

EE320 (2/2012)

INTRODUCTORY MATHEMATICAL ECONOMICS

STATIC (or EQUILIBRIUM) ANALYSIS

(Part 2)

Topics

- Equilibrium in national-income analysis
 - Keynesian model
 - IS-LM model
 - Comparative statics

Simplest Keynesian National-Income Model

- Consider a *closed* economy (i.e. no trade).

- The equilibrium is: $Y = C + I_0 + G_0$

where $C = a + bY$ ($a > 0, 0 < b < 1$)

- At equilibrium, $Y = a + bY + I_0 + G_0$

➔ $Y^* = \frac{a + I_0 + G_0}{1 - b}$

- Corresponding consumption: $C^* = a + bY^*$

➔ $C^* = \frac{a + b(I_0 + G_0)}{1 - b}$

Closed Economy with Proportional Income Tax

- Equilibrium condition: $Y = C + I_0 + G_0$

where $C = a + bY_d \quad (a > 0, 0 < b < 1)$

$$Y_d = Y - T$$

$$T = tY \quad (0 < t < 1)$$

➤ At equilibrium, $Y = a + bY_d + I_0 + G_0$

$$Y = a + b(1-t)Y + I_0 + G_0$$

$$\Rightarrow Y^* = \frac{a + I_0 + G_0}{1 - b(1-t)}$$

$$\Rightarrow C^* = \frac{a + b(1-t)(I_0 + G_0)}{1 - b(1-t)}$$

Open Economy with Proportional Income Tax

- Equilibrium condition: $Y = C + I_0 + G_0 + X_0 - M$

where

$$C = a + bY_d \quad (a > 0, 0 < b < 1)$$

$$T = tY \quad (0 < t < 1)$$

$$M = mY$$

- At equilibrium, $Y = a + b(1-t)Y + I_0 + G_0 + X_0 - mY$



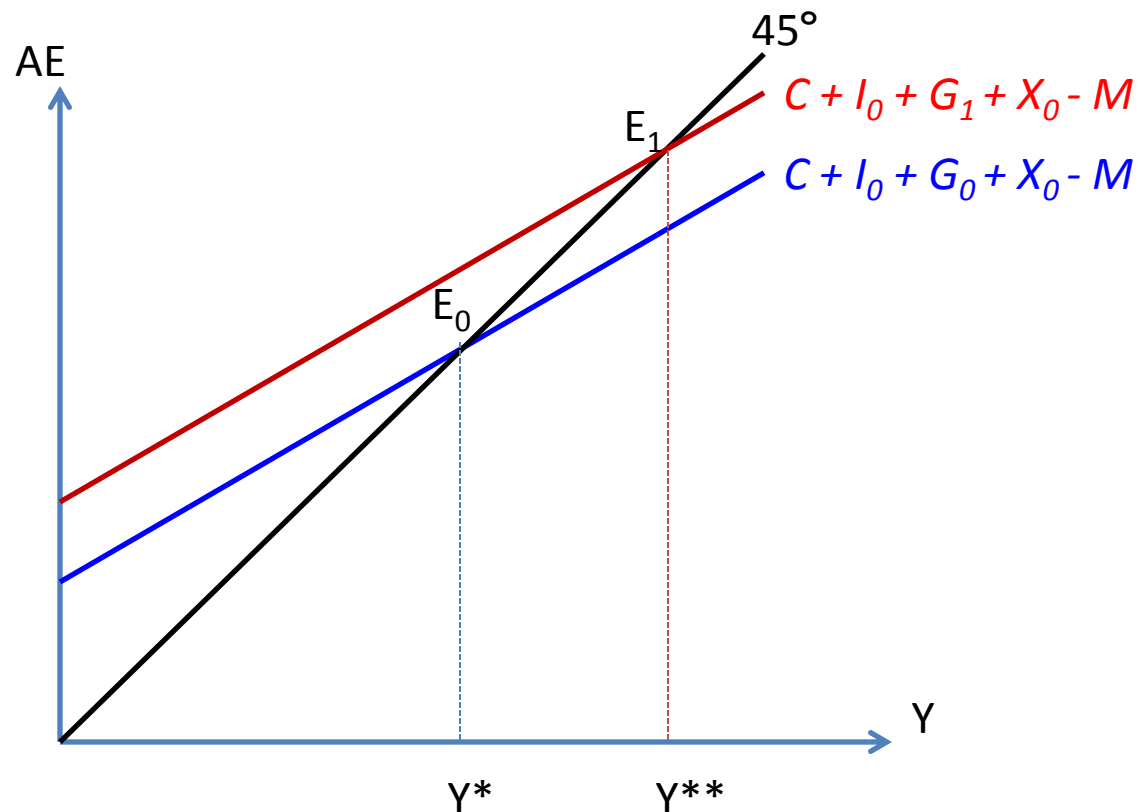
$$Y^* = \frac{a + I_0 + G_0 + X_0}{1 - b(1-t) + m}$$



$$C^* = \frac{a(1+m) + b(1-t)(I_0 + G_0 + X_0)}{1 - b(1-t) + m}$$

- If $M = mY_d$, $Y^* = ?$ (own practice!)

Graph: Keynesian National-Income Model



Suppose G increases. What is the new value of Y^* ?

Comparative Statics Analysis

- Suppose government increases its expenditure, what's the impact on the equilibrium national income (i.e. $\Delta Y^*/\Delta G=?$)
- Recall: $Y^* = \frac{a + I_0 + G_0 + X_0}{1 - b(1-t) + m}$, and suppose $\Delta G = G_1 - G_0$.

$$\triangleright Y^{*1} - Y^{*0} = \frac{a + I_0 + G_1 + X_0}{1 - b(1-t) + m} - \frac{a + I_0 + G_0 + X_0}{1 - b(1-t) + m}$$

$$\triangleright \Delta Y^* = Y^{*1} - Y^{*0} = \frac{\Delta G_1}{1 - b(1-t) + m}$$

$$\Rightarrow \boxed{\frac{\Delta Y^*}{\Delta G_1} = \frac{1}{1 - b(1-t) + m} > 0}$$

Thus, if $1 - b(1-t) + m < 1$, then $\Delta Y^*/\Delta G > 1$.

IS-LM Model

- The previous Keynesian model only deals with the equilibrium in the goods (or commodity) market. It seeks to determine the equilibrium level of national income.
- The **IS-LM model** deals with not only the **commodity market** but also the **money market**. It determines **the equilibrium levels of national income and interest rate** in the economy.
- The **IS curve** represents the equilibrium in the commodity market.
 - **Planned “investment” = Planned “savings”**
- The **LM curve** represents the equilibrium in the money market.
 - **Liquidity preference (money demand) = Money supply**

IS-LM Model: Closed Economy & No Tax

- Commodity Market:

$$Y = C + I + G_0$$

$$C = a + bY \quad (a > 0, 0 < b < 1)$$

$$I = I_0 - ir \quad (I_0, i > 0, \text{ and } r \text{ is interest rate})$$

$$\Rightarrow Y = a + bY + I_0 - ir + G_0 \quad \Rightarrow \boxed{Y = \frac{(a + I_0 + G_0) - ir}{1 - b}} : IS$$

- Money Market:

$$M^S = M_0$$

$$M^D = kY - hr \quad (k, h > 0)$$

$$\Rightarrow \text{At equilibrium, } M_0 = kY - hr \quad \Rightarrow \boxed{Y = \frac{M_0}{k} + \frac{h}{k}r} : LM$$

- From the two equilibrium conditions, we can solve for Y^* and r^* :

$$\boxed{Y^* = \frac{(a + I_0 + G_0)h + iM_0}{ik + h(1 - b)}} \quad \text{and} \quad \boxed{r^* = \frac{(a + I_0 + G_0)k - (1 - b)M_0}{ik + h(1 - b)}}$$

IS-LM Model: Closed Economy with Tax

- Commodity Market:

$$Y = C + I + G_0$$

$$C = a + bY_d \quad (a > 0, 0 < b < 1)$$

$$I = I_0 - ir \quad (I_0, i > 0)$$

$$Y_d = Y - T \quad \text{where } T = tY \quad (0 < t < 1)$$

$$\Rightarrow Y = a + b(1-t)Y + I_0 - ir + G_0$$

$$\Rightarrow Y = \frac{(a + I_0 + G_0) - ir}{1 - b(1-t)} : IS$$

- Money Market:

$$M^S = M_0$$

$$M^D = kY - hr \quad (k, h > 0)$$

$$\Rightarrow M_0 = kY - hr$$

$$\Rightarrow Y = \frac{M_0}{k} + \frac{h}{k} r : LM$$

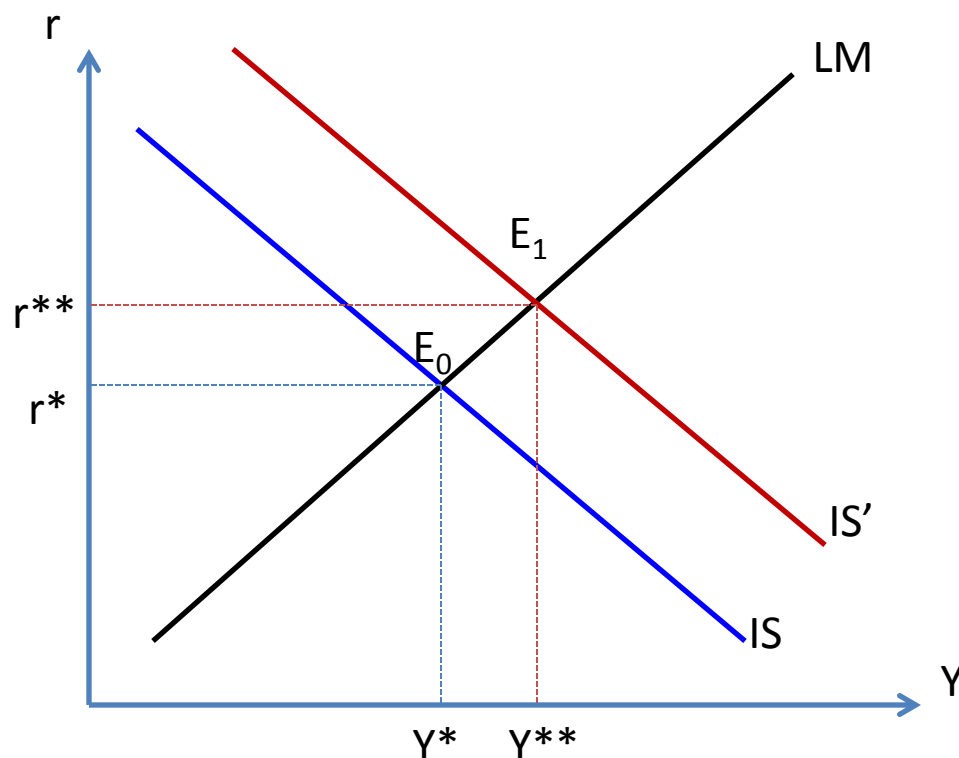
- Equilibrium values for Y^* and r^* :

$$Y^* = \frac{(a + I_0 + G_0)h + iM_0}{ik + h[1 - b(1-t)]}$$

and

$$r^* = \frac{(a + I_0 + G_0)k - [1 - b(1-t)]M_0}{ik + h[1 - b(1-t)]}$$

Graph: Equilibrium in IS-LM Model



- When the government increases its expenditure, what's the impact on the equilibrium Y and r ?

Comparative Statics Analysis: IS-LM Model

- In a closed economy with tax, suppose G increases ($\Delta G = G_1 - G_0$), $\Delta Y^*/\Delta G = ?$ and $\Delta r^*/\Delta G = ?$
- Recall: $Y^* = \frac{(a + I_0 + G_0)h - iM_0}{ik + h[1 - b(1 - t)]}$ and $r^* = \frac{(a + I_0 + G_0)k - [1 - b(1 - t)M_0]}{ik + h[1 - b(1 - t)]}$

$$\Delta Y^* = Y^{*1} - Y^{*0} = \frac{h}{ik + h[1 - b(1 - t)]} \Delta G \quad \Rightarrow \quad \frac{\Delta Y^*}{\Delta G} = \frac{h}{ik + h[1 - b(1 - t)]} > 0$$

$$\Delta r^* = r^{*1} - r^{*0} = \frac{k}{ik + h[1 - b(1 - t)]} \Delta G \quad \Rightarrow \quad \frac{\Delta r^*}{\Delta G} = \frac{k}{ik + h[1 - b(1 - t)]} > 0$$