FN211 Risk and Return

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Outline

- Asset Return and Stand-Alone Risk
- Return and Risk in Portfolio Context
- Diversification
- Introduction to VaR

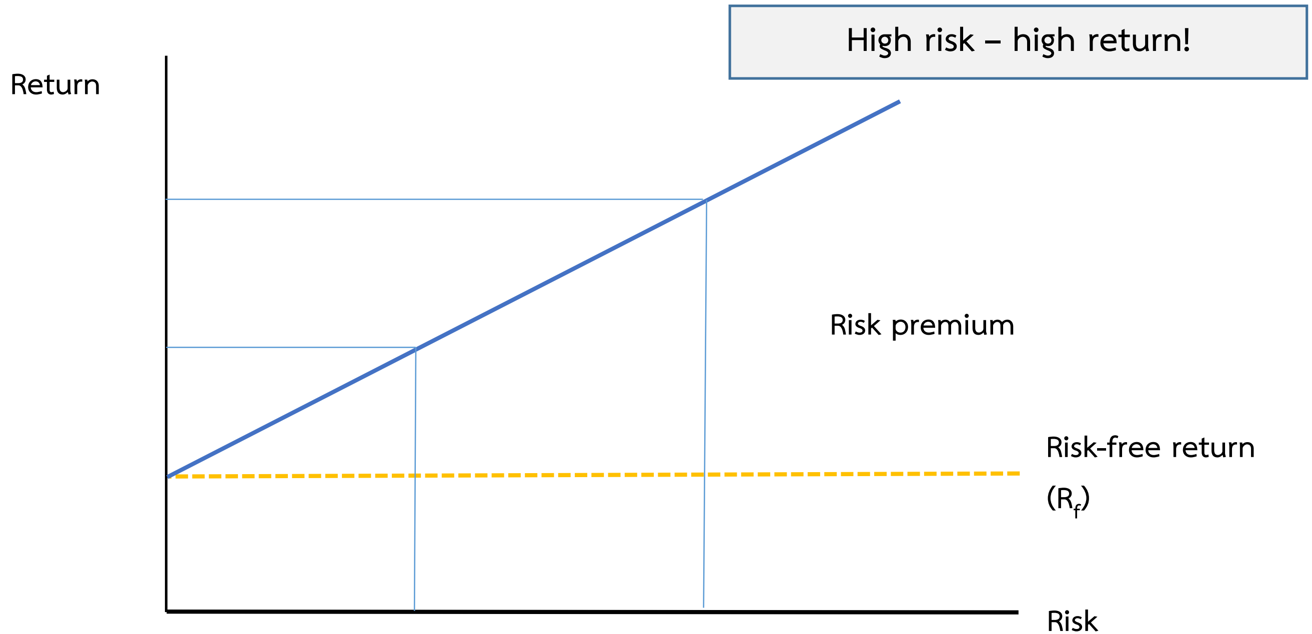
What are investment returns?

- Investment returns measure financial results of an investment.
- Returns may be historical or prospective (anticipated).
- Returns can be expressed in:
  - (\$) dollar terms.
  - (%) percentage terms.

Example: An investment costs $1,000 and is sold after 1 year for $1,100.

Risk-Return Tradeoff

The relationship between risk and return



Required rate of return = Risk-free return + Risk premium

= R_f + Risk premium

What is investment risk?

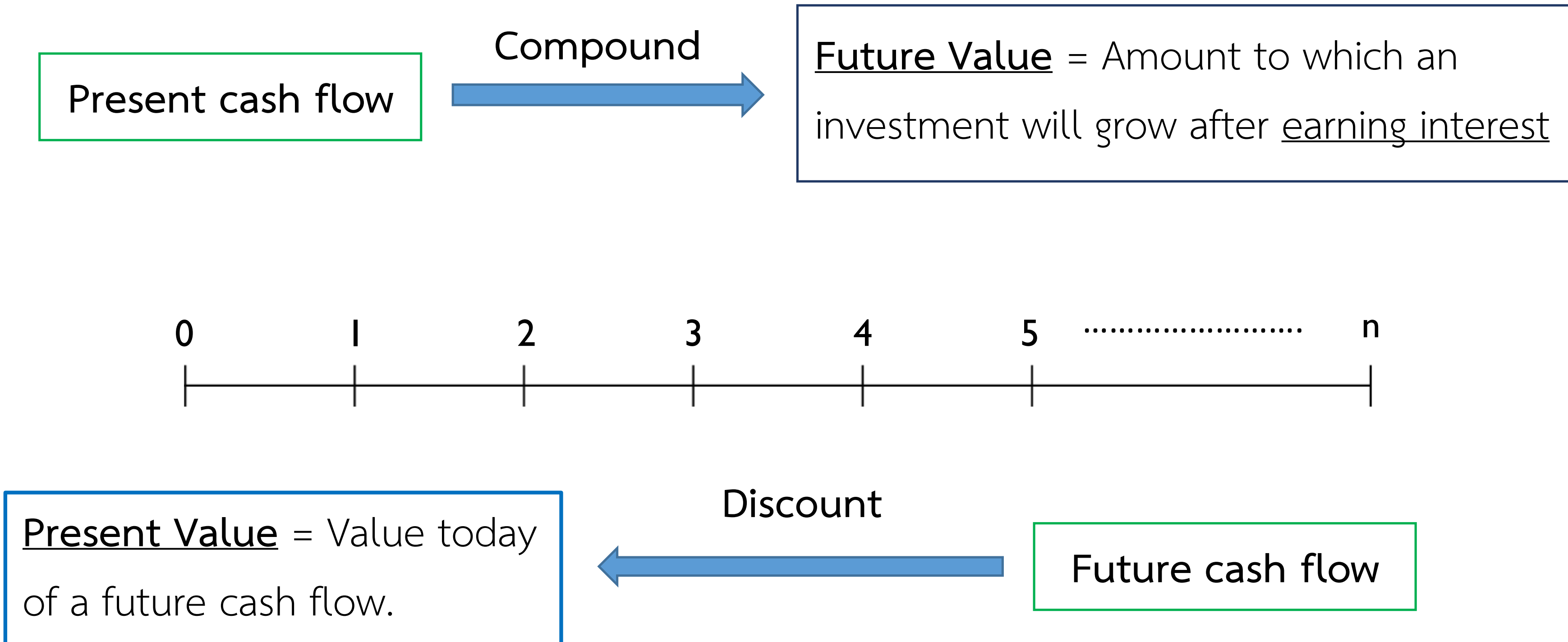
- Typically, investment returns are not known with certainty.
- Investment risk pertains to the probability of earning a return less than expected.
- Greater the chance of a return far below the expected return, greater the risk.

Stand-Alone Risk	Risk in Portfolio Context

Asset Return and Stand-Alone Risk

Return: Future Value vs. Present Value

<u>Investment time line</u>



Definition

(1) Discrete Compounding

- If the interest rate is simple (no compounding takes place), then the future value of any investment can be written as:

$$\boxed{\text{Future Value (FV)} = PV \times (1 + r)^{t}}$$

How to compute return?

Definition

(2) Continuous Compounding

- Continuous compounding introduces the concept of the natural logarithm. This is the constant rate of growth for all naturally growing processes. It's a figure that developed out of physics.

$$\boxed{FV = P * e^{rt}}$$

How to compute return?

What is difference?

Period	0	1	2
Asset Value	100	90	100

What are returns and total returns?

Expected Returns

- Expected returns are based on the probabilities of possible outcomes.
- In this context, “expected” means average if the process is repeated many times.
- The “expected” return does not even have to be a possible return.

$$E(R) = \sum_{i=1}^{n} p_i R_i$$

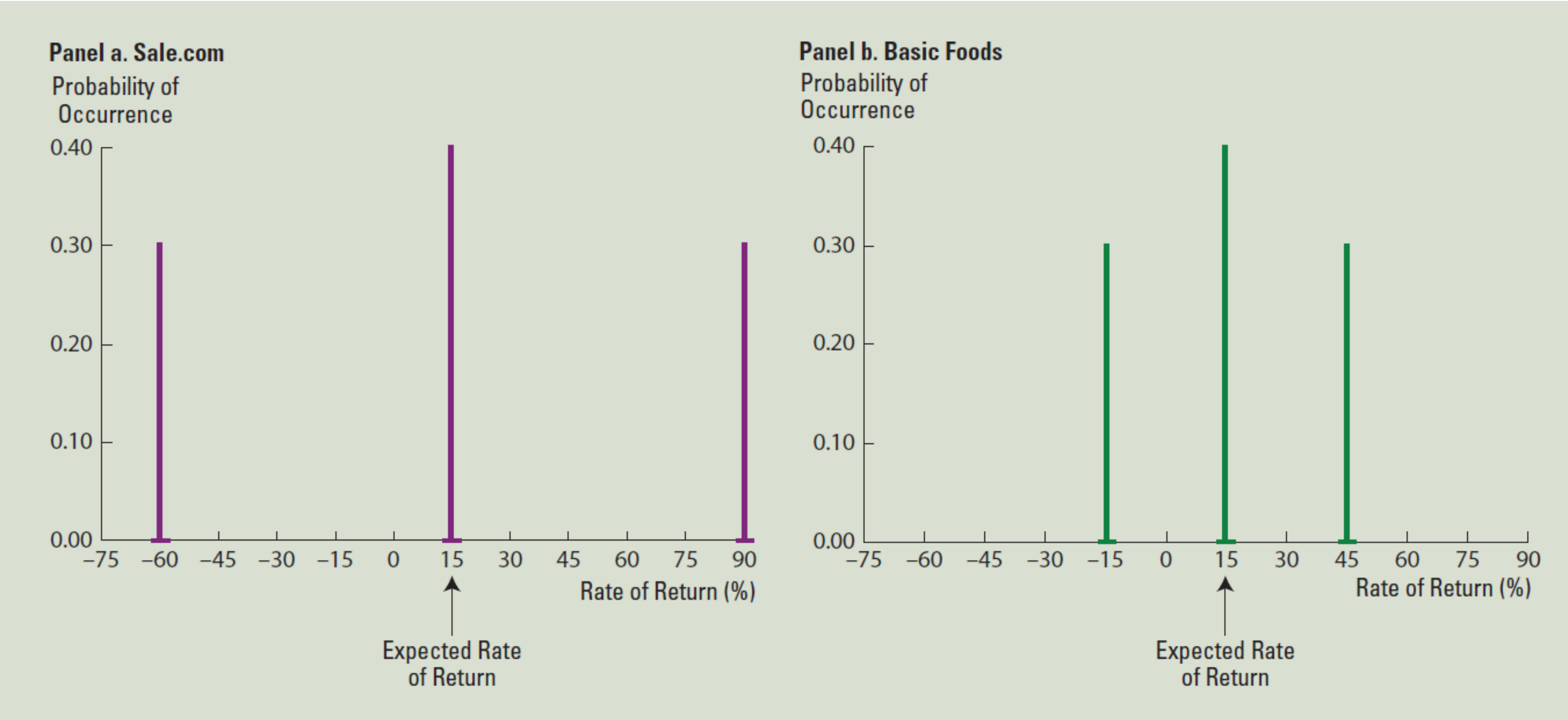
Example: Expected Return

- Consider the following information:

State	Probability	ABC, Inc. Return
Boom	.25	0.15
Normal	.50	0.08
Slowdown	.15	0.04
Recession	.10	-0.03

- What is the expected return?

Consider which is better?



Variance and Standard Deviation

- Variance and standard deviation measure the volatility of returns.
- Using unequal probabilities for the entire range of possibilities
- Weighted average of squared deviations

$$\sigma^2 = \sum_{i=1}^{n} p_i (R_i - E(R))^2$$

Example: Variance and Standard Deviation

- Consider the following information:

State	Probability	ABC, Inc. Return
Boom	.25	0.15
Normal	.50	0.08
Slowdown	.15	0.04
Recession	.10	-0.03

- What is the variance?
- What is the standard deviation?

Example (2)

- Suppose you have predicted the following returns for stocks C and T in three possible states of the economy. What are the expected returns?

State	Probability	C	T
Boom	0.3	0.15	0.25
Normal	0.5	0.10	0.20
Recession	???	0.02	0.01

What are the variance and standard deviation for each stock?

The Reward-to-Volatility (Sharpe) Ratio

- ***Excess Return:*** The difference in any particular period between the actual rate of return on a risky asset and the actual risk-free rate
- ***Risk Premium:*** The difference between the expected HPR on a risky asset and the risk-free rate

- *Sharpe Ratio =* $$\frac{\text{Risk premium}}{\text{SD of excess returns}}$$

Time Series Analysis of Past Rates of Return

- True means and variances are unobservable
  - Possible scenarios like the one in the examples are unknown

- Means and variances must be estimated

Returns Using Arithmetic and Geometric Averaging

- **Arithmetic Average:** $$E(r)=\sum_{s=1}^{n} p(s)r(s)=\frac{1}{n}\sum_{s=1}^{n} r(s)$$

- **Geometric (Time-Weighted) Average:**
  - Terminal value of the investment: $TV_n=(1+r_1)(1+r_2)...(1+r_n)$
  - Geometric Average: $g=TV_n^{1/n}-1$

Estimating Variance and Standard Deviation

- Estimated Variance
  - Expected value of squared deviations

$$\hat{\sigma}^2 = \frac{1}{n}\sum_{s=1}^{n}\left[r(s) - \bar{r}\right]^2$$

- Unbiased estimated standard deviation

$$\hat{\sigma} = \sqrt{\frac{1}{n-1}\sum_{j=1}^{n}\left[r(s) - \bar{r}\right]^2}$$

Covariance

Covariance and correlation are both measures of the extent to which two random variables move together.

- Covariance is the expected value of the product of each variable's deviation from its respective mean.

$$\sigma_{X,Y} = E\{[X - E(X)][Y - E(Y)]\}$$

$$\sigma_{R_i,R_j} = \sum_{i=1}^{n} P(R_i)[R_i - E(R_i)][R_j - E(R_j)]$$

Correlation

Covariance and correlation are both measures of the extent to which two random variables move together.

- Correlation is a scaled transformation of covariance wherein the extent of comovement is measured along a scale from exactly the same movement in the same direction to exactly the same movement in opposite directions.
  - When two variables move the same degree in opposing directions, they are said to be perfectly negatively correlated.
  - When two variables move the same degree in the same direction, they are said to be perfectly positively correlated.
  - When there is absolutely no commonality of movement, the variables are said to be uncorrelated.

$$\rho_{X,Y} = \frac{\sigma_{X,Y}}{\sigma_X \sigma_Y}$$

$$-1 \leq \rho_{X,Y} \leq 1$$

Example (2-continued)

- Suppose you have predicted the following returns for stocks C and T in three possible states of the economy.

State	Probability	C	T
Boom	0.3	0.15	0.25
Normal	0.5	0.10	0.20
Recession	???	0.02	0.01

What are covariance and correlation of stock C and T returns?

Return and Risk in Portfolio Context

Portfolios

- A portfolio is a collection of assets.
- An asset's risk and return are important in how they affect the risk and return of the portfolio.
- The risk-return trade-off for a portfolio is measured by the portfolio expected return and standard deviation, just as with individual assets.

Example: Portfolio Weights

- Suppose you have \$15,000 to invest and you have purchased securities in the following amounts. What are your portfolio weights in each security?
  - \$2000 of C
  - \$3000 of KO
  - \$4000 of INTC
  - \$6000 of BP

Portfolio Expected Returns

- The expected return of a portfolio is the weighted average of the expected returns of the respective assets in the portfolio.

$$E(R_P) = \sum_{j=1}^{m} w_j E(R_j)$$

- You can also find the expected return by finding the portfolio return in each possible state and computing the expected value as we did with individual securities.

Example: Expected Portfolio Returns

- Consider the portfolio weights computed previously. If the individual stocks have the following expected returns, what is the expected return for the portfolio?
  - C: 19.69%
  - KO: 5.25%
  - INTC: 16.65%
  - BP: 18.24%

Portfolio Variance

- Compute the portfolio variance and standard deviation using the same formulas as for an individual asset.

$$\boxed{\text{Portfolio Variance} = x_1^2\sigma_1^2 + x_2^2\sigma_2^2 + 2(x_1 x_2 \rho_{12} \sigma_1 \sigma_2)}$$

Example: Portfolio Variance

- Consider the following information on returns and probabilities:
  - Invest 50% of your money in Asset A.

State	Probability	A	B	Portfolio
Boom	.4	30%	-5%	12.5%
Bust	.6	-10%	25%	7.5%

- What are the expected return and standard deviation for each asset?
- What are the expected return and standard deviation for the portfolio?

Example (2): Portfolio Variance

- Consider the following information on returns and probabilities:

State	Probability	X	Z
Boom	.25	15%	10%
Normal	.60	10%	9%
Recession	.15	5%	10%

- What are the expected return and standard deviation for a portfolio with an investment of \$6,000 in asset X and \$4,000 in asset Z?

Example (3)

Use the following scenario analysis for Stocks X and Y to answer CFA Problems 3 through 6 (round to the nearest percent).

	Bear Market	Normal Market	Bull Market
Probability	0.2	0.5	0.3
Stock X	−20%	18%	50%
Stock Y	−15%	20%	10%

3. What are the expected rates of return for Stocks X and Y?
4. What are the standard deviations of returns on Stocks X and Y?
5. Assume that of your \$10,000 portfolio, you invest \$9,000 in Stock X and \$1,000 in Stock Y. What is the expected return on your portfolio?
6. Probabilities for three states of the economy and probabilities for the returns on a particular stock in each state are shown in the table below.

State of Economy	Probability of Economic State	Stock Performance	Probability of Stock Performance in Given Economic State
Good	0.3	Good	0.6
		Neutral	0.3
		Poor	0.1
Neutral	0.5	Good	0.4
		Neutral	0.3
		Poor	0.3
Poor	0.2	Good	0.2
		Neutral	0.3
		Poor	0.5

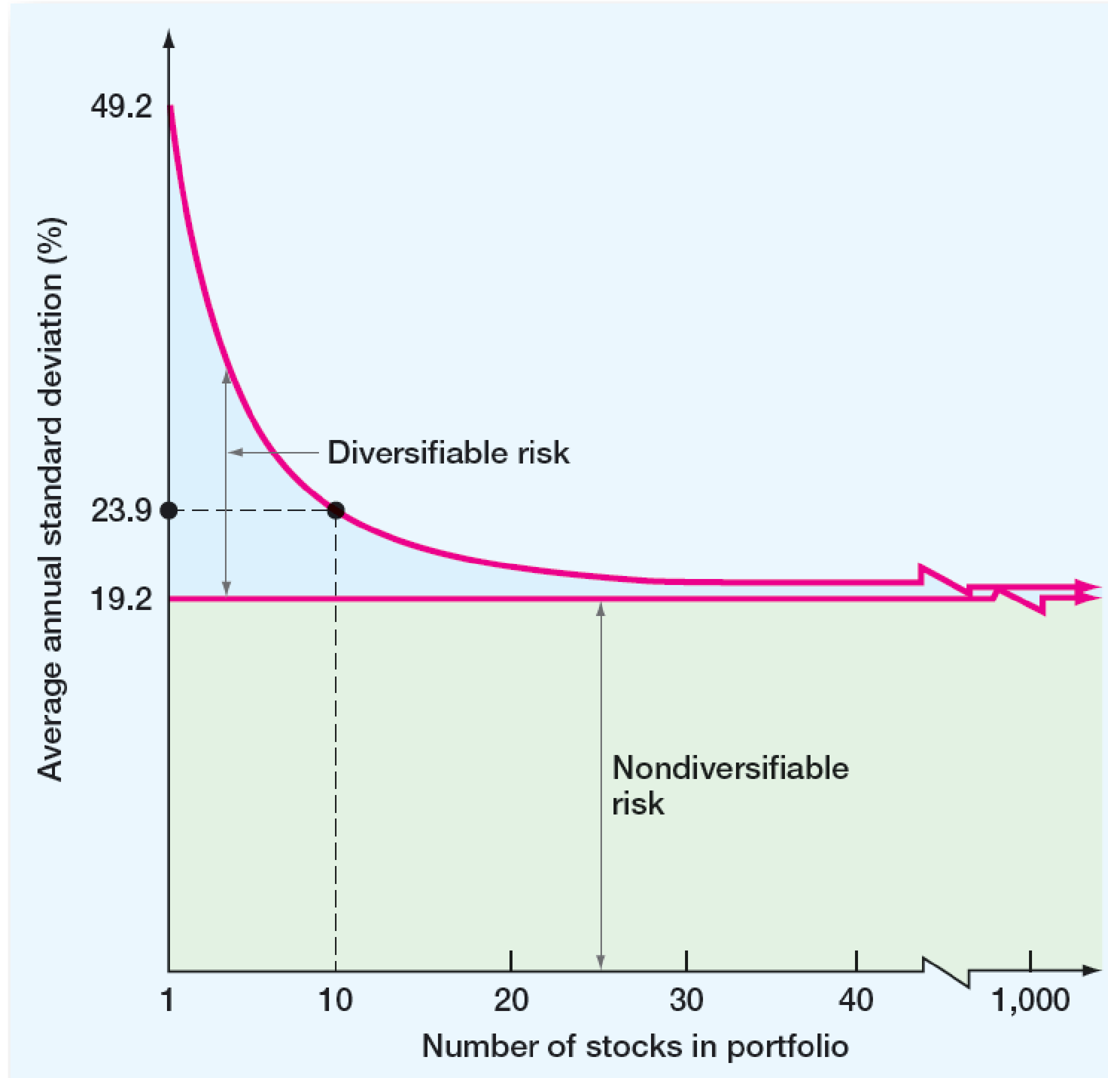
What is the probability that the economy will be neutral *and* the stock will experience poor performance?

Diversification

The Principle of Diversification

- Diversification can substantially reduce the variability of returns without an equivalent reduction in expected returns.
- This reduction in risk arises because worse than expected returns from one asset are offset by better than expected returns from another.
- However, there is a minimum level of risk that cannot be diversified away and that is the systematic portion.

Portfolios



Average annual standard deviation (%)
49.2
23.9
19.2
Diversifiable risk
Nondiversifiable risk
1
10
20
30
40
1,000
Number of stocks in portfolio

Total Risk

- Total risk = systematic risk + unsystematic risk
- The standard deviation of returns is a measure of total risk.
- For well-diversified portfolios, unsystematic risk is very small.
- Consequently, the total risk for a diversified portfolio is essentially equivalent to the systematic risk.

Do we have diversification benefits from the portfolio investment?

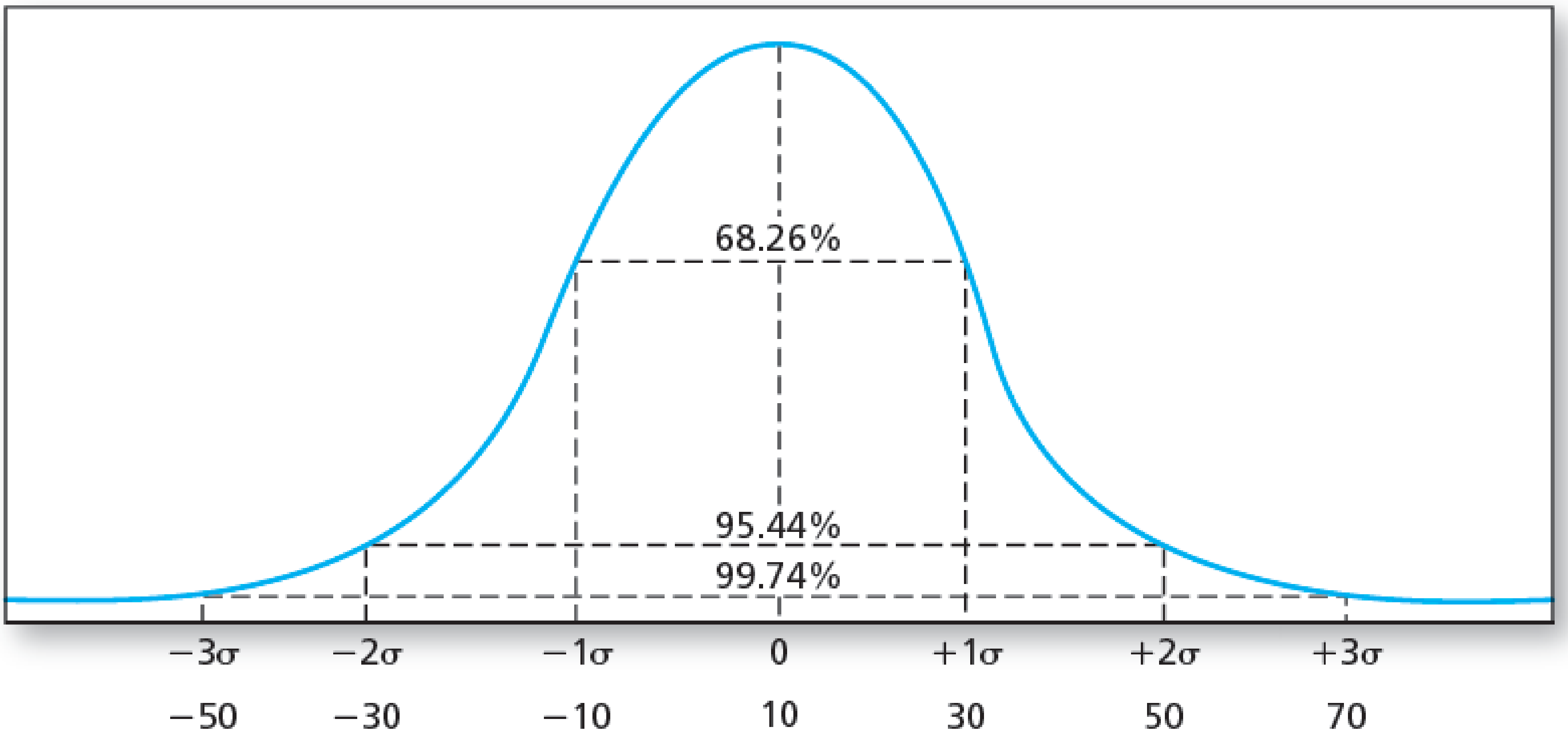
- Consider the following information on returns and probabilities:

State	Probability	X	Z
Boom	.25	15%	10%
Normal	.60	10%	9%
Recession	.15	5%	10%

- What are the expected return and standard deviation for a portfolio with an investment of $6,000 in asset X and $4,000 in asset Z?

Introduction to VaR

The Normal Distribution



Mean = 10%, SD = 20%

What if excess returns are not normally distributed?

- STD is no longer a complete measure of risk
  - ***Skew:*** Ratio of the average cubed deviations from the sample average, also called the third moment

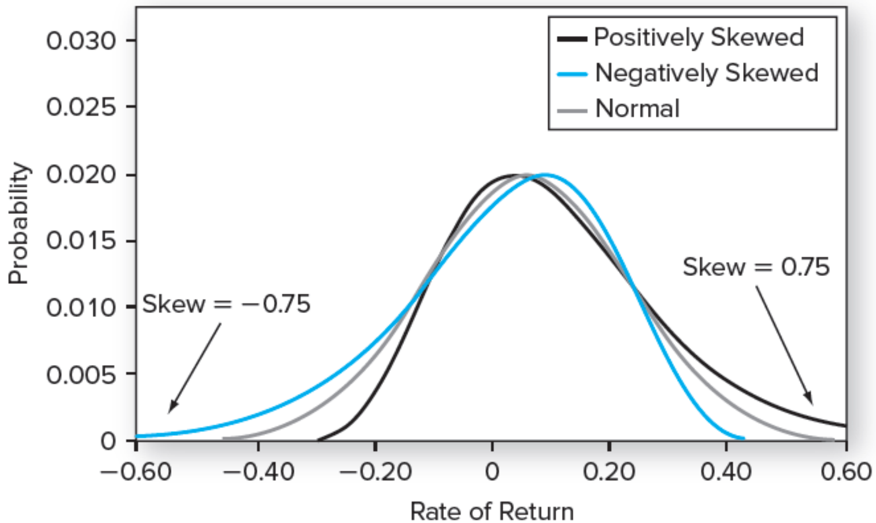
$$Skew = Average\left[\frac{(R-\bar{R})^3}{\hat{\sigma}^3}\right]$$

  - ***Kurtosis:*** The likelihood of extreme values on either side of the mean

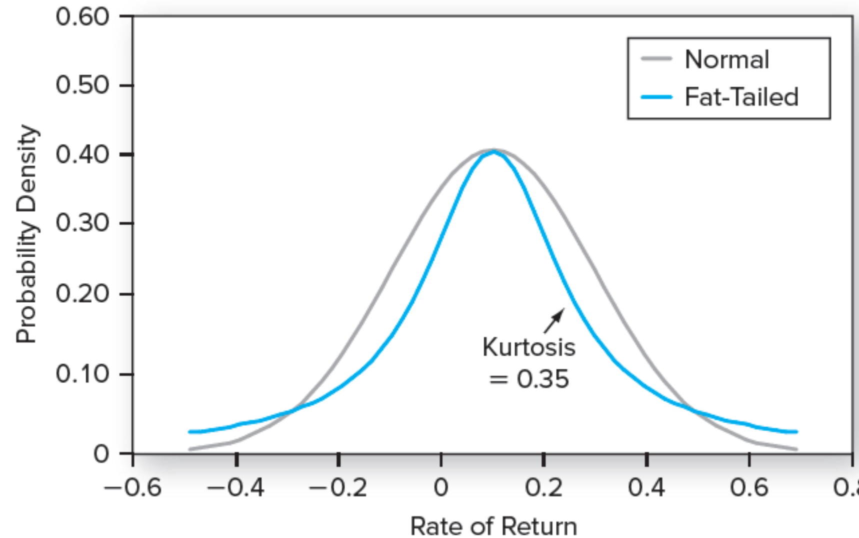
$$Kurtosis = Average\left[\frac{(R-\bar{R})^4}{\hat{\sigma}^4}\right]-3$$

- Sharpe ratio is not a complete measure of portfolio performance

Normal and Skewed Distributions



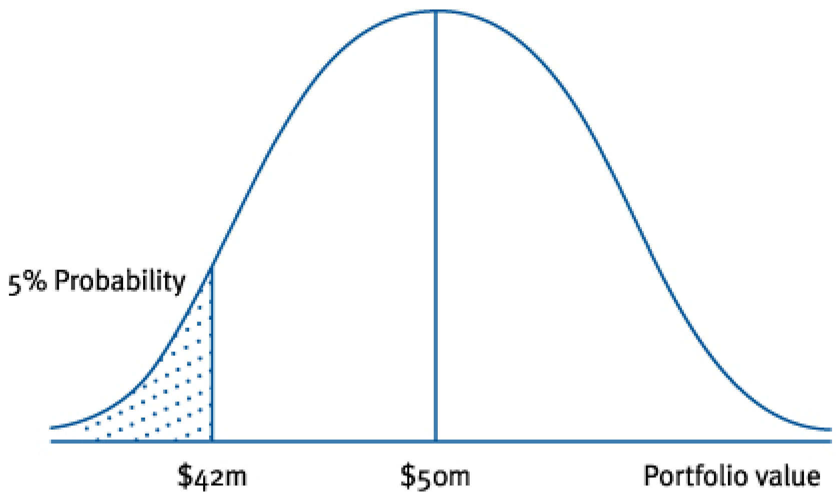
Normal and Fat-Tailed Distributions



The Question Being Asked in VaR

- ***Value at Risk (VaR)***

"What loss level is such that we are X% confident it will not be exceeded in N business days?"



Introducing the VaR risk measure

- VaR is often defined in dollars, denoted by \$VaR
- \$VaR loss is implicitly defined from the probability of getting an even larger loss as in

$$\Pr(\$Loss > \$VaR) = p$$

- Note by definition that (1–p)100% of the time, the \$Loss will be smaller than the VaR.
- Define VaR with the unit of percentage:

$$\Pr(-R_{PF} > VaR) = p \Leftrightarrow$$
$$\Pr(R_{PF} < -VaR) = p$$

How these two VaR are related to another?

Example

- Data of the daily IPC stock returns during 1/95 and 12/96 showed that variance was 0.000324.
- What could we expect to lose with confidence at 99% of the daily time?

Question?