BACHELOR OF ECONOMICS
THAMMASAT UNIVERSITY

EE325 Introductory Econometrics, Semester 1/2019 (Section 046402)

Due Date: Thursday 27th February 2020 by 09.30 via Assignment Submission in Moodle.

Instruction: Do all questions with your own handwriting and your own attempt.

Use 4 decimal places for numerical answers

1. In Table 1. $X_i$is total econometrics exam point (total points are 100) and Y_i is GPA of each BE student.

Table 1

Student	Y_i	X_i
1	2.8	63
2	3.4	72
3	3.0	78
4	3.5	81
5	3.6	87
6	3.0	75
7	2.7	75
8	3.7	90

To find estimator of $\hat{\beta}_1, \hat{\beta}_2$, $(X_i-\bar{X})$, $(Y_i-\bar{Y})$, $(X_i-\bar{X})^2$, $(X_i-\bar{X})(Y_i-\bar{Y})$,

$\bar{Y}, \bar{X}$

$Y_i = \hat{Y}_i + \hat{u}_i$

1.1 Now consider the two-variable model $Y_i = \beta_1 + \beta_2 X_i + u_i,\ u_i \sim NIID(0, \sigma^2)$

Use OLS to find the estimator of $\beta_1 and \beta_2$. Interpret the regression.

$\hat{\beta}_1, \hat{\beta}_2$

1.2 Find $\hat{Y}_i$ and $\hat{u}_i$ and show that $\sum_{i=1}^{n} \hat{u}_i \approx 0$

$\sum \hat{u}_i = 0$

1.3 Find $var(\hat{u}_i), var(\hat{\beta}_1), and\ var(\hat{\beta}_2)$

1.

student	Y_i	X_i	$(X_i-\bar{X})$	$(X_i-\bar{X})^2$	$(Y_i-\bar{Y})$	$(X_i-\bar{X})(Y_i-\bar{Y})$
1	2.8	63	-14.625	213.8906	-0.4125	6.0328
2	3.4	72	-5.625	31.6406	0.1875	-1.0547
3	3.0	78	0.375	0.1406	-0.2125	-0.0797
4	3.5	81	3.375	11.3906	0.2875	0.9703
5	3.6	87	9.375	87.8906	0.3875	3.6328
6	3.0	75	-2.625	6.8906	-0.2125	0.5578
7	2.7	75	-2.625	6.8906	-0.5125	1.3453
8	3.7	90	12.375	153.1406	0.4875	6.0328
Sum	25.7	621	0	511.8748	0	17.4374

$\bar{X} = 77.625$

$\bar{Y} = 3.2125$

$\sum X_i^2 = 48,717$

1.2.

$$\hat{\beta}_2 = \frac{\sum (X_i-\bar{X})(Y_i-\bar{Y})}{\sum (X_i-\bar{X})^2} = \frac{17.4374}{511.8748} = 0.0341$$

$$\hat{\beta}_1 = \bar{Y} - \hat{\beta}_2\bar{X} = 3.2125 - 0.0341(77.625) = 0.5655$$

$$\hat{Y}_i = \hat{\beta}_1 + \hat{\beta}_2 X_i$$

∴ To interpret the regression, the estimator of β_2 shows that as the total econometrics exam point increases by 1 unit, GPA of the student increases by 0.0341 unit.

1.2.

$$\hat{Y}_i = \hat{\beta}_1 + \hat{\beta}_2 X_i \;;$$

$$\hat{Y}_i = 0.5655 + 0.0341(X_i)$$

$$Y_i = \hat{Y}_i + \hat{u}_i$$

$$Y_i - \hat{Y}_i = \hat{u}_i$$

Student	X_i	$\hat{Y}_i$	$\hat{u}_i$
1	63	2.7138	0.0862
2	72	3.0207	0.3793
3	78	3.2253	-0.2253
4	81	3.3276	0.1724
5	87	3.5322	0.0678
6	75	3.123	-0.123
7	75	3.123	-0.423
8	90	3.6345	0.0655

$\sum_{i=1}^{8} \hat{u}_i^2 = 0.4347$

$\sum_{i=1}^{8} \hat{u}_i = -0.001 \approx 0$

1.3.

$$\operatorname{var}(\hat{\beta}_2) = \frac{\sigma^2}{\sum_{i=1}^{n}(X_i - \bar{X})^2} = \frac{0.0725}{511.8748} = 0.0001$$

$$\sigma^2 = \frac{RSS}{n-2} = 0.0725$$

$$\operatorname{var}(\hat{\beta}_1) = \frac{\sigma^2 \sum_{i=1}^{n} X_i^2}{n \sum_{i=1}^{n}(X_i - \bar{X})^2} = \frac{0.0725\,(48{,}717)}{8\,(511.8748)} = \frac{3531.9825}{4094.9984} = 0.8625$$

$$\operatorname{var}(\hat{u}_i) = \frac{\sum \hat{u}_i^2}{n-2} = \frac{0.4347}{6} = 0.0725$$

2. Data is listed in the table

	X_i	Y_i
1	10	0
2	12	2
3	14	5
4	16	6
5	18	7
6	22	10
7	24	10
8	26	15
9	28	16
10	30	20

To find estimator of $\hat{\beta}_1, \hat{\beta}_2$,
$(X_i - \bar{X})$, $(Y_i - \bar{Y})$, $(X_i - \bar{X})^2$, $(X_i - \bar{X})(Y_i - \bar{Y})$,

2.1 From the simple regression model $Y_i = \beta_1 + \beta_2 X_i + u_i, u_i \sim NIID(0, \sigma^2)$

Find estimators of β_1 and β_2 from the OLS method and interpret the meaning.

2.2 Find the value of $\hat{Y}_i$ and $\hat{u}_i$. Show that $\sum \hat{u}_i \approx 0$

2.3 Plot graph and draw regression line. Does the line pass $(\bar{X}, \bar{Y})$?

2.4 If $X_i = 18$, what is the predicted Y?

2.5 Find $var(\hat{u}_i)$, $var(\hat{\beta}_1)$, $var(\hat{\beta}_2)$

3. Consider the below regression function: consider the two-variable model

$$\boldsymbol{Y_i = \beta_1 + \beta_2 X_i + u_i,\ u_i \sim NIID(0, \sigma^2)}$$

Find an OLS estimator of β_1. Then, provide a proof that this is an unbiased estimator. Please state the assumption(s) of CLRM when used (pages 66-75 in Gujarati).

“Practice makes Perfect.”

X_i	Y_i	$(x_i-\bar{x})$	$(x_i-\bar{x})^2$	$(y_i-\bar{y})$	$(x_i-\bar{x})(y_i-\bar{y})$
10	0	-10	100	-9.1	91
12	2	-8	64	-7.1	56.8
14	5	-6	36	-4.1	24.6
16	6	-4	16	-3.1	12.4
18	7	-2	4	-2.1	4.2
22	10	2	4	0.9	1.8
24	10	4	16	0.9	3.6
26	15	6	36	5.9	35.4
28	16	8	64	6.9	55.2
30	20	10	100	10.9	109

$\bar{x} = 20$

$\bar{y} = 9.1$

To find estimator of $\hat{\beta}_1, \hat{\beta}_2$,
$(x_i-\bar{x}), (Y_i-\bar{Y}), (x_i-\bar{x})^2, (x_i-\bar{x})(Y_i-\bar{Y})$,

$\sum_{i=1}^{10} X_i^2 = 4440$

$\sum x = 200 \quad \sum y = 91$

$\sum (x_i-\bar{x})(y_i-\bar{Y}) = 394$

$\sum (x_i-\bar{x})^2 = 440$

2.1

$$\hat{\beta}_2 = \frac{\sum (x_i-\bar{x})(y_i-\bar{y})}{\sum (x_i-\bar{x})^2} = \frac{394}{440} = 0.8955$$

∴ As X increases by 1 unit
Y increases by 0.8955 unit.

$$\hat{\beta}_1 = \bar{Y} - \hat{\beta}_2 \bar{x} = 9.1 - 0.8955(20) = -8.81$$

2.2. $Y_i = \hat{Y}_i + \hat{u}_i \;;\quad Y_i - \hat{Y}_i = \hat{u}_i$

x_i	$\hat{Y}_i$	$\hat{u}_i$
10	0.145	-0.145
12	1.936	0.064
14	3.727	1.273
16	5.518	0.482
18	7.309	-0.309
22	10.891	-0.891
24	12.682	-2.682
26	14.473	0.527
28	16.264	-0.264
30	18.055	1.945

$\sum \hat{u}_i^2 = 14.09$

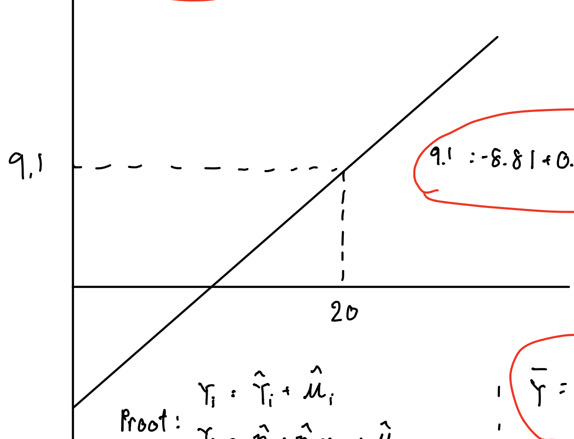
$\sum \hat{u}_i = 0$

2.3

Yes the line passes $\bar{x}, \bar{y}$.

$\hat{Y}_i = -8.81 + 0.8955 x_i$

SRF: $\hat{Y} = \hat{\beta}_1 + \hat{\beta}_2 X_i$



$9.1 = -8.81 + 0.8955(20)$

Proof: $Y_i = \hat{Y}_i + \hat{u}_i$

$Y_i = \hat{\beta}_1 + \hat{\beta}_2 x_i + \hat{u}_i$

$\sum Y_i = \sum \hat{\beta}_1 + \hat{\beta}_2 \sum x_i + \sum \hat{u}_i$

$\sum Y_i = n \cdot \hat{\beta}_1 + \hat{\beta}_2 \sum x_i$

$\frac{\sum Y_i}{n} = \hat{\beta}_1 + \hat{\beta}_2 \frac{\sum x_i}{n}$

$\bar{Y} = \hat{\beta}_1 + \hat{\beta}_2 \bar{X}$

2.4. $\hat{Y}_i = -8.81 + 0.8955(18)$

$\hat{Y}_i = 7.309$

2.5. $var(\hat{\beta}_2) = \dfrac{\sigma^2}{\sum_{i=1}^{n}(x_i-\bar{x})^2} = \dfrac{1.7613}{440} = 0.004$

$var(\hat{\beta}_1) = \dfrac{\sigma^2 \sum_{i=1}^{n} x_i^2}{n \sum_{i=1}^{n}(x_i-\bar{x})^2} = \dfrac{1.7613(\cancel{4440})}{10(\cancel{440})} = \dfrac{3,380.172}{4440}$

$= 0.7613$

$\sigma^2 = \dfrac{RSS}{n-2} = 1.7613$

$var(\hat{u}_i) = \dfrac{RSS}{n-2} = \dfrac{\sum \hat{u}_i^2}{8} = \dfrac{14.09}{8} = 1.7613$

3. To prove that $\hat{\beta}_1$ is an unbiased estimator of β_1, find $E(\hat{\beta}_1) = \beta_1$.

$Y_i = \beta_1 + \beta_2 X_i + u_i$ — (1)

$\hat{\beta}_1 = \bar{Y} - \hat{\beta}_2 \bar{X}$ — (2)

$\bar{Y} = \beta_1 + \beta_2 \bar{X} + \bar{u}$ — (3)

(2) → put in (3) → $\hat{\beta}_1 = \beta_1 + \beta_2 \bar{x} + \bar{u} - \hat{\beta}_2 \bar{x}$

$E(\bar{u}) = 0$

$E(\beta_2 - \hat{\beta}_2) = 0$

$\hat{\beta}_1 = \beta_1 + (\beta_2 - \hat{\beta}_2)\bar{x} + \bar{u}$

$E(\hat{\beta}_1) = E(\beta_1) + \bar{x}E(\beta_2 - \hat{\beta}_2) + E(\bar{u})$

$E(\hat{\beta}_1) = \beta_1$