Time Series

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Part 1: OLS with time series data

Chayanee Chawanote

Overview

- Stochastic Processes
  - Stationary processes
  - Non-stationary processes
- Concept of Basic Time Series
  - Unit Root test
  - Spurious regression
  - Cointegration

Example of time series model

- A static model: look at relationship between contemporaneous variables (same time period)
  $y_t = \beta_0 + \beta_1 x_t + u_t$
- A finite distributed lag (FDL) model: a independent variable in other time periods also determine the dependent variable
  $y_t = \alpha_0 + \delta_0 x_t + \delta_1 x_{t-1} + \delta_2 x_{t-2} + u_t$
  - δ_0 - impact propensity, reflecting the immediate change in y
  - δ_1 - effect from x in the last period that determine y in the current period
  - $\delta_0 + \delta_1 + ... + \delta_q$ is long-run propensity (LRP) measures the long-run change in the expected value of y given a one-unit, permanent increase in x.

Stochastic processes

- OLS assumes that all explanatory variables are nonstochastic process variables
- Stochastic process: a collection of random variables ordered in time
- The probability structure of a sequence of random variables is determined by the join distribution of a stochastic process
    - Stationary stochastic process: probability distribution is stable over time
    - Non-stationary stochastic process

Stationary processes

- Any set of values will have the same joint distribution as any other set of values measured at a different point in time.
- The stationary process is identically distributed, with the following properties
  - Mean of series must be stationary: $E(y_t) = \mu$
  - Variance of series must be stationary: $Var(y_t) = \sigma^2$
  - Covariance of series must be stationary: $Cov(y_t, y_{t+s}) = E(y_t - \mu)(y_{t+s} - \mu) = \gamma$

(weakly dependent: $Corr(y_t, y_{t+s}) \to 0$ and $s \to \infty$)

Moving average process (MA)

- MA is an example of a weakly dependent series.
- Form for MA(1): $y_t = u_t + \alpha_1 u_{t-1}, t = 1, 2, ...$
- $u_t \sim (0, \sigma_u^2)$
- y is a weighted average of u_t and u_{t-1}
- $Cov(y_t, y_{t-1}) = \alpha_1 \sigma_u^2$, $Corr(y_t, y_{t-1}) = \frac{\alpha_1}{1+\alpha_1^2}$
- MA(1) has a finite memory of one period: the last period matters, but observations prior to that have no effect on the current value of the process.

Autoregressive process (AR)

- Form for AR(1): $y_t = \rho_1 y_{t-1} + u_t, t = 1, 2, ..$
- $u_t \sim (0, \sigma_u^2)$
- y will be a stable stochastic process if $|\rho_1| < 1$.
- AR(1) process has infinite memory: every past value of the process affects the current value. As the effects of past values are weighted by powers of fraction ρ_1, those effects damp to zero.
- Under covariance stationarity and $\rho_1^2 < 1$, $\sigma_y^2 = \frac{\sigma_u^2}{1-\rho_1^2}$
- $Cov(y_t, y_{t+s}) = \rho_1^s \sigma_y^2$, $Corr(y_t, y_{t+s}) = \rho_1^s$

OLS regression with time series

- TS1: Linearity and weak dependence
  - Assume that a set of variables $[\mathbf{x}, y]$ is stationary and weakly dependent
- TS2: No perfect collinearity
- TS3: Zero conditional mean
  - $\mathbf{x}_t = (x_{1t}, x_{2t}, ..., x_{kt})$ are contemporaneously exogenous, $E[u_t|\mathbf{x}_t] = 0$
  - This conditional mean does not rule out correlation between, say, u_{t-1} and x_{1t}
- Under TS1-3, OLS estimators are consistent, but not necessarily unbiased.

OLS regression with time series

- To use standard inference procedures, we need the following assumptions:
- TS4: Homoskedasticity
  - The errors are contemporaneously homoskedastic, $Var(u_t|\mathbf{x}_t) = \sigma^2$
- TS5: No serial correlation
  - For all $t \neq s, E(u_t, u_s|\mathbf{x}_t, \mathbf{x}_s) = 0$
- It's quite difficult to guarantees that unobservables u_t are uncorrelated over time.
- Under TS1-5, OLS estimators are asymptotically normally distributed. The usual OLS standard errors, t and F stats are asymptotically valid.

Highly persistent time series

- This means 'strongly dependent': time-varying mean or time-varying variance $\rightarrow$ nonstationary processes
- For example, AR(1) with $\rho_1 = 1$: Random walk model
  - Random walk without drift:$y_t = y_{t-1} + u_t$
    $E(y_t) = E(y_0 + \sum u_t) = y_0$, $Var(y_t) = t\sigma^2$
  - Random walk with drift: $y_t = \delta + y_{t-1} + u_t$
    $E(y_t) = y_0 + t\delta$, $Var(y_t) = t\sigma^2$

Highly persistent time series

- Highly persistent: value of y today is important for determining the value of y in the very distant future
  - $E(y_{t+s}|y_t) = y_t, \forall s \geq 1$
- A random walk is a special case of what's known as a unit root process (a root of the polynomial)
- Trending and persistence are different things - a series can be trending but weakly dependent, or a series can be highly persistent without any trend.
- A random walk with drift is an example of a highly persistent series that is trending.