

$$\begin{aligned}
 1.) \quad E_0 [M_{02} f_2] &= E_0 [M_{02} (S_2 - F_{02})] \\
 &= E [M_{02} S_2] - E [M_{02} F_{02}] \\
 &= S_0 - D_0 - \frac{1}{R_{rf}^2} F_0 = 0 \quad \text{dueto } \frac{F_{02}}{R_{rf}^2} = S_0 - D_0
 \end{aligned}$$

$$2.) \quad V(C_t, t) = -\delta^t e^{-\alpha t}$$

$$V(C_t, t) = \alpha \delta^t e^{-\alpha t}$$

$$P_0 \cdot E_0 \left[ \sum_{t=1}^{\infty} \frac{V_c(C_t, t)}{V_c(C_0, 0)} dt \right] = E_0 \left[ \sum_{t=1}^{\infty} \delta^t e^{-\alpha d_t - d_0} dt \right]$$

$$3.) \quad \frac{P_t}{dt} = E_t \left[ \sum_{j=1}^{\infty} \delta^j \left( \frac{C_{t+j}}{C_t} \right)^{r-1} \left( \frac{d_{t+j}}{dt} \right) \right] = E_t \left[ \sum_{j=1}^{\infty} \delta^j e^{(r-1) \ln(C_{t+j}/C_t) + \ln(d_{t+j}/dt)} \right]$$

$$\frac{P_t}{dt} = E_t \left[ \sum_{j=1}^{\infty} \delta^j e^{(r-1) X_j \mu_c + \sigma_c \sum_{i=1}^j \eta_{t+i}} + j \mu_d + \sigma_d \sum_{i=1}^j \xi_{t+i} \right]$$

$$= \sum_{j=1}^{\infty} e^{j(\ln \delta - (1-r)\mu_c + \mu_d + \frac{1}{2}(1-r)^2 \sigma_c^2 + \sigma_d^2) - (1-r) \rho \sigma_c \sigma_d j}$$

$$= \frac{1}{1 - \delta e^{-(1-r)\mu_c + \mu_d + \frac{1}{2}(1-r)^2 \sigma_c^2 + \sigma_d^2 - (1-r) \rho \sigma_c \sigma_d}}^{-1}$$

$$\text{Then } P_t = dt \frac{\delta e^a}{1 - \delta e^a} \quad ; \quad a = \mu_d - (1-r)\mu_c + \frac{1}{2} [(1-r)^2 \sigma_c^2 + \sigma_d^2] - (1-r) \rho \sigma_c \sigma_d$$

$$4) a) E_t(b_{t+1}) = \frac{R_f}{q_t} b_t + E_t[e_{t+1}] q_t + (1 - q_t) E_t[z_{t+1}] = R_f b_t$$

$$b.) E_t[b_{t+1}] = R_f b_t, \quad \lim_{i \rightarrow \infty} E_t[b_{t+i}] = \begin{cases} +\infty & \text{if } b_t > 0 \\ -\infty & \text{if } b_t < 0 \end{cases}$$

$P_t$  is a limited then  $b_t > 0$ , Rational speculate bubble not exist since  $P_t$  cannot exceed  $P_{sales}$  and  $b_t$  cannot exceed  $P_{sales} - P_t^*$

c.) Rational bubble can't be existed, since bond price must be  $P_t = d_t$  at maturity and equal to 0 when  $P_t = P_t^*$  (rational price)

b.) Due to  $P_t = f_t + b_t$ ;  $b_t \neq 0 \rightarrow$  impossible.  
 $P_t = f_t = d_t$  at date T