

EE325 Introductory Econometrics, Semester 1/2019 (Section 046402)

Due Date: Thursday 27th February 2020 by 09.30 via Assignment Submission in Moodle.

Instruction: Do all questions with your own handwriting and your own attempt.

Use 4 decimal places for numerical answers

1. In Table 1. X_i is total econometrics exam point (total points are 100) and Y_i is GPA of each BE student.

Table 1

Student	Y_i	X_i
1	2.8	63
2	3.4	72
3	3.0	78
4	3.5	81
5	3.6	87
6	3.0	75
7	2.7	75
8	3.7	90

$$\begin{aligned} \bar{Y} &= 3.2125 & \bar{X} &= 77.625 \\ x_i - \bar{x} & & y_i - \bar{y} & & (x_i - \bar{x})(y_i - \bar{y}) & & \sum (x_i - \bar{x})^2 \\ & & & & & & = 511.875 \\ 1 & -14.625 & -0.4125 & & 6.0328 & & \\ 2 & -5.625 & 0.1875 & & -1.0547 & & \\ 3 & 0.375 & -0.2125 & & -0.0797 & & \\ 4 & 3.375 & 0.2875 & & 0.9703 & & \\ 5 & 4.375 & 0.3875 & & 3.6928 & & \\ 6 & -2.625 & -0.2125 & & 0.5578 & & \\ 7 & -2.625 & -0.5125 & & 1.3453 & & \\ 8 & 12.375 & 0.4875 & & 6.0328 & & \\ & & & & \sum & & = 17.4347 \end{aligned}$$

- 1.1 Now consider the two-variable model $Y_i = \beta_1 + \beta_2 X_i + u_i$, $u_i \sim NIID(0, \sigma^2)$

Use OLS to find the estimator of β_1 and β_2 . Interpret the regression.

- 1.2 Find $\hat{Y}_i$ and $\hat{u}_i$ and show that $\sum_{i=1}^n \hat{u}_i \approx 0$

- 1.3 Find $var(\hat{u}_i)$, $var(\hat{\beta}_1)$, and $var(\hat{\beta}_2)$

$$OLS: \hat{\beta}_2 = \frac{\sum (Y_i - \bar{Y})(X_i - \bar{X})}{\sum (X_i - \bar{X})^2} = \frac{17.4347}{511.875} = 0.0341$$

$$\begin{aligned} \hat{\beta}_1 &= \bar{Y} - \hat{\beta}_2 \bar{X} = 3.2125 - (0.0341)(77.625) \\ &= 0.5653 \end{aligned}$$

$$Y_i = \beta_1 + \beta_2 X_i; \quad Y_i = 0.5653 + 0.0341 X_i + u_i$$

1.2) $\hat{Y}_i, \hat{U}_i$

$$\hat{Y}_i = 0.5655 + 0.0347 x_i$$

$$\hat{Y}_1 = 2.7138$$

$$\hat{Y}_2 = 3.0207$$

$$\hat{Y}_3 = 3.2253$$

$$\hat{Y}_4 = 3.3276$$

$$\hat{Y}_5 = 3.5322$$

$$\hat{Y}_6 = 3.1230$$

$$\hat{Y}_7 = 3.1230$$

$$\hat{Y}_8 = 3.6345$$

$$\hat{U}_i = Y_i - \hat{Y}_i \quad \sum \hat{U}_i^2 = 0.4347$$

$$3.2125$$

$$\hat{U}_1 = 0.0862$$

$$\hat{U}_5 = 0.0678$$

$$0.4347$$

$$\hat{U}_2 = 0.3793$$

$$\hat{U}_6 = -0.123$$

$$\hat{U}_3 = -0.2253$$

$$\hat{U}_7 = -0.423$$

$$\hat{U}_4 = 0.1724$$

$$\hat{U}_8 = 0.0655$$

$$\sum_{i=1}^8 \hat{U}_i = -0.0001 \approx 0$$

1.3) $\text{var}(u) = \frac{\sum (Y_i - \hat{Y}_i)^2}{n-2} = \frac{\sum \hat{U}_i^2}{n-2} \xrightarrow{\text{SSR}} = \frac{0.4347}{8-2} = 0.0725$

$$\sum x_i^2 = 48717$$

$$\begin{aligned} \text{var}(\hat{\beta}_1) &= \frac{\sum (x_i^2)}{n \sum x_i^2} \sigma^2 \quad ; \quad x_i = x_i - \bar{x} \quad ; \quad \sum x_i^2 = 511.875 \\ &= \frac{48717}{511.875(8)} (0.0725) = 0.8625 \end{aligned}$$

$$\text{var}(\hat{\beta}_2) = \frac{\sigma^2}{\sum x_i^2} = \frac{0.0725}{511.875} = 0.000141645$$

2. Data is listed in the table

$\bar{X} = 20, \bar{Y} = 9.1$

	X_i	Y_i	$x_i - \bar{x}$	$y_i - \bar{y}$	$(x_i - \bar{x})(y_i - \bar{y})$	$\sum (x_i - \bar{x})(y_i - \bar{y})$
1	10	0	-10	-9.1	91	= 394
2	12	2	-8	-7.1	56.8	
3	14	5	-6	-4.1	24.6	
4	16	6	-4	-3.1	12.4	
5	18	7	-2	-2.1	4.2	
6	22	10	2	0.9	1.8	
7	24	10	4	0.9	3.6	
8	26	15	6	5.9	35.4	
9	28	16	8	6.9	55.2	
10	30	20	10	10.9	109	

2.1 From the simple regression model $Y_i = \beta_1 + \beta_2 X_i + u_i, u_i \sim NIID(0, \sigma^2)$

Find estimators of β_1 and β_2 from the OLS method and interpret the meaning.

2.2 Find the value of $\hat{Y}_i$ and $\hat{u}_i$. Show that $\sum \hat{u}_i \approx 0$

2.3 Plot graph and draw regression line. Does the line pass $(\bar{X}, \bar{Y})$?

2.4 If $X_i = 18$, what is the predicted Y ?

2.5 Find $var(\hat{u}_i), var(\hat{\beta}_1), var(\hat{\beta}_2)$

3. Consider the below regression function: consider the two-variable model

$$Y_i = \beta_1 + \beta_2 X_i + u_i, u_i \sim NIID(0, \sigma^2)$$

Find an OLS estimator of β_1 . Then, provide a proof that this is an unbiased estimator.

Please state the assumption(s) of CLRM when used (pages 66-75 in Gujarati).

$$OLS: \hat{\beta}_2 = \frac{\sum (Y_i - \bar{Y})(X_i - \bar{X})}{\sum (X_i - \bar{X})^2} = \frac{394}{440} = 0.8955$$

$$\hat{\beta}_1 = \bar{Y} - \hat{\beta}_2 \bar{X} = 9.1 - (0.8955)(20) = -8.81$$

$$\hat{Y}_i = \hat{\beta}_1 + \hat{\beta}_2 X_i; \quad \hat{Y}_i = -8.81 + 0.8955 X_i$$

$$2.2) \hat{Y}_1 = 0.145$$

$$\hat{Y}_2 = 1.936$$

$$\hat{Y}_6 = 10.891$$

$$\hat{Y}_7 = 12.682$$

"Practice makes Perfect."

$$\hat{Y}_3 = 8.727$$

$$\hat{Y}_4 = 5.518$$

$$\hat{Y}_5 = 7.309$$

$$\hat{Y}_8 = 14.473$$

$$\hat{Y}_9 = 16.264$$

$$\hat{Y}_{10} = 18.055$$

$$\hat{u}_i = Y_i - \hat{Y}_i$$

$$\hat{u}_1 = -0.145$$

$$\hat{u}_2 = 0.064$$

$$\hat{u}_3 = 1.273$$

$$\hat{u}_4 = 0.482$$

$$\hat{u}_5 = -0.309$$

$$\hat{u}_6 = -0.891$$

$$\hat{u}_7 = -2.682$$

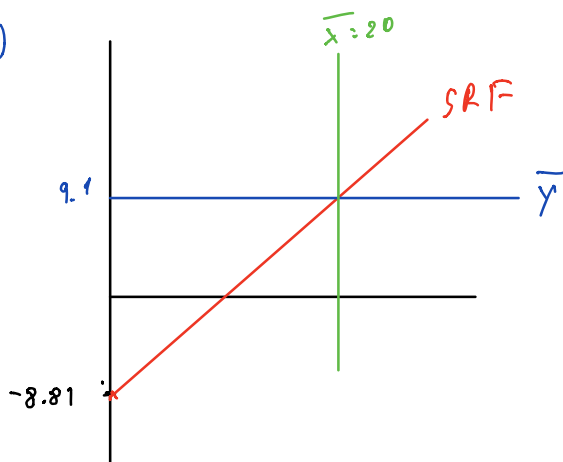
$$\hat{u}_8 = 0.527$$

$$\hat{u}_9 = -0.264$$

$$\hat{u}_{10} = 1.945$$

$$\sum \hat{u}_i = 0 //$$

2.3)



$$\hat{Y}_i = -8.81 + 0.8955x_i$$

$$\bar{Y} = 9.1, \bar{x} = 20$$

$$\hat{Y}_i = -8.81 + 0.8955(20)$$

$$= 9.1$$

$\therefore$ it passes the SRF line

$$\begin{aligned}
 2.4) \hat{Y}_i &= -8.81 + 0.8955x_i \quad \text{If } x_i = 18 \\
 &= -8.81 + 0.8955(18) \\
 &= 7.309
 \end{aligned}$$

$$2.5) \text{ Find } \text{var}(\hat{\mu}_i), \text{var}(\hat{\beta}_1), \text{var}(\hat{\beta}_2)$$

$$\text{var}(\hat{\mu}_i) = \frac{\sum (Y_i - \hat{Y})^2}{n-2} = \frac{14.0909}{8} = 1.7614$$

$$\text{var}(\hat{\beta}_1) = \frac{\sum x_i^2}{n \sum (x_i - \bar{x})^2} = \frac{4440}{10(440)} = 1.0091$$

$$\text{var}(\hat{\beta}_2) = \frac{\sigma^2}{\sum x_i^2} = \frac{1.7614}{4440} = 0.0004$$

$$3) \text{ OLS } \quad Y_i^1 = \beta_1^1 + \beta_2^1 x_i^1 + u_i^1$$

$$\min \sum u_i^1{}^2 \quad u_i^1 = y_i^1 - \beta_1^1 - \beta_2^1 x_i^1$$

$$\sum u_i^1{}^2 = \sum (y_i^1 - \beta_1^1 - \beta_2^1 x_i^1)^2$$

$$-2 \sum (y_i^1 - \beta_1^1 - \beta_2^1 x_i^1) = 0$$

$$\sum y_i^1 - \sum \beta_1^1 - \beta_2^1 \sum x_i^1 = 0$$

$$\sum \beta_1^1 = \sum y_i^1 - \beta_2^1 \sum x_i^1$$

$$n \hat{\beta}_1^1 = \sum y_i^1 - \beta_2^1 \sum x_i^1$$

$$\hat{\beta}_1^1 = \bar{y}_i - \beta_2^1 \bar{x}$$

unbiased when $E(\hat{\beta}_1^1) = \beta_1$

$$\hat{\beta}_1^1 = \bar{y} - \beta_2^1 \bar{x}$$

$$\hat{\beta}_1^1 = \beta_1 + \beta_2^1 \bar{x} - \beta_2^1 \bar{x}$$

$$\hat{\beta}_1^1 = \beta_1$$

$$E(\hat{\beta}_1^1) = \beta_1$$