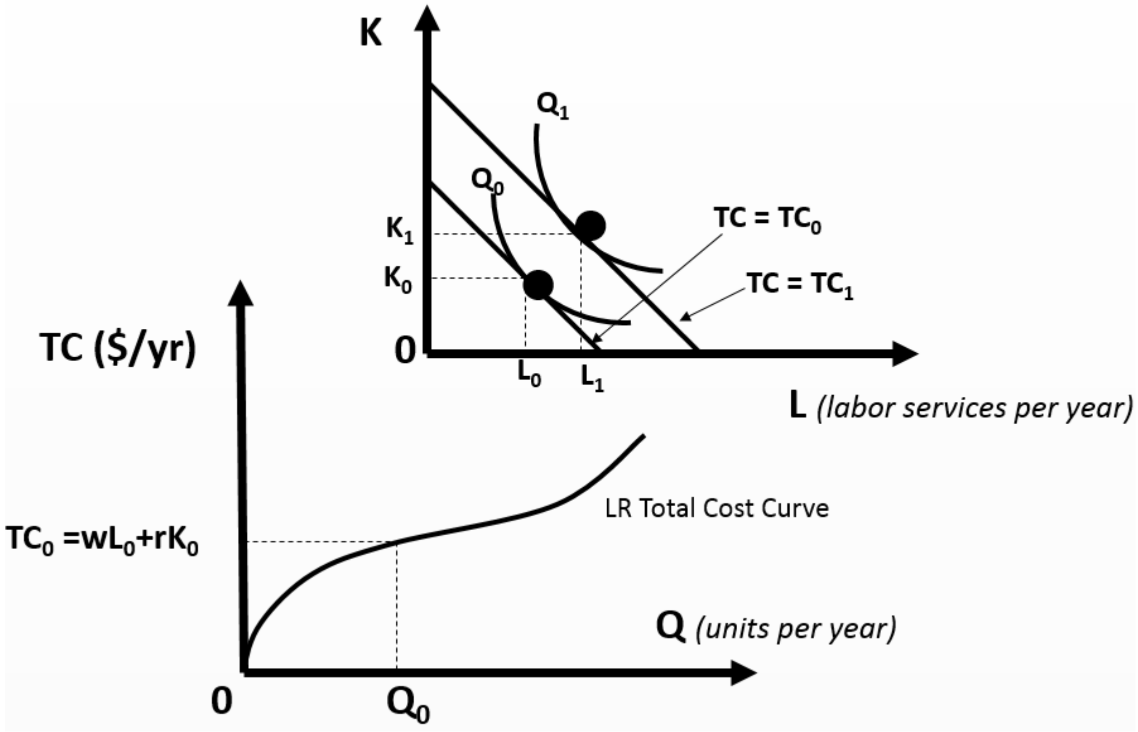


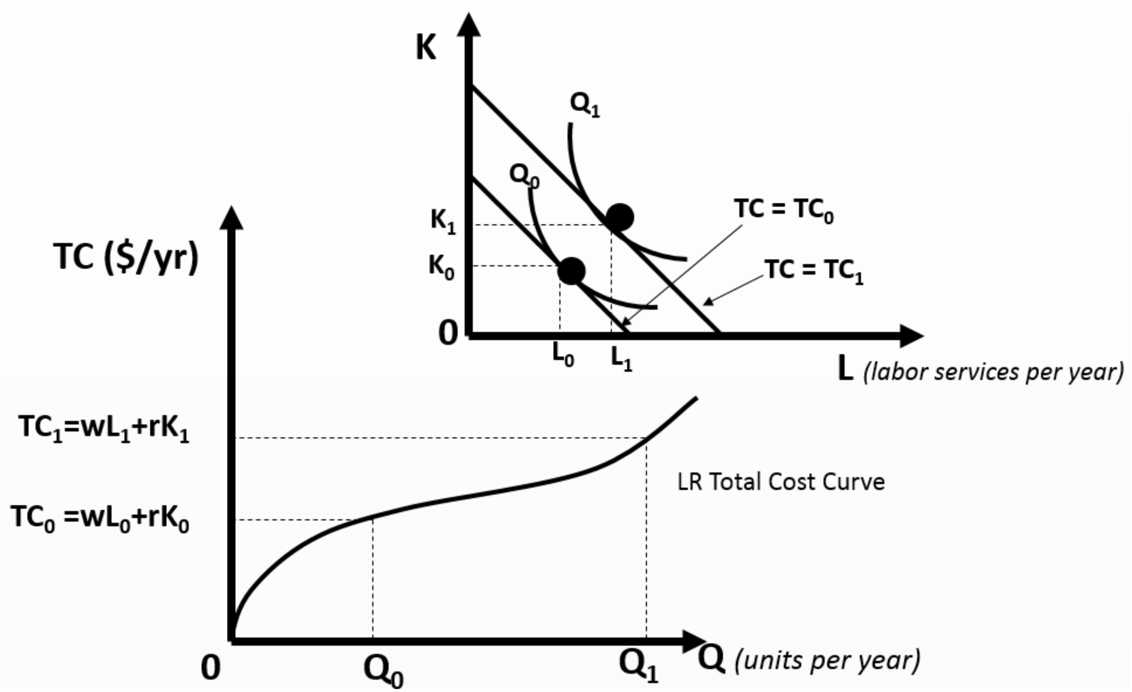
Long Run Total Cost Curve: Tracking Movement Continued





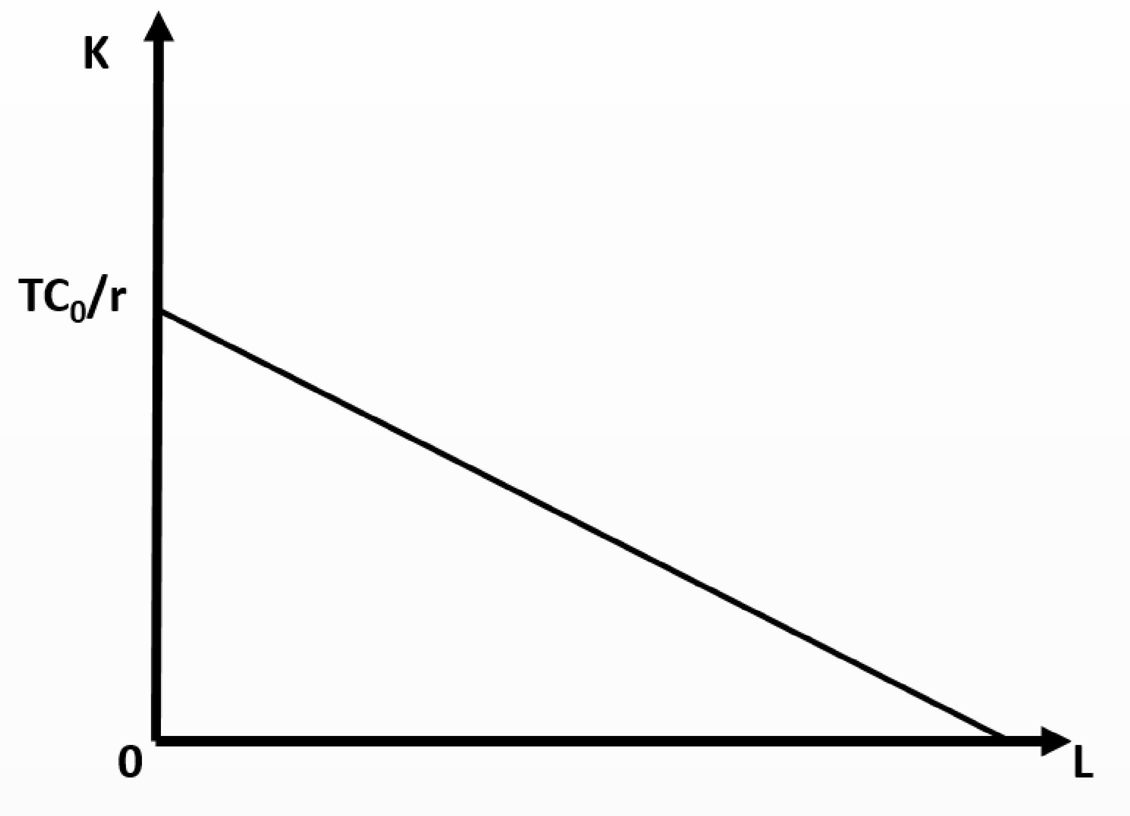
Long Run Total Cost Curve: Tracking Movement Continued



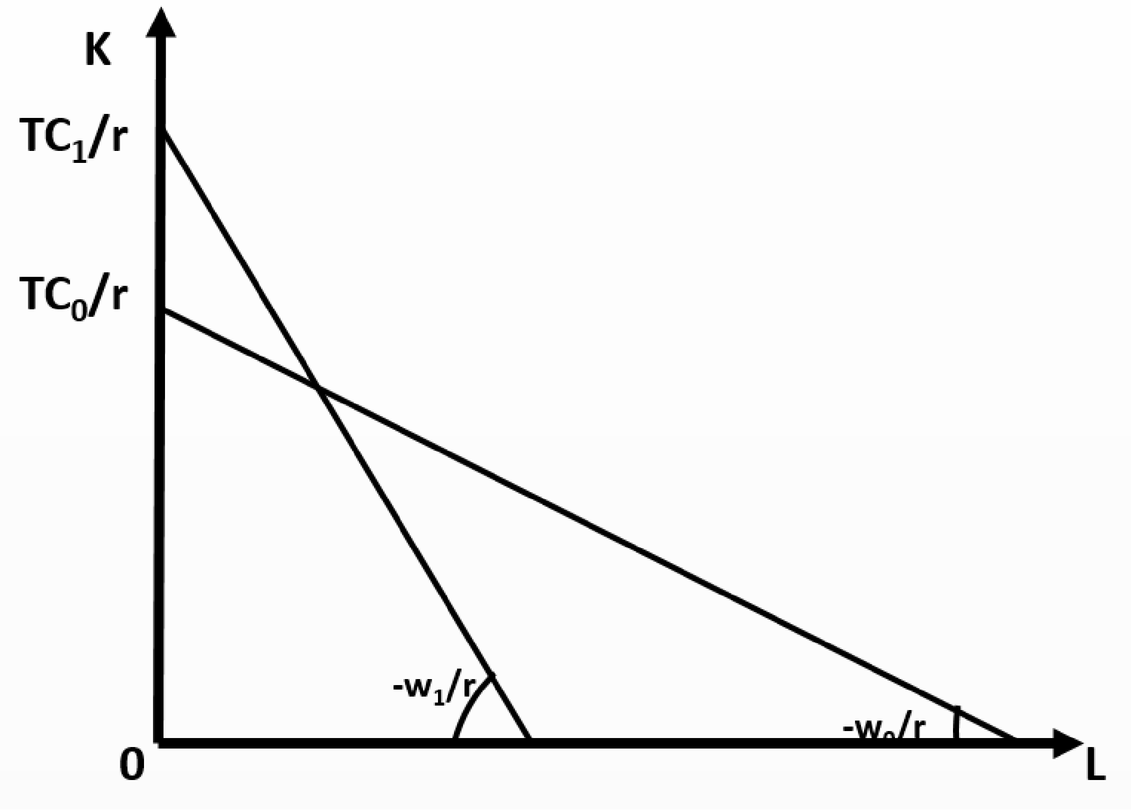


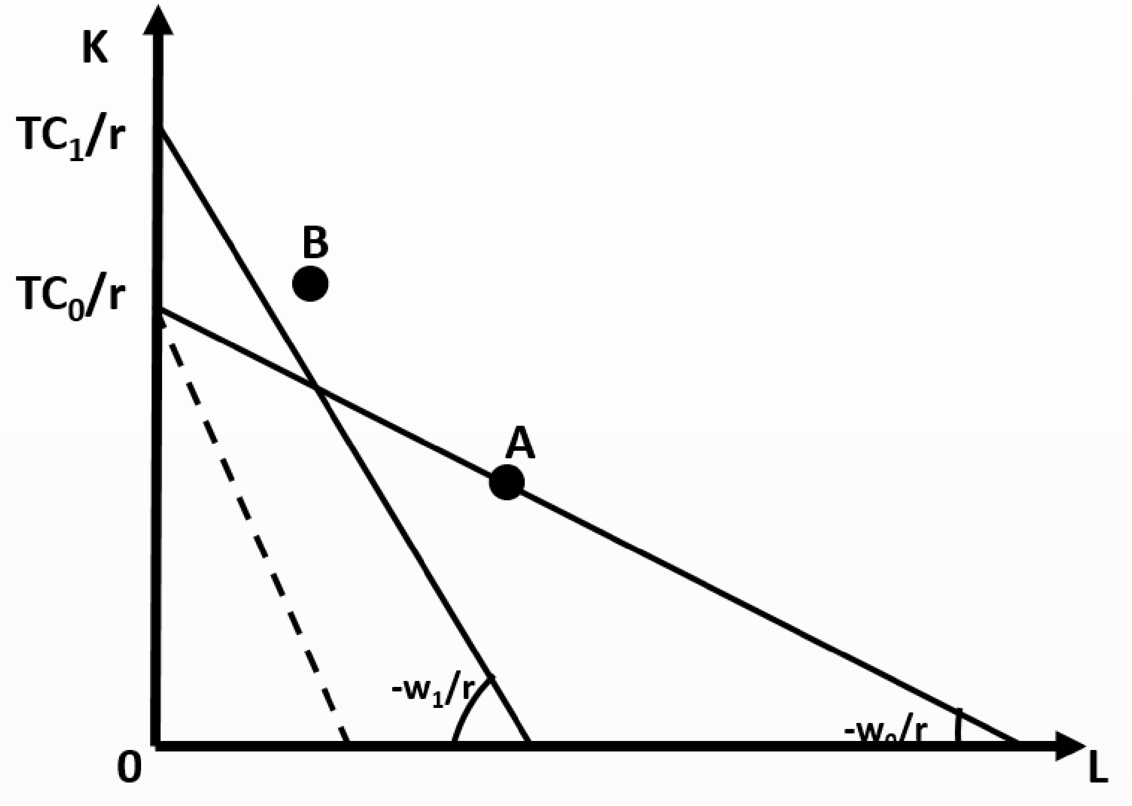
Long Run Total Cost Curve: Identifying Shifts

- Graphically, how does the total cost curve shift if wages rise but the price of capital remains fixed
- Will consider a change in input price

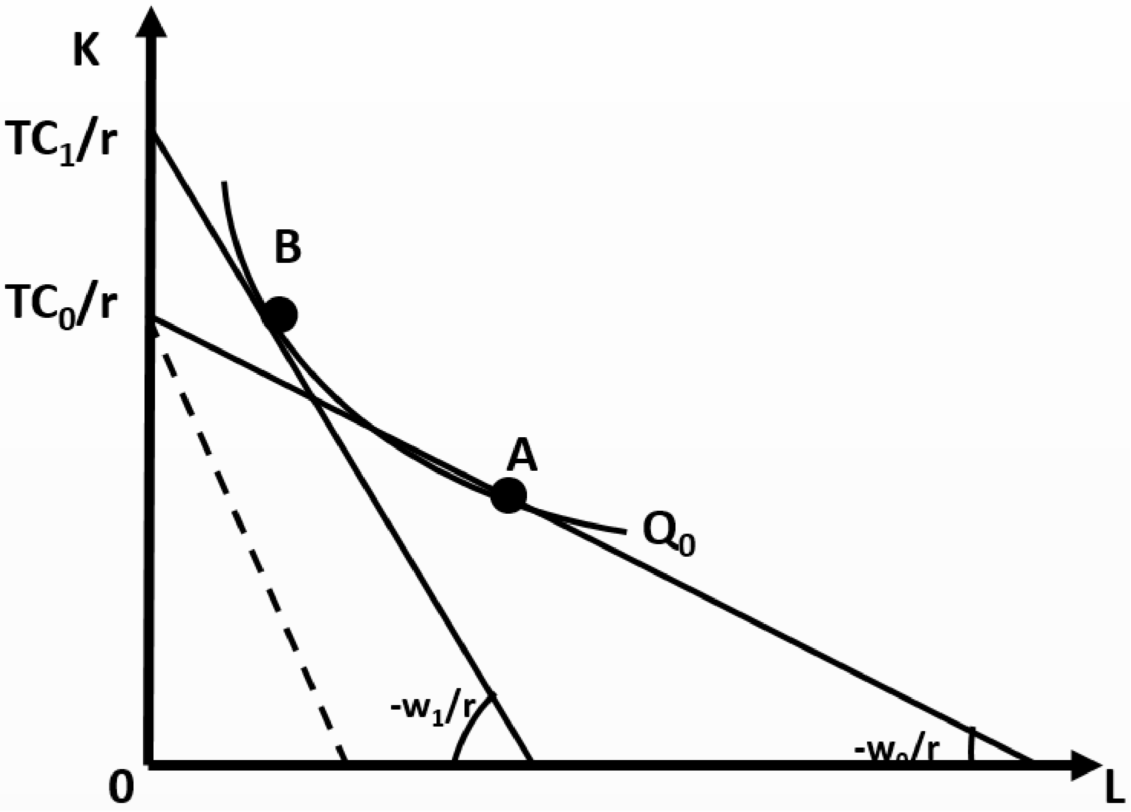


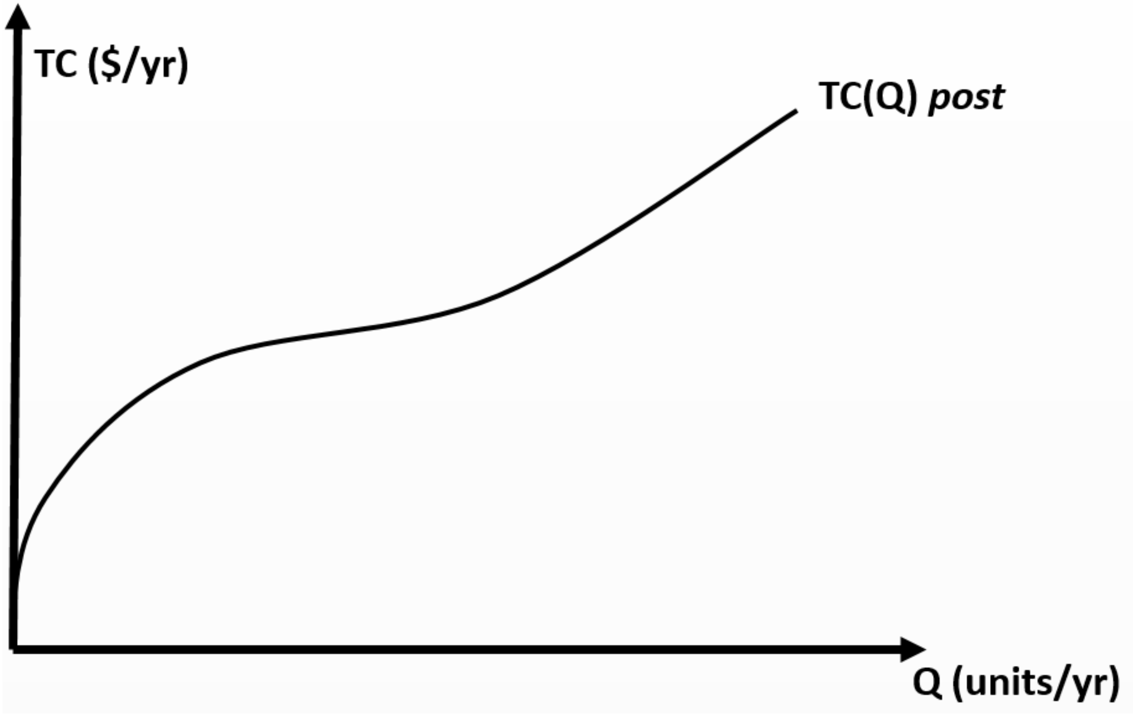
A Change in Input Prices



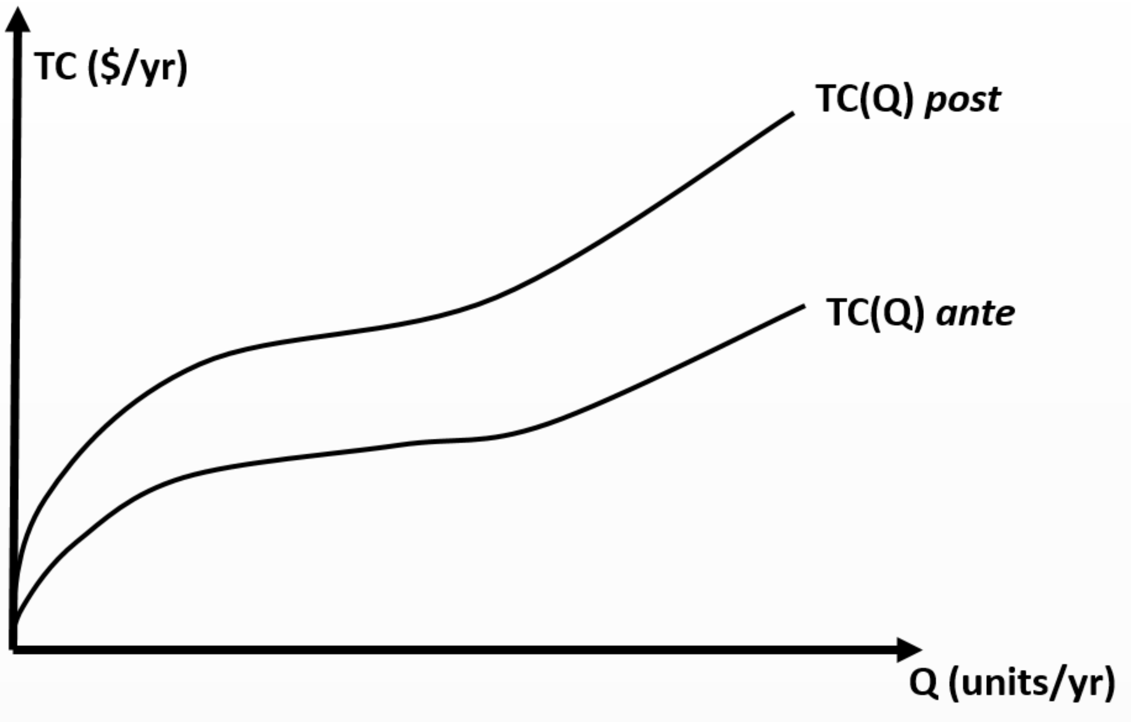


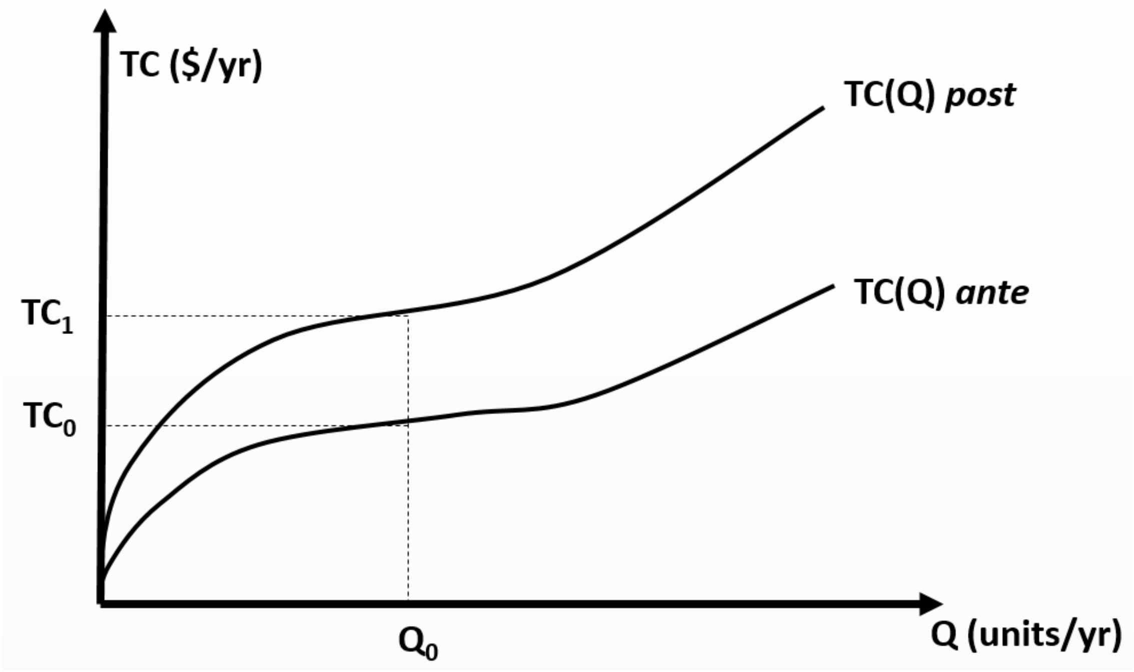
A Shift in the Total Cost Curve





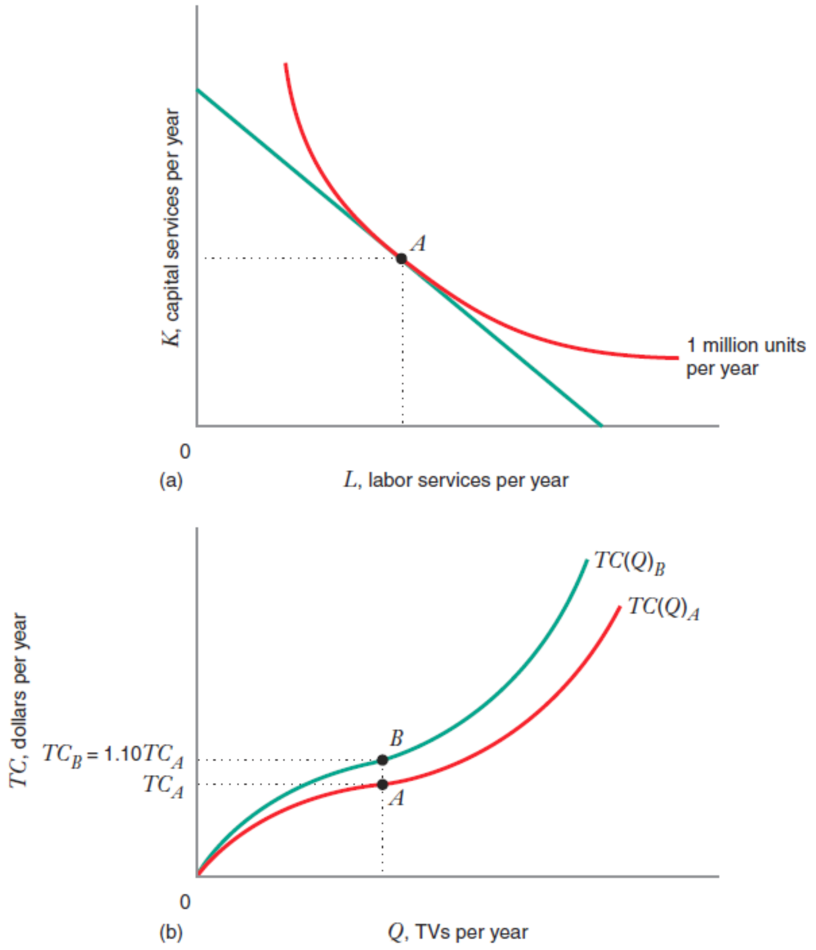
A Shift in the Total Cost Curve Continued





A Proportionate Change in the Prices of All Inputs

- The price of each input increases by 10 percent
- Panel (a) shows that the cost-minimizing input combination remains the same
- Panel (b) shows that the total cost curve shifts up by the same 10 percent



Long Run Cost Functions

- The long run average cost function is the long run total cost function divided by output, Q
- That is, the LRAC function tells us the firm's cost per unit of output
- $AC(Q,w,r) = TC(Q,w,r)/Q$

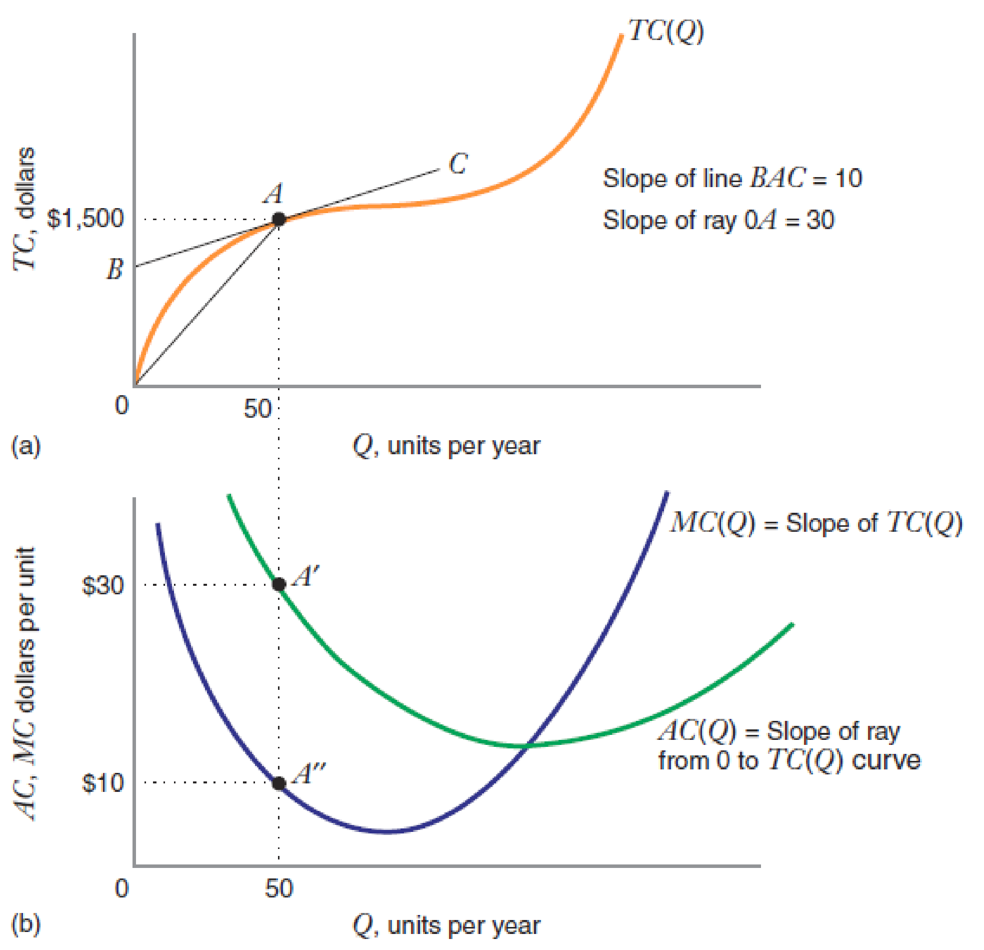
- The long run marginal cost function measures the rate of change of total cost as output varies, holding constant input prices
- $MC(Q,w,r) = \{TC(Q+\Delta Q,w,r) - TC(Q,w,r)\}/\Delta Q$
- $= \Delta TC(Q,w,r)/\Delta Q$
  - where: w and r are constant

Average and Marginal Cost Curves: What is their Relationship

- Suppose that w and r are fixed
- When marginal cost is less than average cost, average cost is decreasing in quantity
- That is, if $MC(Q) < AC(Q)$, $AC(Q)$ decreases in Q
- When marginal cost is greater than average cost, average cost is increasing in quantity
  - That is, if $MC(Q) > AC(Q)$, $AC(Q)$ increases in Q
- When marginal cost equals average cost, average cost does not change with quantity
  - That is, if $MC(Q) = AC(Q)$, $AC(Q)$ is flat with respect to Q.

Deriving Long-Run Average and Marginal Cost Curves from the Long-Run Total Cost Curve

- Panel (a) shows the firm's long-run total cost curve TC(Q)
- Panel (b) shows the long-run average cost curve AC(Q) and the long-run marginal cost curve MC(Q)
  - Both derived from TC(Q)

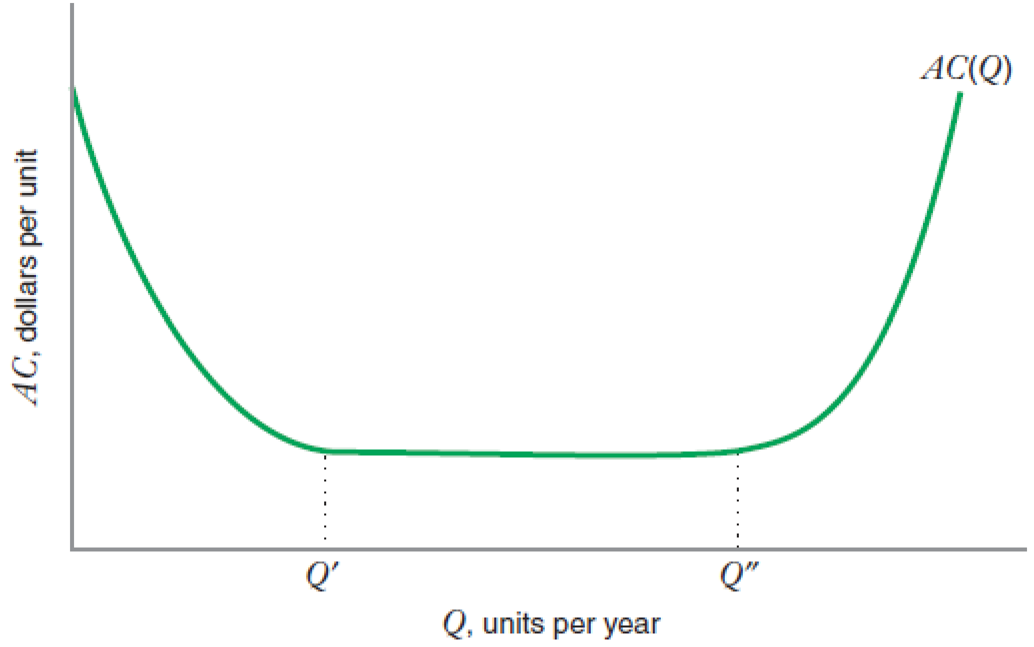


Economies and Diseconomies of Scale

- If average cost decreases as output rises, all else equal, the cost function exhibits economies of scale
- Similarly, if the average cost increases as output rises, all else equal, the cost function exhibits diseconomies of scale
- The smallest quantity at which the long run average cost curve attains its minimum point is called the minimum efficient scale.

Economies and Diseconomies of Scale for a Typical Real-World Average Cost Curve

- There are economies of scale for outputs less than Q
- Average costs are flat between and Q' and Q''
  - There are diseconomies of scale thereafter
- The output level Q' is called the minimum efficient scale



Returns to Scale and Economies of Scale

- When the production function exhibits increasing returns to scale, the long run average cost function exhibits economies of scale so that AC(Q) decreases with Q, all else equal
- When the production function exhibits decreasing returns to scale, the long run average cost function exhibits diseconomies of scale so that AC(Q) increases with Q, all else equal
- When the production function exhibits constant returns to scale, the long run average cost function is flat: it neither increases nor decreases with output

Relationship between Economies of Scale and Returns to Scale

	Production Function		
	$Q = L^2$	$Q = \sqrt{L}$	$Q = L$
Labor requirements function	$L = \sqrt{Q}$	$L = Q^2$	$L = Q$
Long-run total cost	$TC = w\sqrt{Q}$	$TC = wQ^2$	$TC = wQ$
Long-run average cost	$AC = w/\sqrt{Q}$	$AC = wQ$	$AC = w$
How does long-run average cost vary with Q?	Decreasing	Increasing	Constant
Economies/diseconomies of scale?	Economies of scale	Diseconomies of scale	Neither
Returns to scale	Increasing	Decreasing	Constant

Output Elasticity of Total Cost

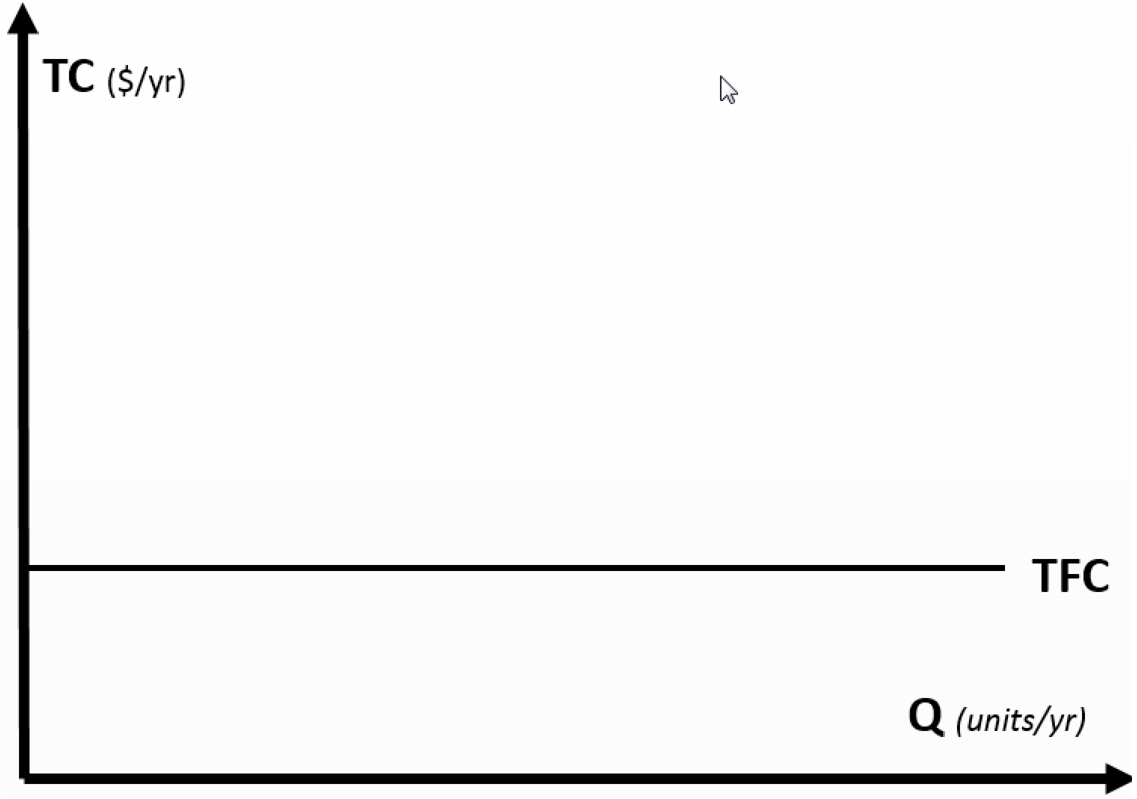
- The percentage change in total cost per one percent change in output is the output elasticity of total cost, $\varepsilon_{TC,Q}$
- $\varepsilon_{TC,Q} = (\Delta TC/TC)(\Delta Q/Q)$
- $= (\Delta TC/\Delta Q)/(TC/Q) = MC/AC$

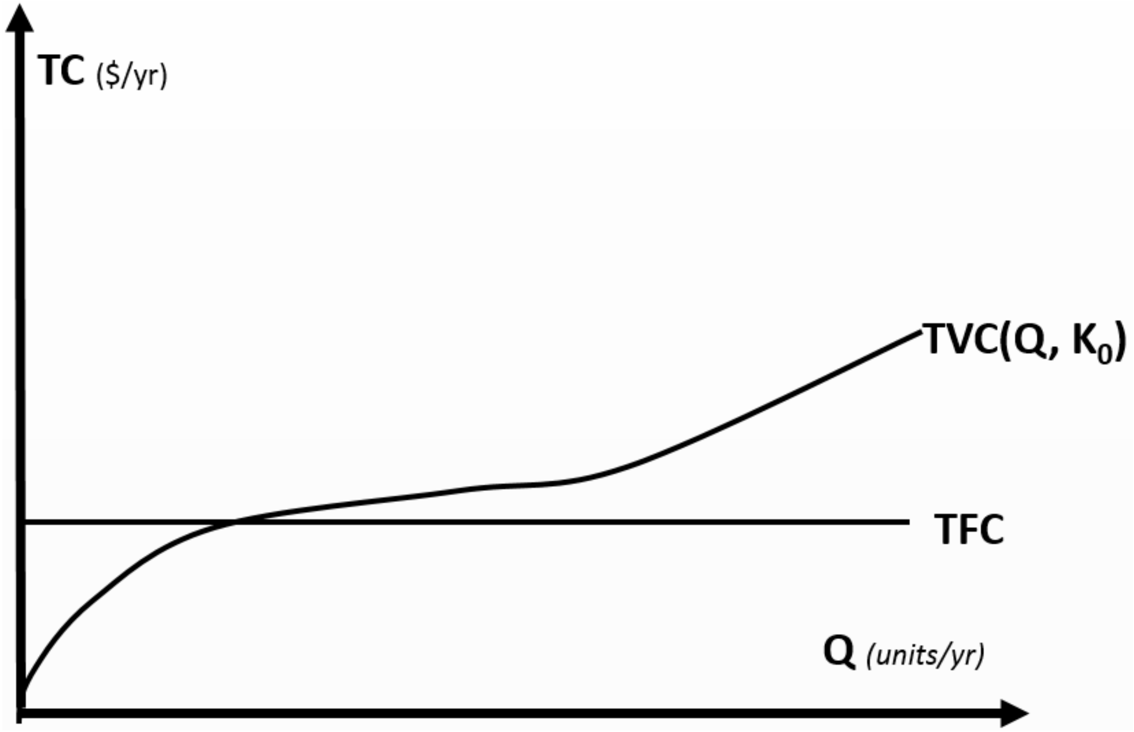
- If $\varepsilon_{TC,Q} < 1$, MC < AC, so AC must be decreasing in Q
  - Therefore, we have economies of scale
- If $\varepsilon_{TC,Q} > 1$, MC > AC, so AC must be increasing in Q
  - Therefore, we have diseconomies of scale
- If $\varepsilon_{TC,Q} = 1$, MC = AC, so AC is just flat with respect to Q

Short Run, Total Variable, and Total Fixed Cost Functions

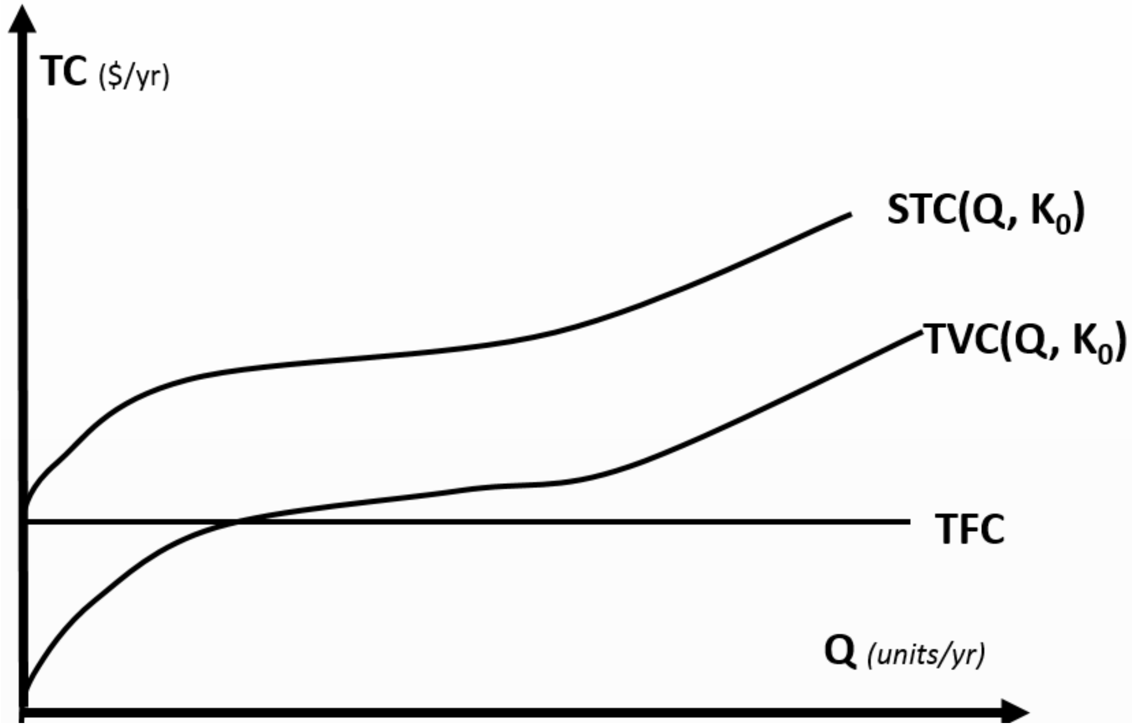
- The short run total cost function tells us the minimized total cost of producing Q units of output, when (at least) one input is fixed at a particular level
- The total variable cost function is the minimized sum of expenditures on variable inputs at the short run cost minimizing input combinations
- The total fixed cost function is a constant equal to the cost of the fixed input(s)
  - $STC(Q,K_0) = TVC(Q,K_0) + TFC(Q,K_0)$
  - Where: K_0 is the fixed input and w and r are fixed (and suppressed as arguments)

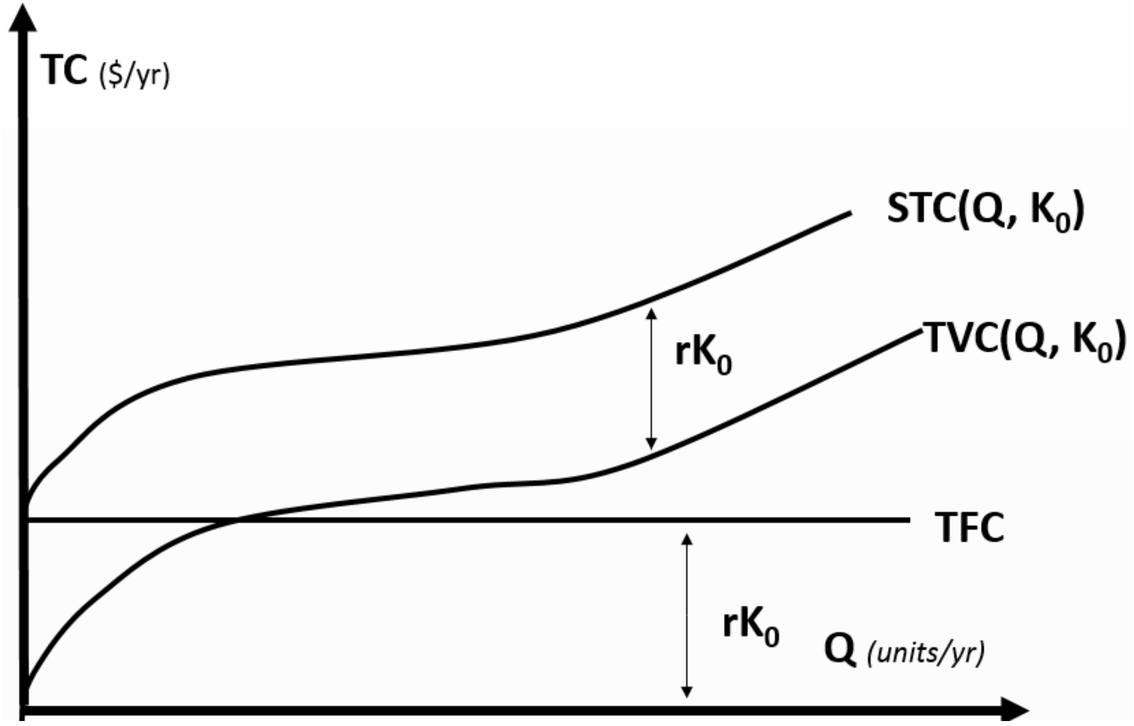
Key Cost Functions Interactions





Key Cost Functions Interactions Continued

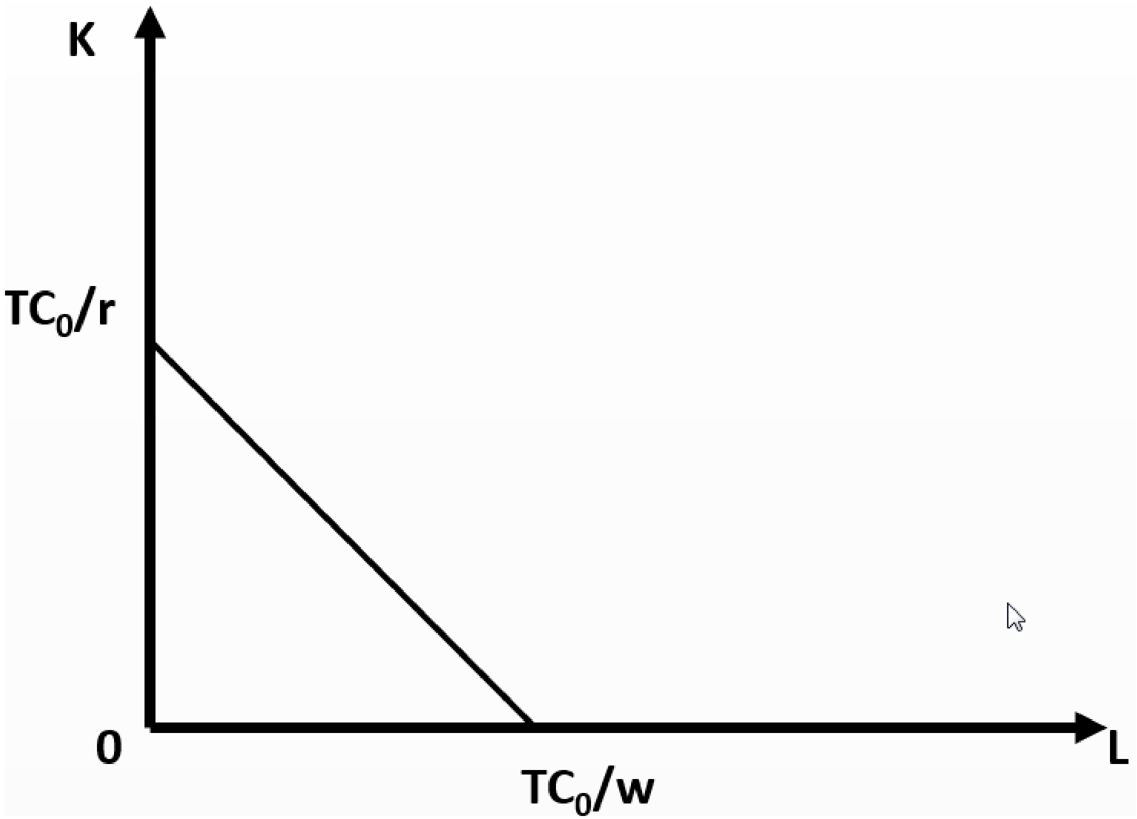


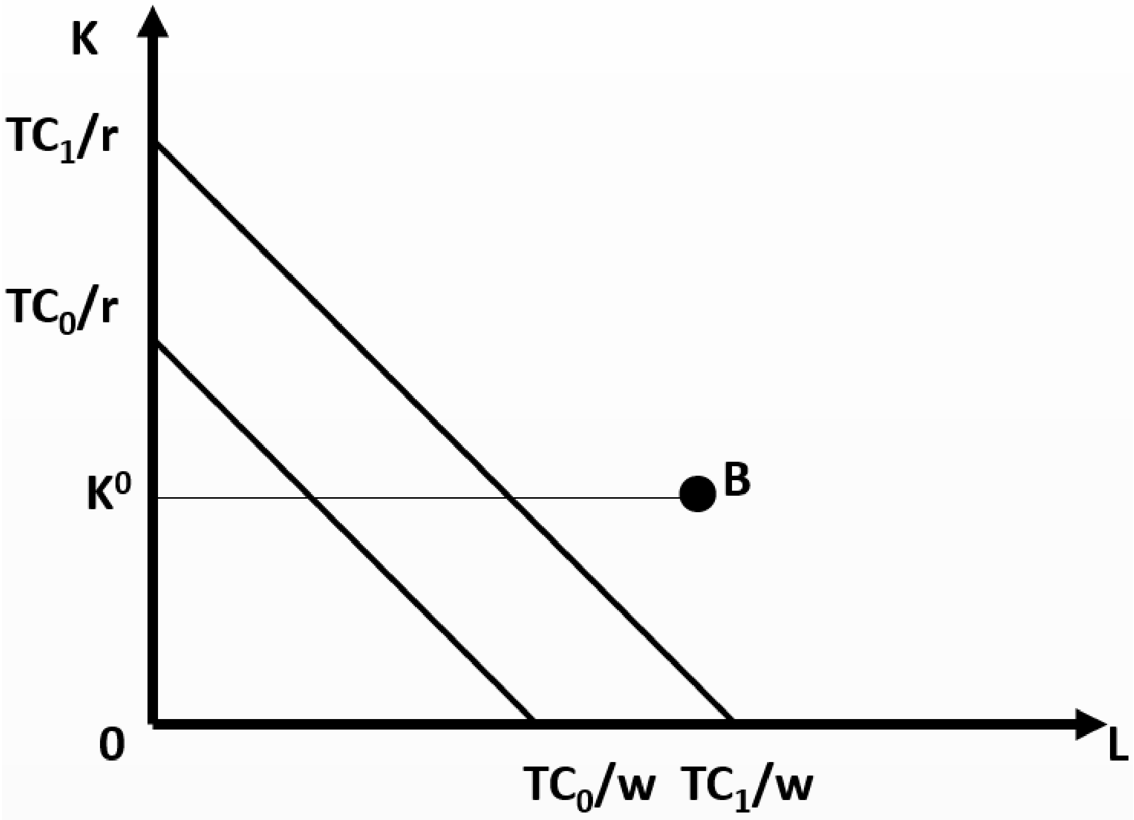


Long and Short Run Total Cost Functions

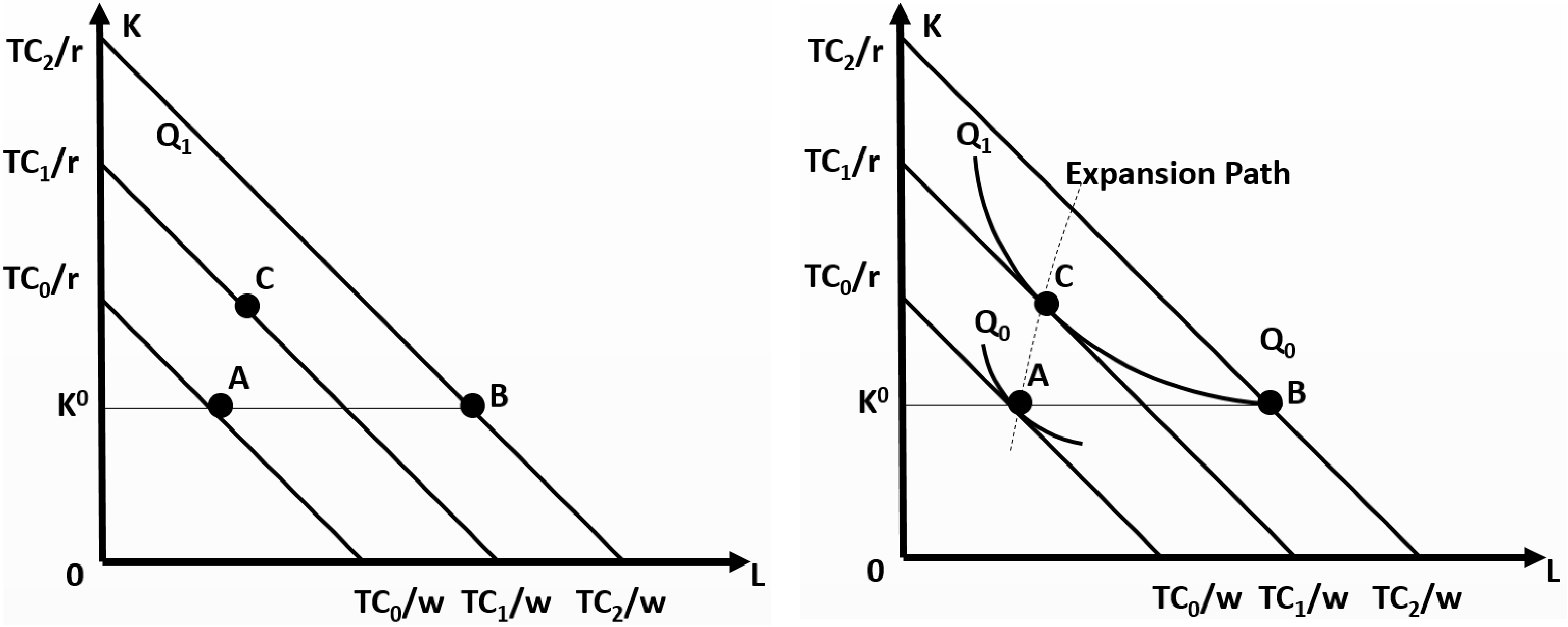
- The firm can minimize costs at least as well in the long run as in the short run because it is “less constrained”
- Hence, the short run total cost curve lies everywhere above the long run total cost curve
- However, when the quantity is such that the amount of the fixed inputs just equals the optimal long run quantities of the inputs, the short run total cost curve and the long run total cost curve coincide

Long and Short Run Total Cost Functions Graphically

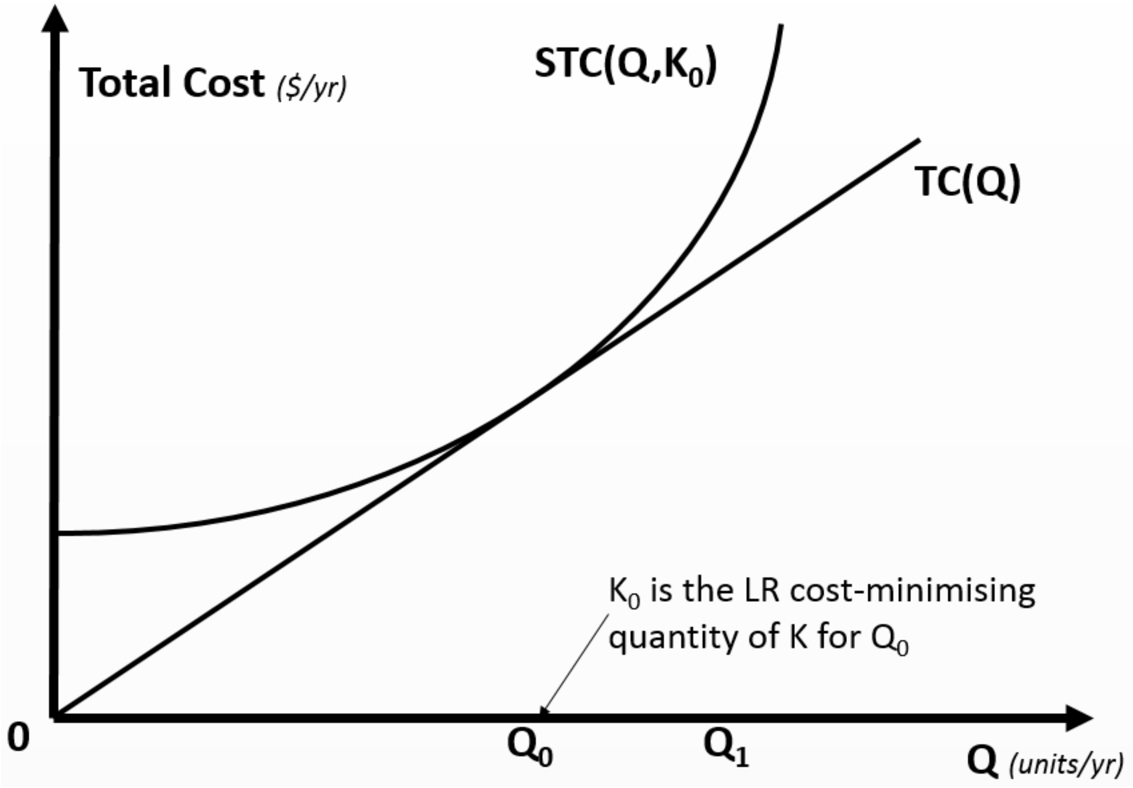


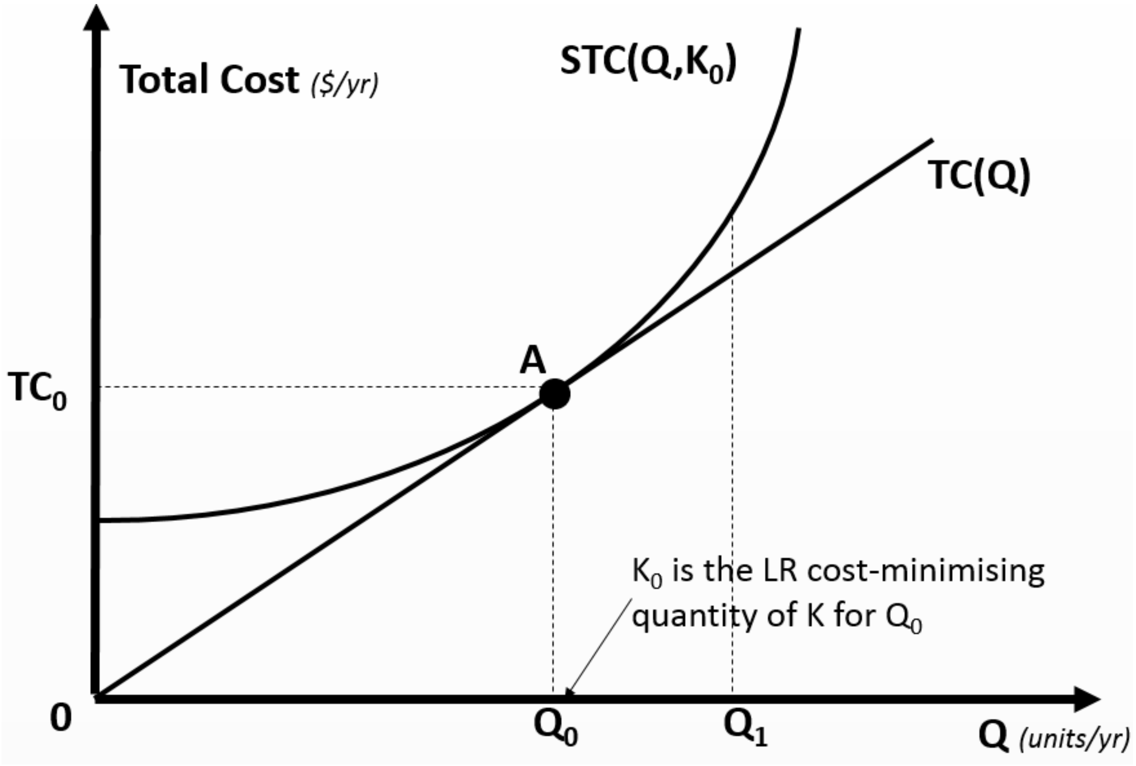


Long and Short Run Total Cost Functions Graphically Continued

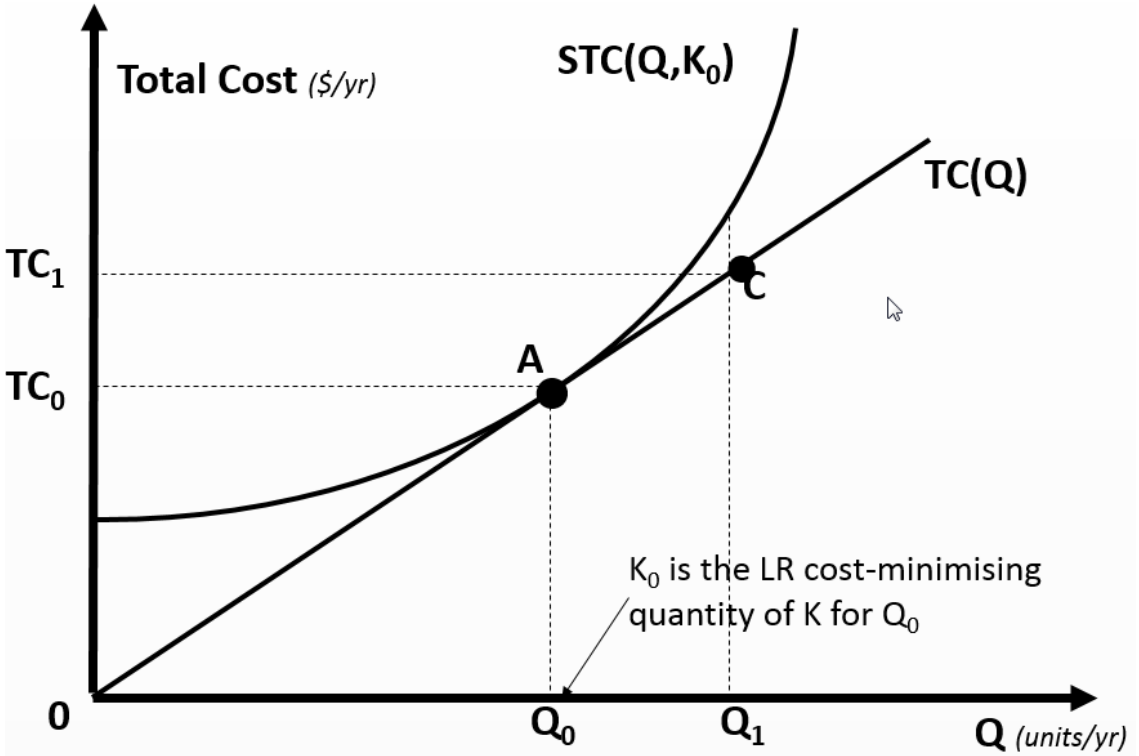


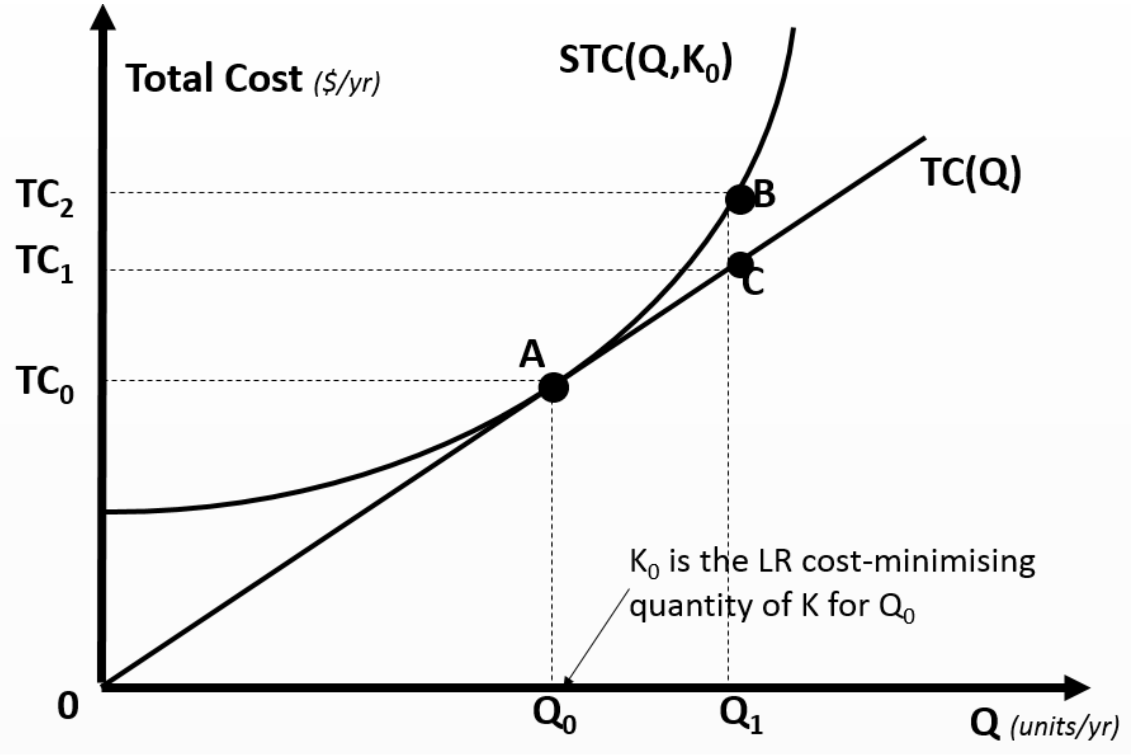
Long and Short Run Total Cost Functions Graphically Continued





Long and Short Run Total Cost Functions Graphically Continued





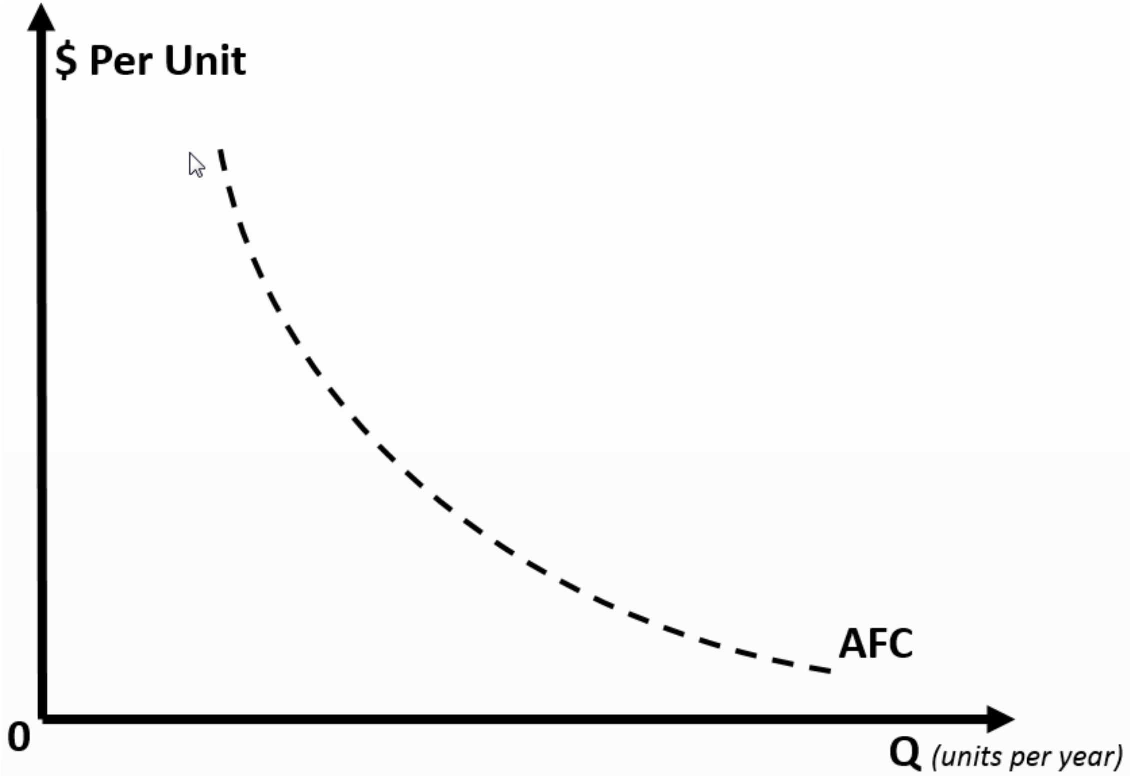
Short Run Average and Marginal Cost Functions

- The short run average cost function is the short run total cost function divided by output, Q
- That is, the SAC function tells us the firm's short run cost per unit of output
- $SAC(Q,K_0) = STC(Q,K_0)/Q$
  - Where: w and r are held fixed

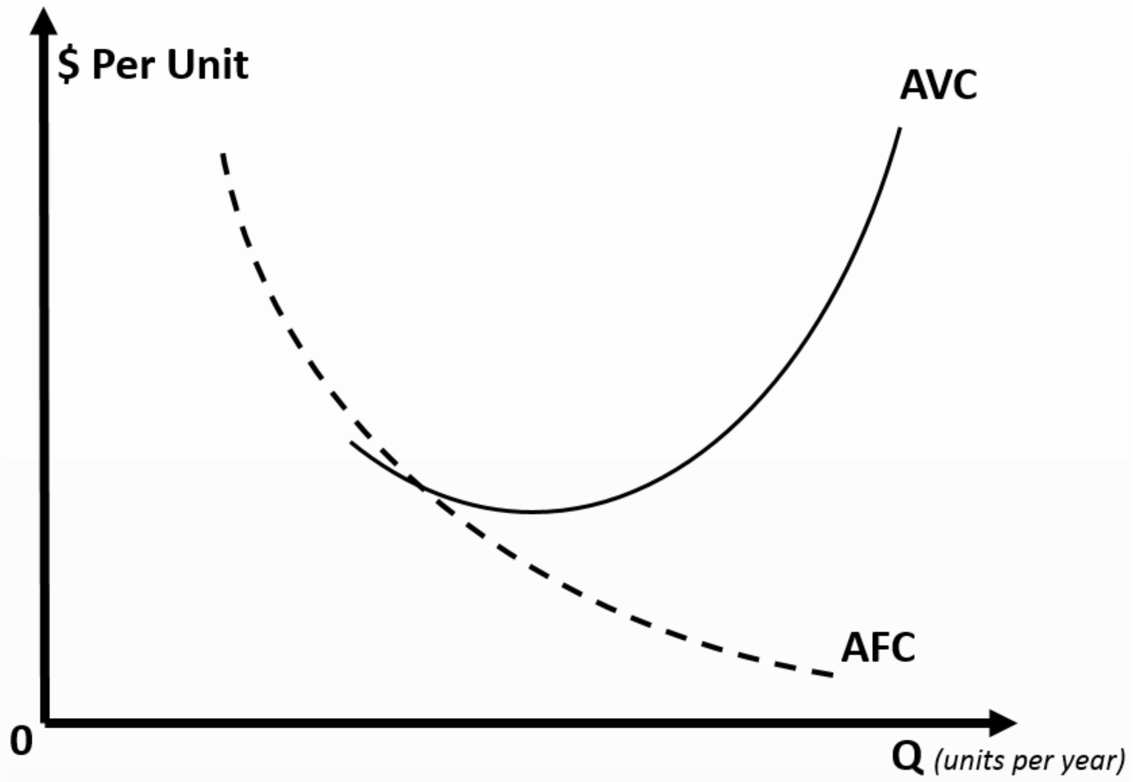
- The short run marginal cost function measures the rate of change of short run total cost as output varies, holding constant input prices and fixed inputs
- $SMC(Q,K_0)=\{STC(Q+\Delta Q,K_0)–STC(Q,K_0)\}/\Delta Q$
- $= \Delta STC(Q,K_0)/\Delta Q$
  - Where: w, r, and K_0 are constant

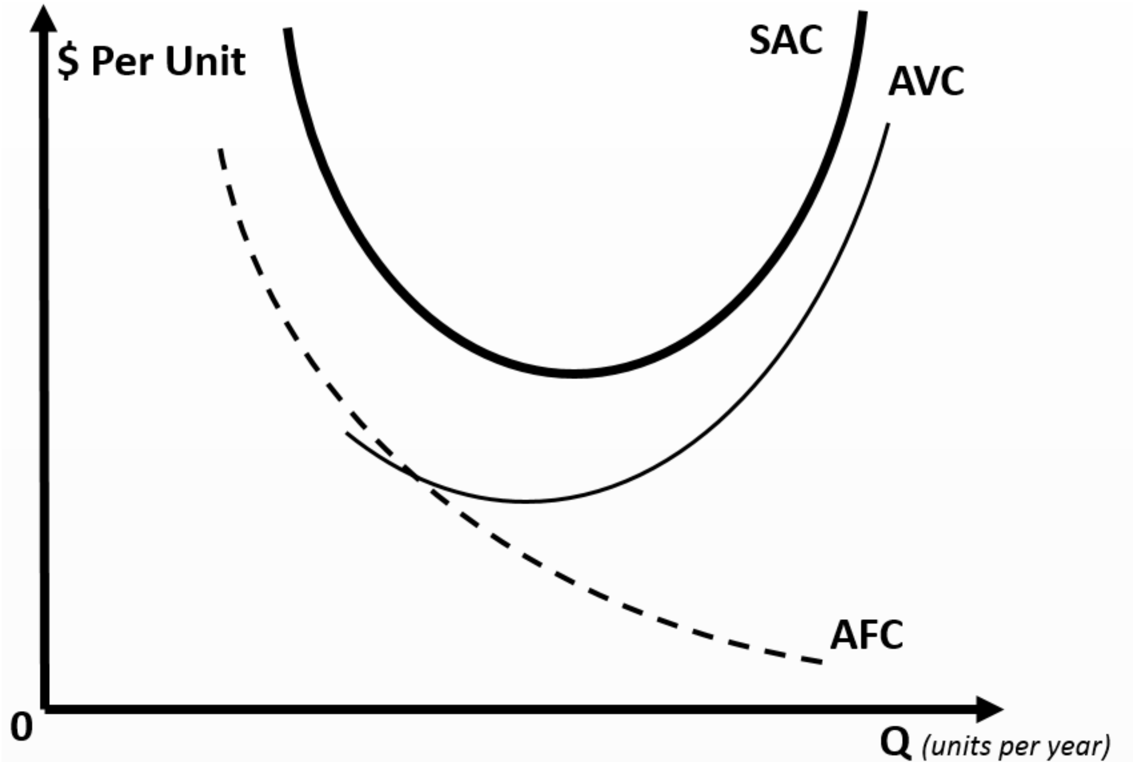
Summary Cost Functions

- When STC = TC, SMC = MC
- STC = TVC + TFC
- SAC = AVC + AFC
- Where:
  – SAC = STC/Q
  – AVC = TVC/Q (“average variable cost”)
  – AFC = TFC/Q (“average fixed cost”)
- The SAC function is the vertical sum of the AVC and AFC functions

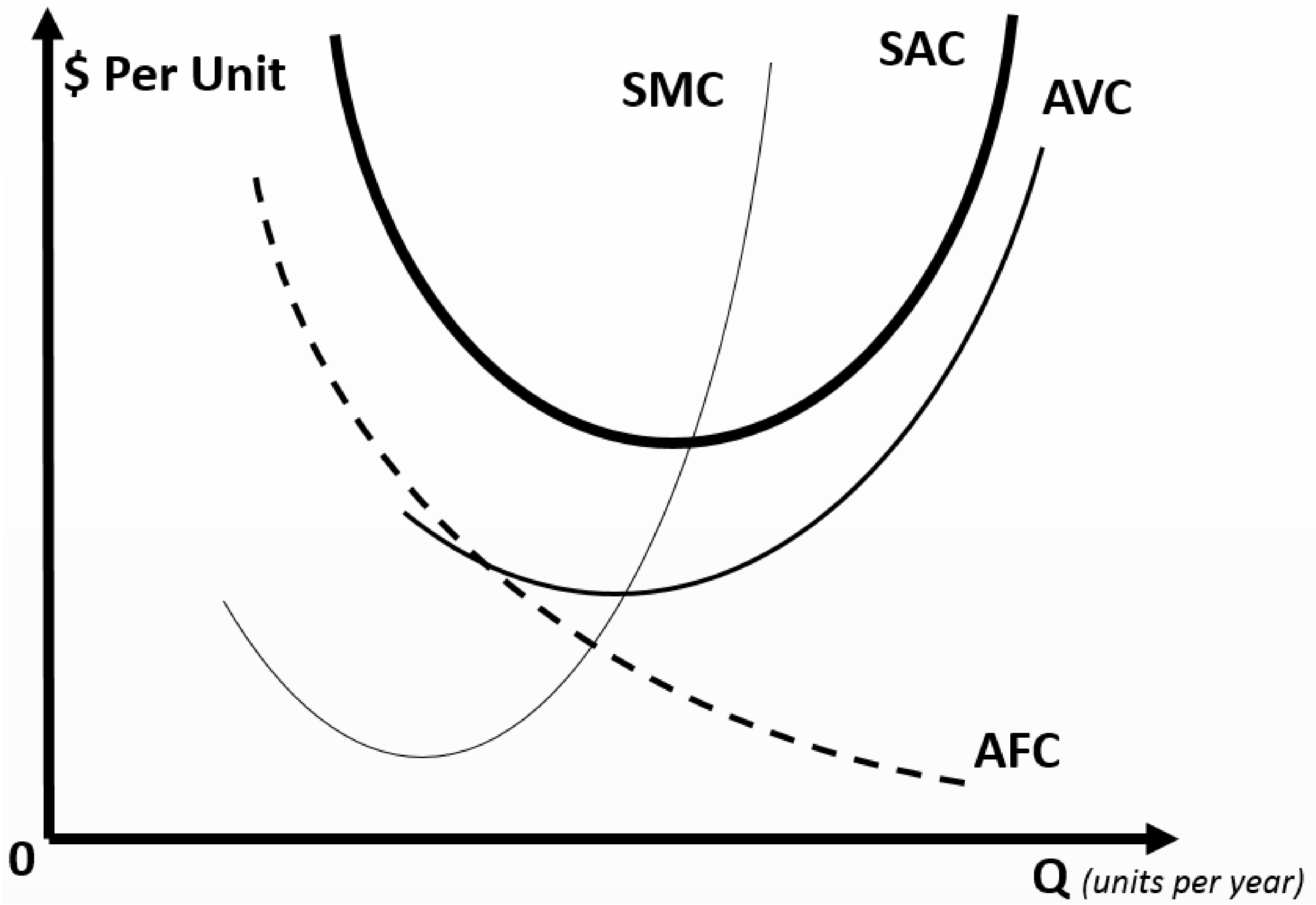


Summary Cost Functions Continued

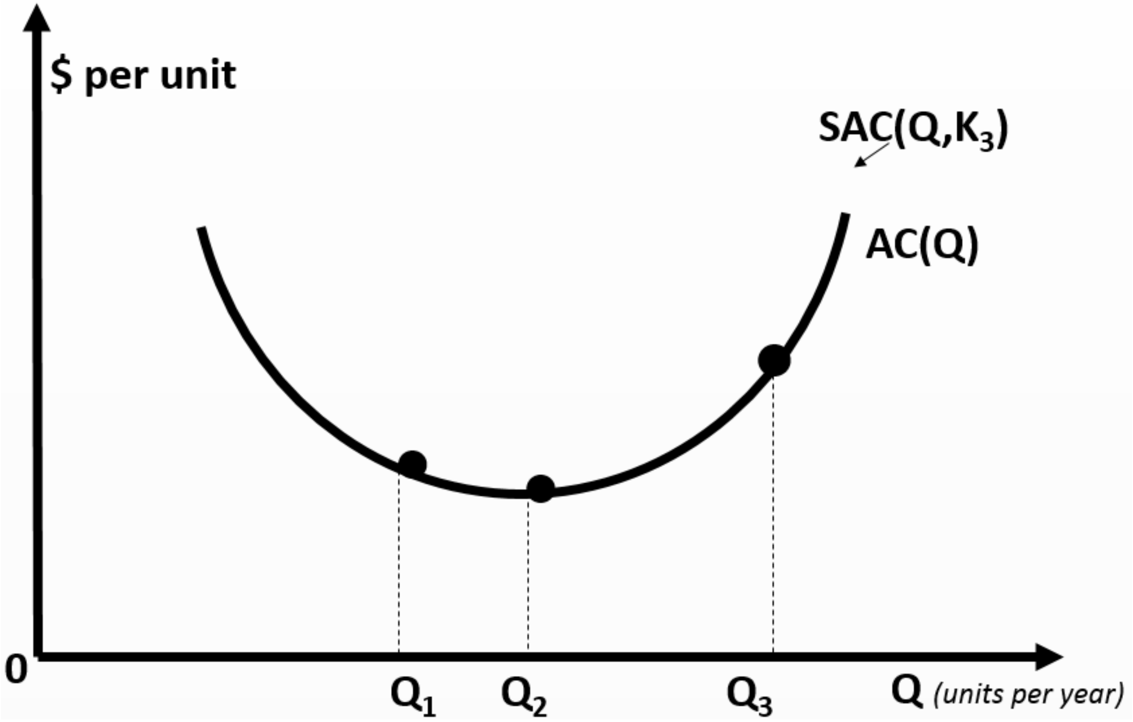


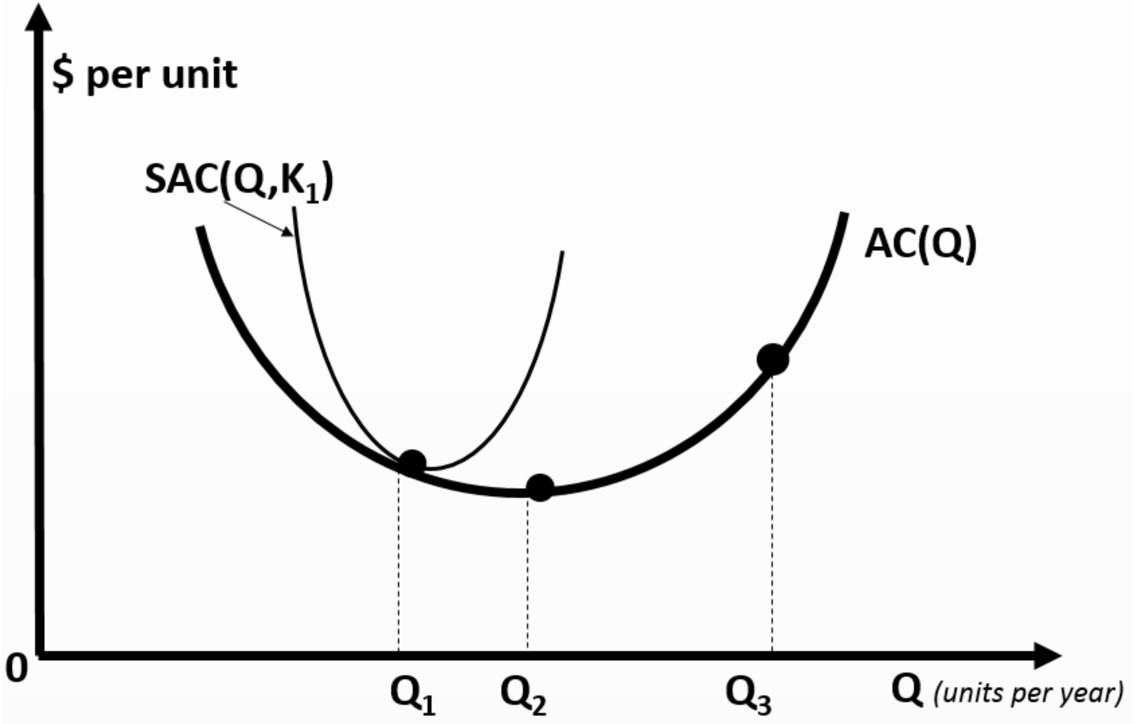


Summary Cost Functions Continued

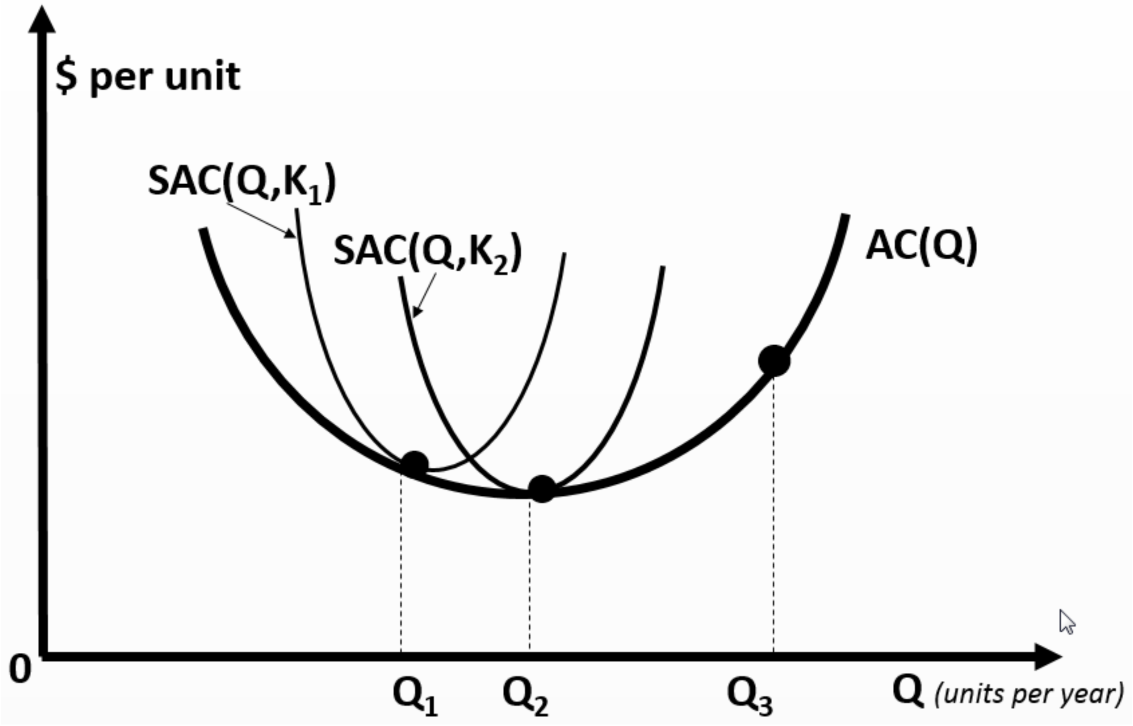


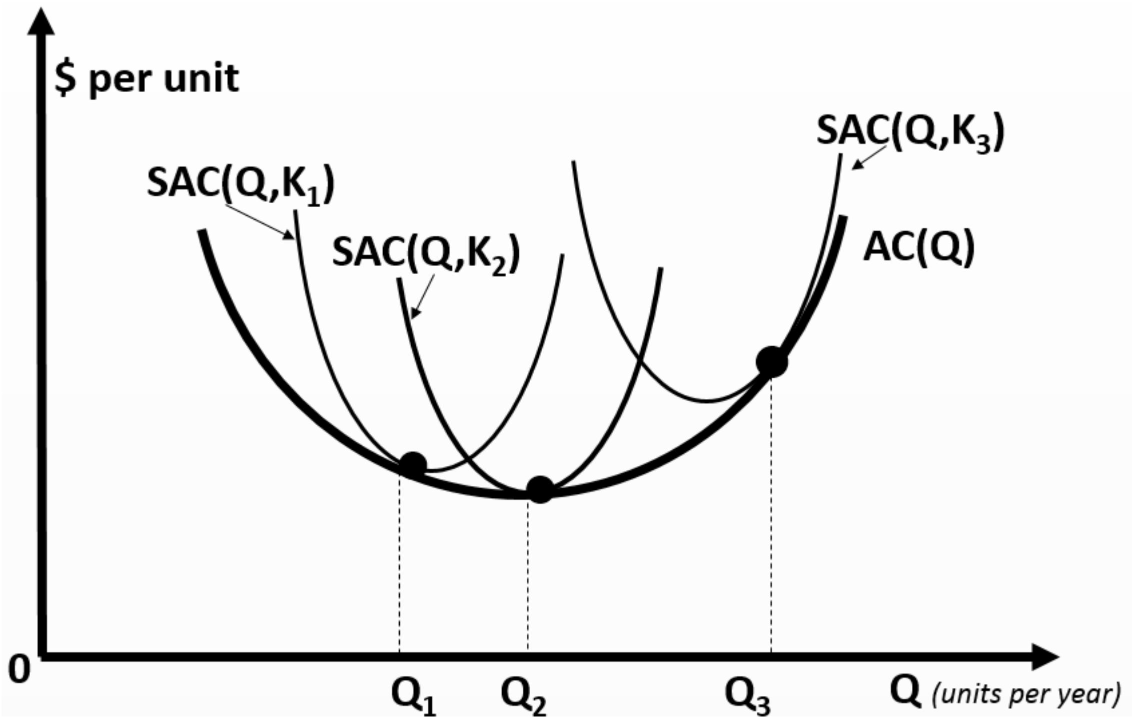
Long-Run Average Cost Function as an Envelope Curve



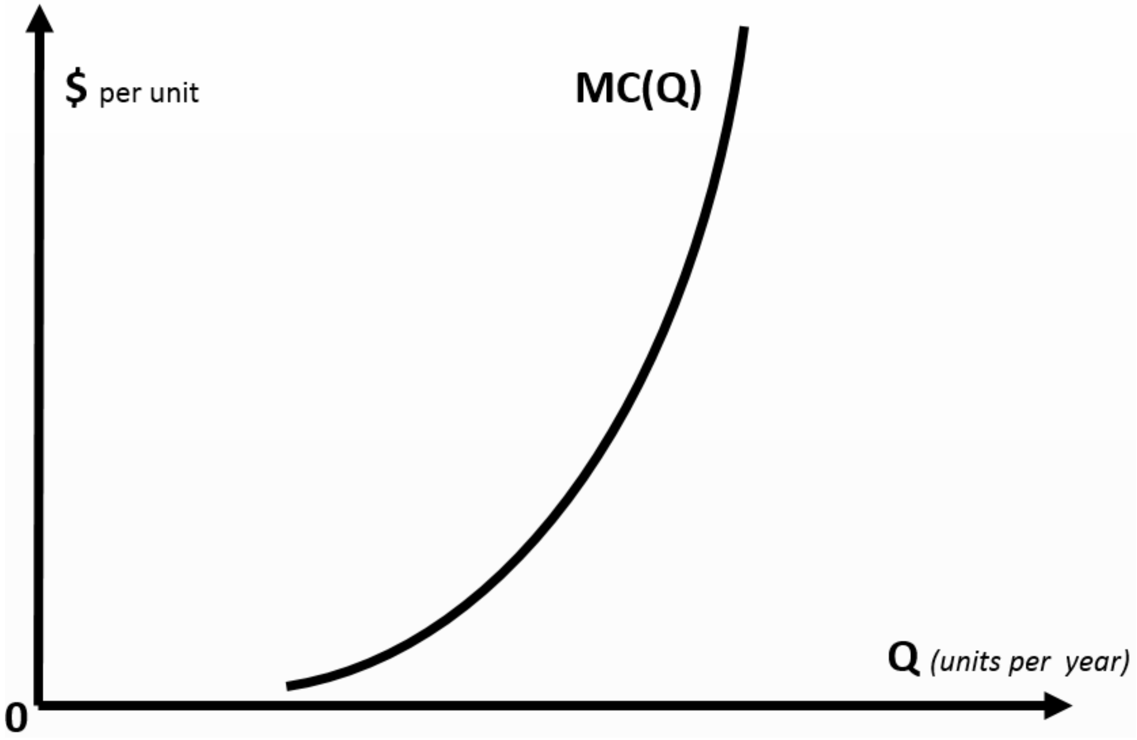


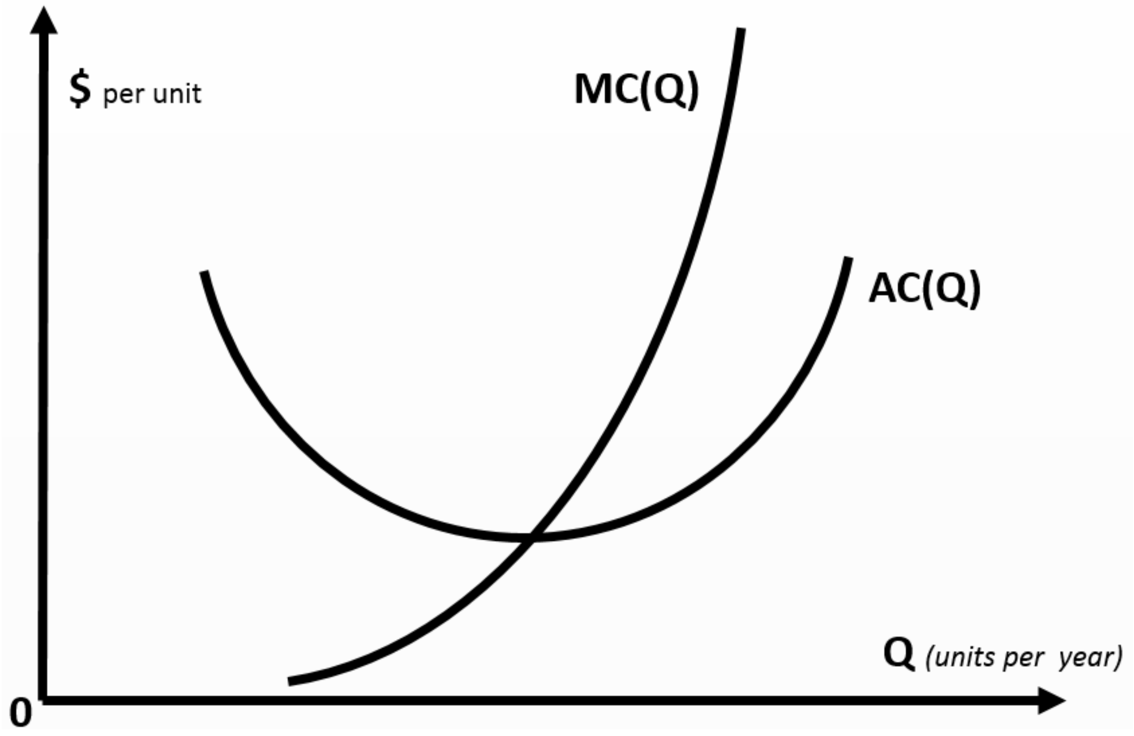
Long-Run Average Cost Function as an Envelope Curve Continued



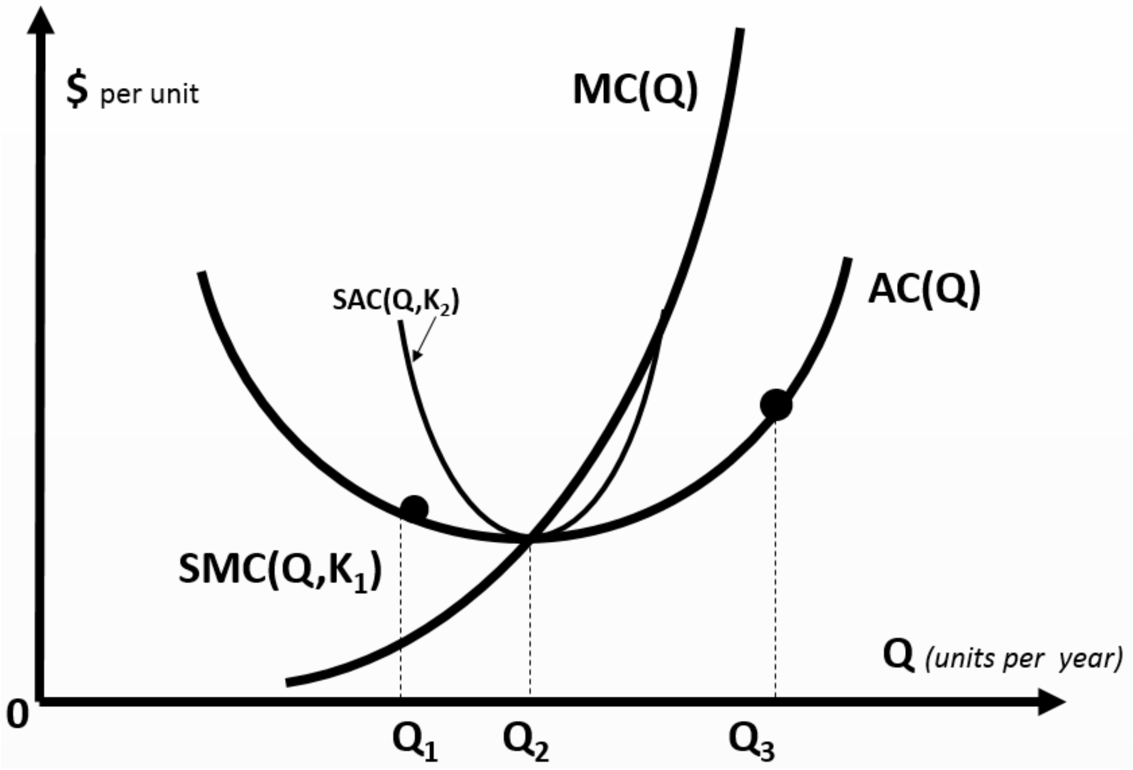


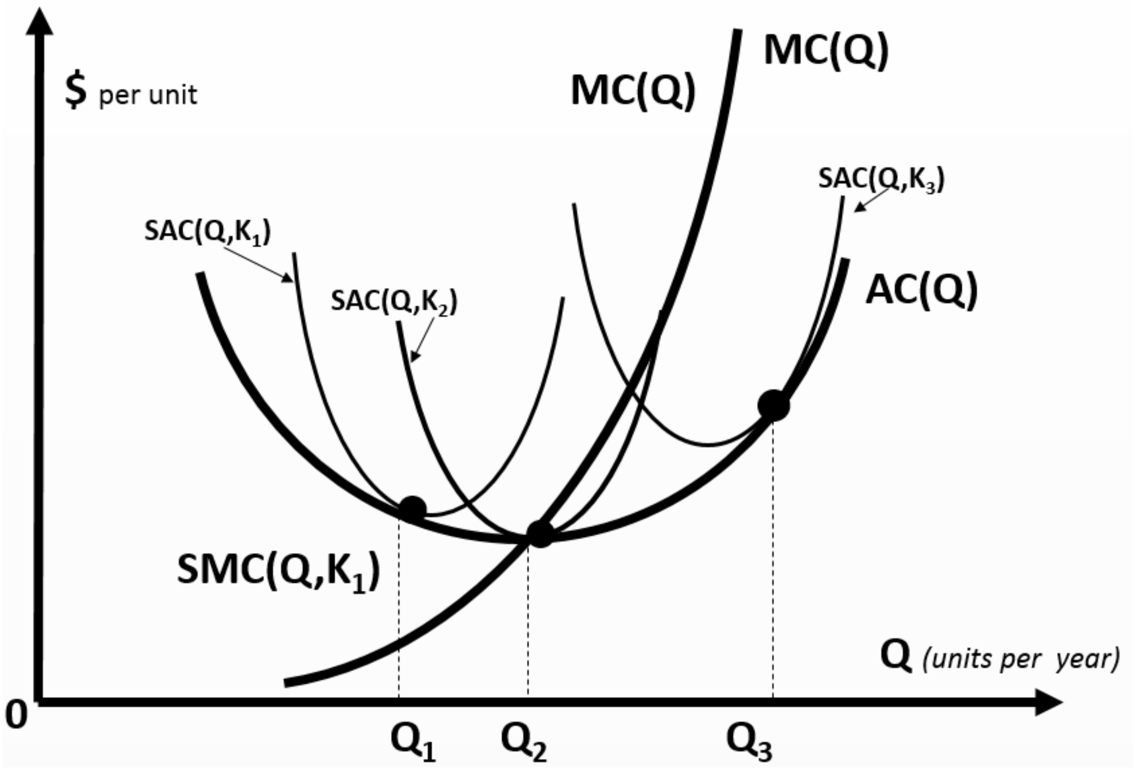
Cost Function Summary



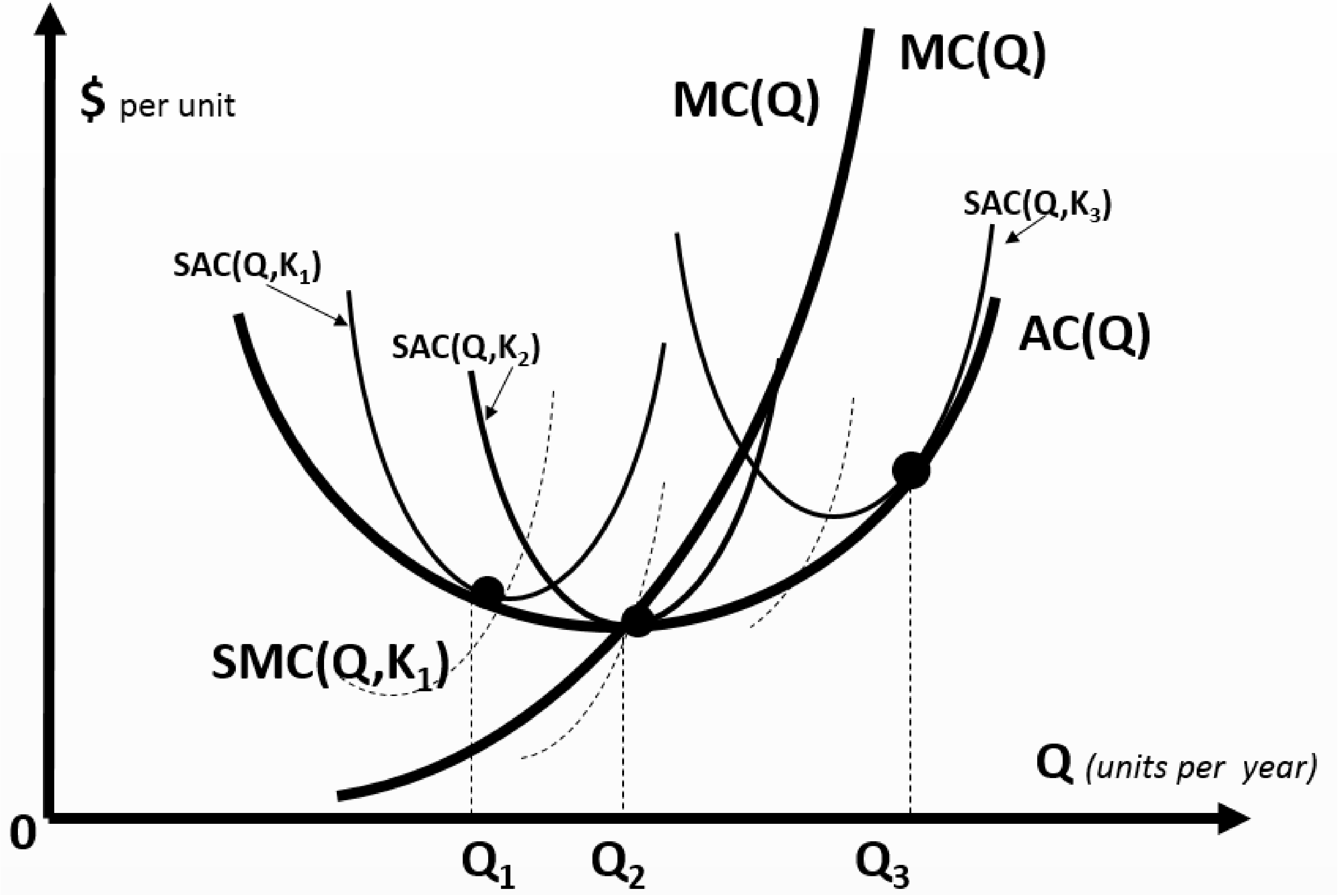


Cost Function Summary Continued





Cost Function Summary Continued



Economies of Scope

- Economies of scope – a production characteristic in which the total cost of producing given quantities of two goods in the same firm is less than the total cost of producing those quantities in two single-product firms
- Mathematically:
- $TC(Q_1, Q_2) < TC(Q_1, 0) + TC(0, Q_2)$
- Stand-alone costs – the cost of producing a good in a single-product firm, represented by each term in the right-hand side of the above equation

Economies of Experience

- Economies of experience – cost advantages that result from accumulated experience, or, learning-by-doing
- Experience curve – a relationship between average variable cost and cumulative production volume
  - Used to describe economies of experience
  - Typical relationship is $AVC(N) = AN^B$
  - Where N – cumulative production volume,
  - $A > 0$ – constant representing AVC of first unit produced,
  - $-1 < B < 0$ – experience elasticity (% change in AVC for every 1% increase in cumulative volume
  - Slope of the experience curve tells us how much AVC goes down (as a % of initial level), when cumulative output doubles

Estimating Cost Functions

- Total cost function – a mathematical relationship that shows how total costs vary with factors that influence total costs, including the quantity of output and prices of inputs
- Cost driver – A factor that influences or “drives” total or average costs
- Constant elasticity cost function – A cost function that specifies constant elasticity of total cost with respect to output and input prices.
- Translog cost function – A cost function that postulates a quadratic relationship between the log of total cost and the logs of input prices and output