4.3 Application examples

Note: Mapping the linear economic model in a matrix form

Example 4.K: Using the form Ax = d, write the following market model in the matrix form.

$$Qd = Qs$$

$$Qd = a - bP + cY$$

$$Qs = e + fP + gW$$

Q ; P that clear the market

when a, b, c, e, f, g are parameters. Y = income and W = weather condition.

$$Q = a - bp + cy$$
$$Q = e + fp + gw$$

$Ax = d$

$$Q + bp = a + cy$$
$$Q - fp = e + gW$$

$$x = \begin{pmatrix} Q \\ p \end{pmatrix} \Rightarrow \underbrace{\begin{pmatrix} 1 & b \\ 1 & -f \end{pmatrix}}_{A} \underbrace{\begin{pmatrix} Q \\ p \end{pmatrix}}_{x} = \underbrace{\begin{pmatrix} a+cy \\ e+gw \end{pmatrix}}_{d}$$

$$\begin{pmatrix} Q \\ p \end{pmatrix} = \begin{pmatrix} 1 & b \\ 1 & -f \end{pmatrix}^{-1} \begin{pmatrix} a+cy \\ e+gw \end{pmatrix}$$

#

Q = ? ; P = ?

$$\begin{pmatrix} Q \\ P \end{pmatrix} = \begin{pmatrix} 1 & b \\ 1 & -f \end{pmatrix}^{-1} \begin{pmatrix} a + cy \\ e + gw \end{pmatrix}$$

$$\begin{pmatrix} 1 & b \\ 1 & -f \end{pmatrix}^{-1} = \begin{bmatrix} -f & -b \\ -1 & 1 \end{bmatrix} \cdot \frac{1}{\det(A)}$$

$$\hookrightarrow (-f-b) = -(f+b)$$

$$\operatorname{Cof}(A) = \begin{bmatrix} (-1)^{1+1} \cdot (-f) & (-1)^{1+2} \cdot 1 \\ (-1)^{2+1} (b) & (-1)^{2+2} (1) \end{bmatrix} = \begin{bmatrix} -f & -1 \\ -b & 1 \end{bmatrix}$$

$$\begin{pmatrix} Q \\ P \end{pmatrix} = -\frac{1}{f+b} \begin{pmatrix} -f & -b \\ -1 & 1 \end{pmatrix} \begin{pmatrix} a+cy \\ e+gw \end{pmatrix}$$

$$\begin{pmatrix} Q \\ P \end{pmatrix} = -\frac{1}{f+b} \begin{pmatrix} -f(a+cy) - b(e+gw) \\ -1(a+cy) + 1(e+gw) \end{pmatrix}$$

$$Q^* = \frac{f(a+cy) + b(e+gw)}{f+b} \; ; \quad \frac{\Delta Q^*}{\Delta y} = \frac{f \cdot c}{f+b} > 0$$

$$P^* = \frac{(a+cy) - (e+gw)}{f+b} \qquad \frac{\Delta Q^*}{\Delta w} = \frac{b \cdot g}{f+b} > 0$$

$$\frac{\Delta P^*}{\Delta y} = \frac{c}{f+b} > 0 \; ; \quad \frac{\Delta P^*}{\Delta w} = -\frac{g}{f+b} < 0$$

(Demand Effect) (Supply Effect)

Example 4.L

Consider a simple macroeconomic model.

$C = a + bY_d; \quad 0 < b < 1$

$I = I_a + iY; \quad 0 < i < 1$

$G = G_0$

$T = T_0 + tY; \quad 0 < t < 1$

$R = R_0$

$Y_d = Y - T + R$

$\Rightarrow Y = C + I + G$

Default you need to know!

where R is the government transfer and G is the government purchase. All the remainings are defined as usual.

- State all the endogenous variables in the model

Endo: C, I, T, Y_d, Y

Exo: G, R, I_a, T_0, a

b, (MPC) i (MPI)
t (MPT)
-parameters
-coefficient

- Is the above system solvable?

5-equations / 5-Endos

Solution of all the variables ~~at the~~ at the same time.

"Solvable" "if solve for the ~~[illegible]~~ set"

$$x = \begin{pmatrix} C \\ I \\ T \\ Y_d \\ Y \end{pmatrix}$$

$A x = d$

$5\times5 \quad 5\times1 \quad 5\times1$

- Simplify the system to a 3-variable system that includes Y, C and I.

$y = C + I + G \Rightarrow G_0$

$C = a + b\,y_d \Rightarrow C = a + b(Y - T + R)$

$I = I_0 + i \cdot y$ $\quad C = a + b(y - T_0 - ty + R)$ $\quad R \Rightarrow R_0$

$G = G_0$

$R = R_0$

$\cancel{y_d = Y - T + R.}$

$T = T_0 + t \cdot y$

(3) $y = C + I + G_0$

(3) $C = a + b(Y - T_0 - ty + R_0)$

$I = I_0 + i\,y.$

Simplified 3-3 system of Equation of the original macroeconomic model.

Exact-Identified / system of Equations

- Rewrite the system of equations in the matrix form and solve for all the three variables using the Cramer's rule.

$$\begin{cases} y - C - I = G_0 \\ C - b(1-t)\cdot y = a + b(R_0 - T_0) \\ I - i\cdot y = I_0. \end{cases}$$

$$X = \begin{pmatrix} y \\ C \\ I \end{pmatrix} \Rightarrow AX = d$$

$$\underbrace{\begin{bmatrix} 1 & -1 & -1 \\ -b(1-t) & 1 & 0 \\ -i & 0 & 1 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} y \\ C \\ I \end{bmatrix}}_{X} = \underbrace{\begin{bmatrix} G_0 \\ a + b(R_0 - T_0) \\ I_0 \end{bmatrix}}_{d}$$

$$I$$

$$\cancel{\det(A)} = \frac{\begin{vmatrix} 1 & -1 & \cancel{X}G_0 \\ -b(1-t) & 1 & a+b(R_0-T_0) \\ -i & 0 & I_0 \end{vmatrix}}{\det(A)}$$

$$\det(A) = 1 + 0 + 0 - (-1)(-i) - 0 - (1)(-b(1-t))(-1)$$

$$= 1 - i - b(1-t)$$

Vector d

$$\left|\begin{array}{ccc|cc} 1 & -1 & G_0 & 1 & -1 \\ -b(1-t) & 1 & a+b(R_0-T_0) & -b(1-t) & 1 \\ -i & 0 & I_0 & -i & 0 \end{array}\right|$$

$$\frac{\Delta I^*}{\Delta G_0} = \frac{i}{1-i-b(1-t)}$$

assume $i + b(1-t) < 1$

$$\Rightarrow 0 < 1 - (i + b(1-t)) < 1$$

$$\frac{\Delta I^*}{\Delta G_0^*} > 1$$ by our assumption

determinant of A when you Replace #3 with Vector d

$$I_0 + i[a + b(R_0 - T_0)] + 0 - (-i)(G_0) - (0) - (-1)(-b(1-t)).I_0$$

multiplier

$$I_0 + i[a+b(R_0-T_0)] + G_0 i - I_0(b(1-t))$$

$$\Rightarrow I_0(1 - b(1-t)) + \cancel{G_0 i} + [a + b(R_0 - T_0) + G_0]i$$

$$I^* = \frac{I_0(1-b(1-t)) + [a + b(R_0 - T_0) + G_0].i}{1 - i - b(1-t)} \rightarrow \text{Equilibrium investment}$$

$$I^* = \frac{I_0(1-b(1-t)) + [a + b(R_0 - T_0) + G_0]\, i}{1 - i - b(1-t)}$$

$$\frac{\Delta I^*}{\Delta I_0} = \left.\frac{(1-b(1-t))}{1-i-b(1-t)}\right\}$$ multiplier of I_0 on equilibrium investment.

→ Do the rest <u>yourself</u>: solve for Y^* and C^* using the Cramer's Rule; and conduct the sensitivity analysis on the equilibrium functions

Chapter 5: Single-variable optimization: theory and applications

Optimization theory is a mathematical tool developed to solve for the solution of the problem that can be characterized in the following form

Single-variable optimization   maximization problem   highest value of f

$$\max\ (mim)_{x}\ \ f(x)\ ,\ \ x \in D.$$

find x (single value / multiple values) lowest value of f

- Optimization theory has been extensively applied into economics analysis. multivariate optimization   minimization problem.
  - We've seen before that equilibrium economic models include several behavioral equations that capture behavior of groups/persons engaged in the model.
  - All these behavioral equations are actually derived from *the first principle* that agents are rational and choose for *optimal decision*, resulting in the behavior that can be captured by the proposed function.
    - Market demand model
      - Demand equation is the result of maximizing utility, subject to budget set.
      - Supply equation is the result of profit-maximizing decision of firm.
    - National income model
      - Aggregate consumption function is derived from the aggregation of individual consumption function which is the result from intertemporal consumption problem.

5.1 Basic calculus: derivative

What is derivative?

- Derivative reflects the responsiveness of a variable attributed to a variable.

For example, consider a case y=f(x).

- Derivative of y with respect to x is denoted by $\frac{dy}{dx}$ $or\ f'(x)$.
- Very much like to the concept of delta change, derivative considers the case for a small change.

By definition, mathematical notion for derivative is limiting case for

$$\frac{dy}{dx}=\frac{\Delta y}{\Delta x};\ (\Delta x\approx 0)$$

Formally stated, the notion of derivative is given by,

$$\left.\begin{aligned}\frac{dy}{dx}=f'(x_0)&=\lim_{h\to 0}\frac{f(x_0+h)-f(x_0)}{h}\\&=\lim_{x\to x_0}\frac{f(x)-f(x_0)}{x-x_0}\\&=\lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x}\end{aligned}\right\}$$

a small incremental in x ($\Delta x \approx 0$)