Lecture Three

Sequential Games

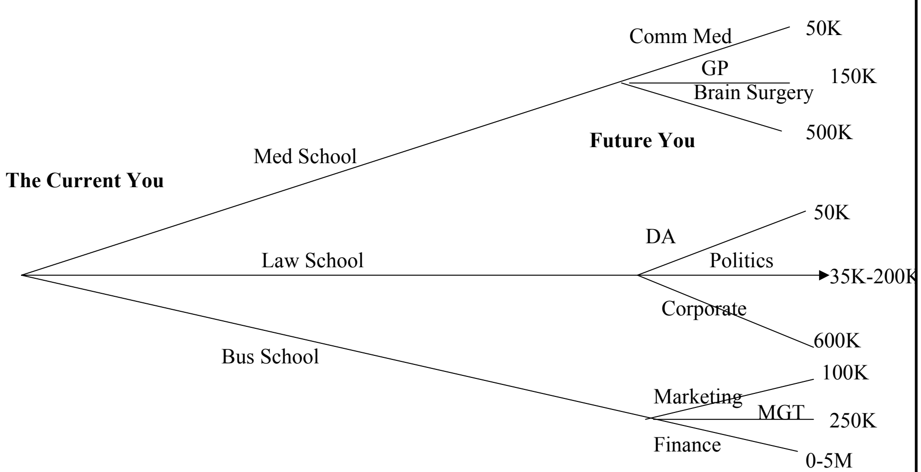
Sequential Games: Outline

- Simple examples of sequential games
- one-person decision problems: are they “Games”?
- Subgames
- Solving Games:Rollback or Backward Induction
- Threats, promises and credibility
- Sequential Market games - quantity or price competition
- First mover advantage?
- Ultimatum games

1-player Games with Perfect Information

- Perfect Information
- Extensive form of a game (tree diagram)
- Features of the extensive form
  - endpoints
  - nodes
  - information sets
  - branches
  - payoffs
- Solving a game by backward induction
- A strategy is a complete plan of auction

A One-Person "Game"



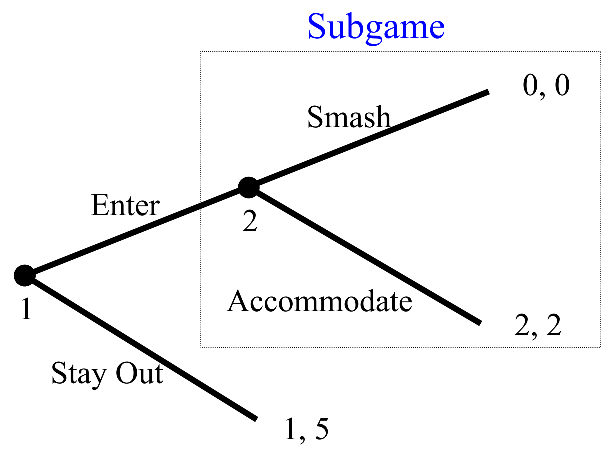
Sequential Games

- recall the distinction between decision problems and strategic problems.
- my simple choice of the best coffee shop to go to is a decision problem
- my (hard) choice of the best coffee shop to go to in order to avoid my boss is a strategic problem (I need to anticipate her choice in order to decide optimally.)

Sequential Games

- strategic problems are the focus of game theory and, as we will see, can be very complicated.
- sequential games with perfect information are the simplest type of strategic problem.
- this is because they are really just a sequence of individual decision problems.
- they are often described in “game trees”.

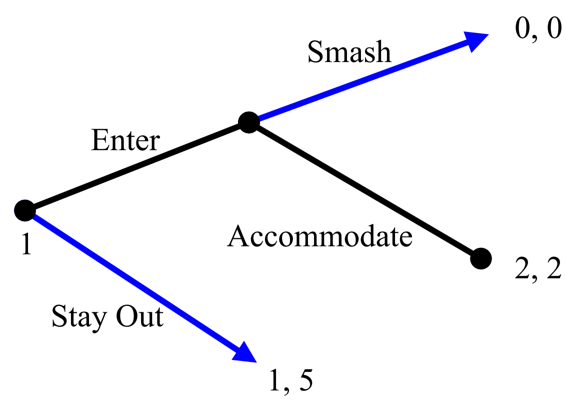
Telex vs. IBM, extensive form: subgame, perfect information



Example of a sequential game with perfect information

- Telex is considering entering the computer business.
- notice that in order for Telex to determine its best strategy, it must be able to anticipate IBM's response.
- How should it approach this problem?

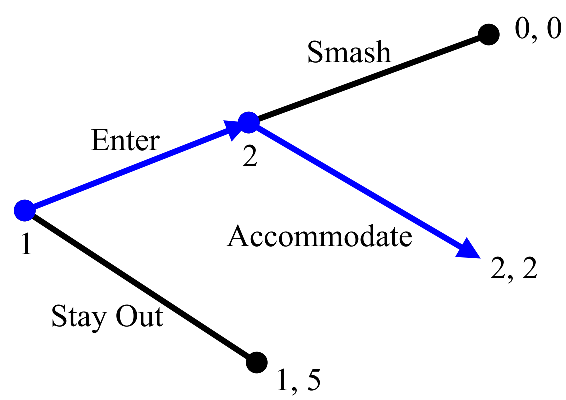
Telex vs. IBM, extensive form: noncredible equilibrium



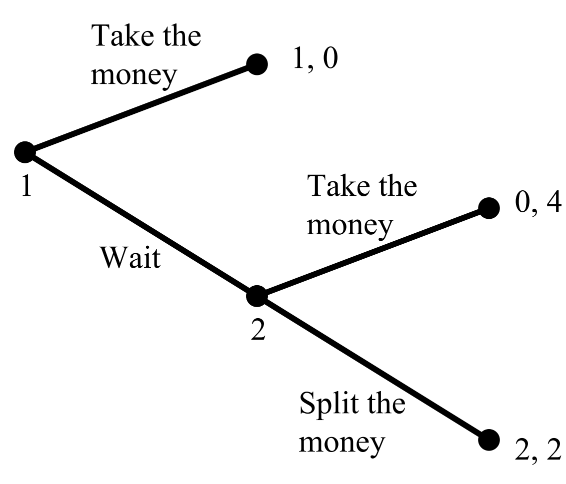
Subgames and their equilibria

- The concept of subgames
- Equilibrium of a subgame
- Credibility problems: threats and promises you have no incentives to carry out when the time comes
- Two important examples
  - Telex vs. IBM
  - Centipede

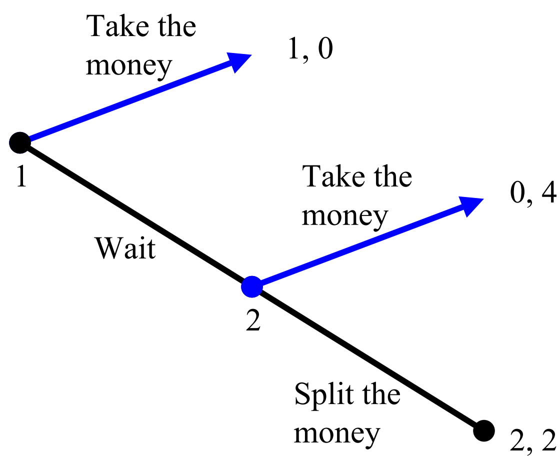
Telex vs. IBM, extensive form: credible equilibrium



Centipede, extensive form



Centipede, extensive form



Maintaining Credibility via Subgame Perfection

- Subgame perfect equilibria: play equilibria on all subgames
- They only make threats and promises that a player does have an incentive to carry out
- Subgame perfection as a necessary condition for solution of games in extensive form

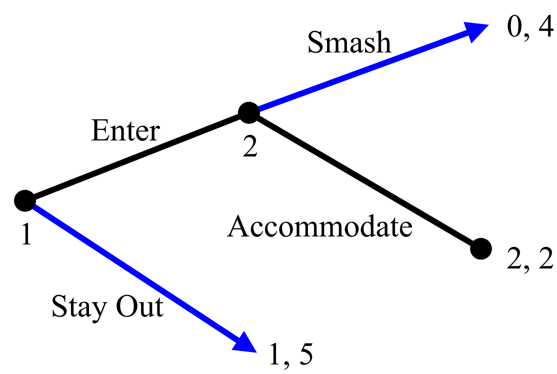
Rollback Equilibrium (Look Ahead and Reason Back)

- This is also called Backward Induction
- Backward induction in a game tree leads to a subgame perfect equilibrium
- In a subgame perfect equilibrium, “best responses” are played in every subgame

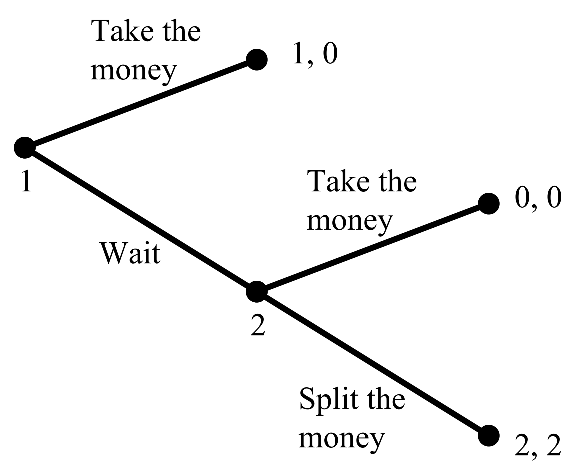
Credible Threats and Promises

- The variation in credibility when money is all that matters to payoff
- Telex vs. Mean IBM
- Centipede with a nice opponent
- The potential value of deceiving an opponent about your type

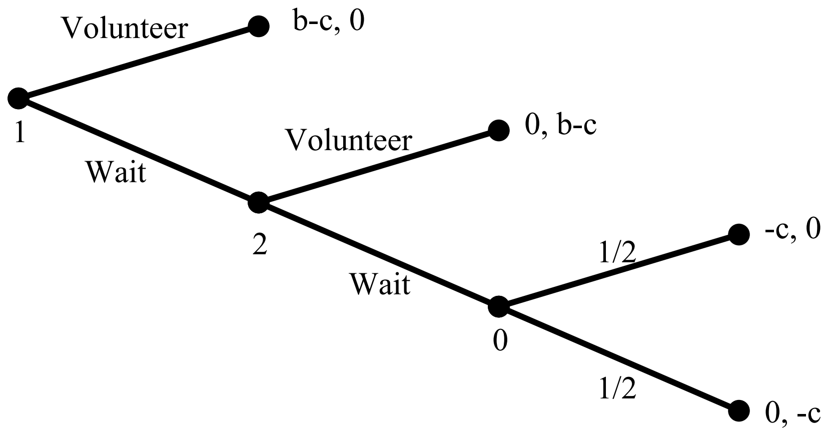
Telex vs. Mean IBM



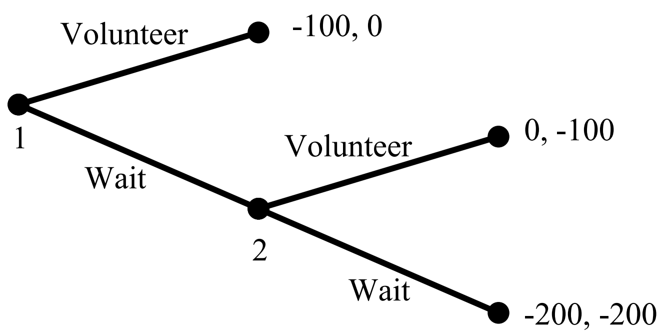
Centipede with a nice opponent, extensive form



Conscription, extensive form



Conscription, b = \$300 and c = \$400



Credible Quantity Competition: Stackelberg Equilibrium

- The first mover advantage in Stackelberg competition
- One firm sends its quantity to the market first. The second firm makes its moves subsequently.
- The strategy for the firm moving second is a function
- Incredible threats and imperfect equilibria

Stackelberg Equilibrium for two firms

Market Price, P = 130 - Q

Market Quantity, $Q = x_1 + x_2$

Constant average variable cost, c = \$10

Firm 1 ships its quantity, x_1, to market first

Firm 2 sees how much firm 1 has shipped and then ships its quantity, x_2, to the market

Stackelberg Equilibrium: Firm 2 wants to maximize its profits **given Firm 1's choice.**

Firm 2's profit function is given by:

$$u_2(x_1, x_2) = [130 - x_1 - x_2 - 10]\, x_2$$

Differentiate wrt x_2 gives:

$$0 = (120 - x_1 - 2x_2)$$

or

$$(120 - x_1)/2 = x_2$$

Stackelberg Equilibrium: Firm 2 wants to maximize its profits **given Firm 1's choice.**

- This gives us Firm 2's Best response given Firm 1' choice of x_1
- We can write this as a function of x_1.
- $g(x_1) = (120 - x_1)/2$

Stackelberg Equilibrium: Firm 1 also wants to maximize its profits

Firm 1's profit function is given by:

$u_1(\mathbf{x}) = [130 - x_1 - g(x_1) - 10]\, x_1$

Substituting $g(x_1)$ into that function:

$u_1(\mathbf{x}) = (120 - x_1 - 60 + x_1/2)\, x_1$

$\therefore$ Firm 1's profits depend only on its shipment

Taking the first order condition for $u_1(\mathbf{x})$:

$0 = 60 - x_1$

The Stackelberg Equilibrium for two firms

The Stackelberg equilibrium value of firm 1's shipments, $x_1^* = 60$

Firm 2's shipments, $x_2^* = 60 - 60/2 = 30$

Market Quantity, Q = 60 + 30 = 90

Market Price, P = 130 - 90 = \$40

We will see that this equilibrium is different from Cournot competition's equilibrium, where $x_1^* = x_2^* = 40$, Q = 80 and P = \$50 (Q's are chosen simultaneously.

Credible Price Competition: Bertrand-Stackelberg Equilibrium

- Firms use prices as the strategic instrument
- The strategy for the firm moving second is a function
- Firm 2 has to beat only firm 1's price which is already posted
- The second mover advantage in Bertrand-Stackelberg competition

Bertrand -Stackelberg Equilibrium for two firms

Market Price, P = 130 - Q and
Constant average variable cost, c = \$10

Firm 1 first announces its price, p_1

Firm 2's profit maximizing response to p_1:

p_2 = \$70	if p_1 is greater than \$70
$p_2 = p_1$ - \$0.01	if p_1 is between \$70 and \$10.02
$p_2 = p_1$	if p_1 = 10.01
p_2 = \$10	otherwise

Get competitive outcome; no extra profits!

Differentiated Products

- Product differentiation mutes both types of mover advantage
- A mover disadvantage can be offset by a large enough cost advantage

Two firms in a Bertrand-Stackelberg competition

The demand function faced by firm 1:

$x_1(\mathbf{p}) = 180 - p_1 - (p_1 - \text{average } p)$

$\Rightarrow \quad x_1 = 180 - 1.5p_1 + 0.5p_2$

Similarly, the demand function faced by firm 2:

$x_2 = 180 + 0.5p_1 - 1.5p_2$

Constant average variable cost, c = \$20

Two firms in a Bertrand-Stackelberg competition: Equilibrium prices

Knowing that firm 2 will determine p_2 by using $g(p_1)$, firm1 tries to maximize its profit:

$$\max\ (p_1 - 20)[180 - 1.5p_1 + 0.5(70 + p_1/6)]$$

Profit maximizes when the first order condition is satisfied: $0 = 215 - (17/12)p_1 + (p_1 - 20)\ (-17/12)$

$\therefore \quad p_1^* = 2920/34 = \$85.88$

Firm 2, which moves last, charges slightly lower price than p_1^*:

$$p_2^* = 70 + p_1^*/6 = 70 + \$14.31 = \$84.31$$

Two firms in a Bertrand-Stackelberg competition: Profits for the two firms

Firm 1 sells less than firm 2 does:

$x_1^* = 93.34$ and $x_2^* = 96.48$

Firm 1's profit, $u_1^* = (93.34)(85.88 - 20)$
$= \$\ 6149.24$

Firm 2's profit, $u_2^* = (96.48)(84.31 - 20)$
$= \$\ 6204.63$

Firm 2, the second mover, makes more money

This Offer is Good for a Limited Time Only

- The credibility problems behind the marketing slogan
- The principle of costly commitment
- Industries where the slogan is credible

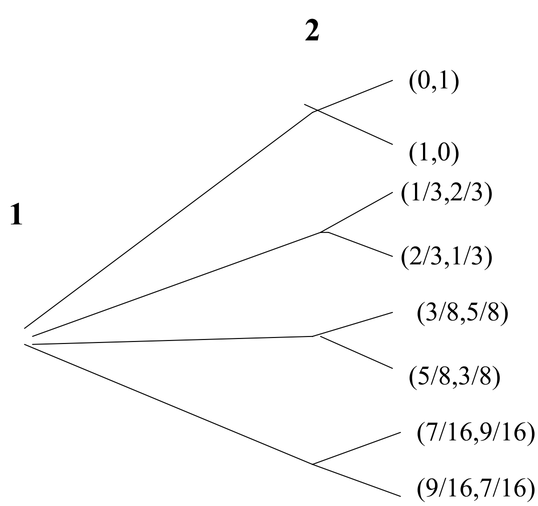
An example of "This offer is good for a limited time only"

- Exploding job offers
  - An early job offer with a very short time to decide on whether to take the job.
  - Risk-averse people often end up accepting inferior job offers

Is it always better to move first?

- it is in the Stackelberg game.
- is it in the Bertrand game?
- what about the cut the cake game?

Cut the Cake



Ultimatum Games

- consider the following game
- I want to buy a car from a seller.
- I know the seller can always sell the car to a used car lot for $1000.
- I value the car at $1500. The seller knows this.
- I need to leave the country on business at the end of the day and have time to make only one offer. The seller knows this as well.
- I can make offers in $1 dollar increments.
- What is the rollback equilibrium of this bargaining game?

Ultimatum Games: Analysis

- This is an example of a take-it-or-leave-it bargaining game.
- What is a subgame? A subgame starts when the seller is in the position to accept or reject my price offer.
- When the seller has a price offer of $1200 on the table, this represents a different subgame than when an offer of $1199 is on the table. Therefore, there are as many possible subgames as there are prices.
- The only interesting ones are those with prices below $1500 and above $1000 so there are 500 different possible subgames.

Ultimatum Games: Solution

- Suppose that I have offered *p* between \$1000 and \$1500. *p* represents the subgame.
- Now the seller must decide Accept or Reject.
- If she accepts, she gets $p-1000$ (over and above what her next best alternative is)
- If she rejects, she gets 0 (over and above what her next best alternative is)
- Conclusion: If $p>1000$ she should accept.

Ultimatum Games: Solution (ctd)

- Now, what is my best strategy.
- I know that for every $p>1000$ I offer, seller will accept, therefore, my payoff is $1500-p$ if $p>1000$ and 0 if $p<1000$. (equality is indeterminate)
- my best response is to offer $p=1001$. The seller should accept, she gets *1* and I get *499*.
- This is quite general, in one shot take-it-or-leave-it games, the offeror gets (almost) all the surplus.
- first mover advantage here is very strong.

Appendix. Ultimatum Games in the Laboratory

- Games with take-it-or-leave-it structure
- In experiments, subjects playing such games rarely play subgame perfect equilibria
- The nice opponent explanation vs. the expected payoff explanation

Modifications

- More players? Easy, as long as we remember to rollback
- More than two strategies? We have already done it. (Cut the cake, (4 choices for player 1), Stackelberg infinite choices.)
- Bottom line. Sequential games with perfect information are interesting but pretty easy.