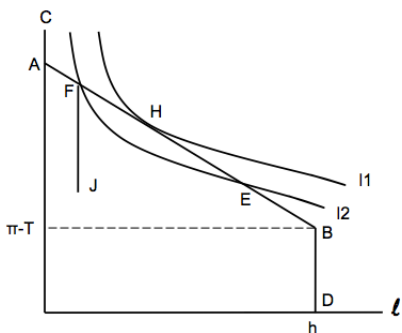


3. Consider a simple one-period, closed-economy model where the representative consumer has utility function $U(C) = C^{1/2}\ell^{1/2}$ and has h available hours to divide between work and leisure. The representative firm has technology given by $Y = zK^{2/3}N^{1/3}$. There is a government that sets its expenditure level at a value $G > 0$.

(a) Define the consumer budget constraint.

.....

(b) Define the consumer's utility maximization condition. Solve for labour supply function $N^s(w)$.



The consumer's utility is maximized when the slope of IC is equal to the slope of the budget line.

The slope of IC = The slope of the budget line

$$MRS_{C,\ell} = \dots\dots\dots$$

$$\frac{MU_\ell}{MU_C} = \dots\dots\dots$$

$$MU_C = \frac{\partial U}{\partial C} = \frac{1}{2}C^{-1/2}\ell^{1/2}$$

$$MU_\ell = \frac{\partial U}{\partial \ell} = \dots\dots\dots$$

The slope of IC = The slope of the budget line

$$\frac{MU_\ell}{MU_C} = w$$

$$\frac{\dots\dots\dots}{\frac{1}{2}C^{-1/2}\ell^{1/2}} = w$$

$$\left(\frac{C}{\ell}\right)^{\dots\dots\dots} = w$$

$$C = \dots\dots\dots$$

Substitute $C = w\ell$ into the equation for the budget line

$$C + w\ell = wh + (\pi - T)$$

$$\dots\dots + w\ell = wh + (\pi - T)$$

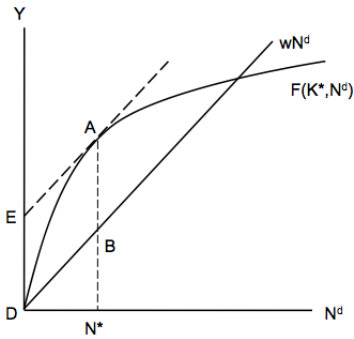
$$\ell = \frac{wh + (\pi - T)}{\dots\dots\dots}$$

From leisure function $\ell(w) = \frac{wh + (\pi - T)}{2w}$. We can find the labour supply function.

$$\begin{aligned}
 N^s(w) &= \dots - \ell(w) \\
 &= \dots - \frac{wh + (\pi - T)}{2w} \\
 &= \frac{-(\pi - T) + \dots h}{2w} \\
 &= \frac{\dots h}{2w} - \frac{\pi - T}{2w} \\
 &= \frac{\dots}{2} - \frac{\pi - T}{2w}
 \end{aligned}$$

This is the labour supply function. $N^s(w) = \frac{h}{2} + \frac{\pi - T}{2w}$. From the labour supply function, $w \uparrow \Rightarrow N^s \dots$ (\uparrow or \downarrow).

(c) Define the firm's profit maximization condition. Solve for labour demand function.



Profit Maximization

- Y = revenue;
- MP_N = marginal revenue;
- wN^d = variable cost;
- w = marginal cost;
- Profit = $Y - wN^d$;
- Max profit = AB where $MP_N = w$.

The firm's profit is maximized when the slope of TR ($zF(K, N^d)$) is equal to the slope of TC (wN^d) or $MR = MC$.

$$Y = zK^{2/3}N^{1/3} = zK^{2/3}N^{d1/3}$$

$$\text{Slope of } Y = \text{Slope of } wN^d$$

$$\frac{\partial Y}{\partial N^d} = \frac{\partial (wN^d)}{\partial N^d}$$

$$\frac{1}{3}zK^{2/3}N^{d\dots\dots} = \dots$$

$$N^d = \dots\dots\dots$$

$$N^d = \left(\frac{3w}{zK^{2/3}} \right)^{-3/2} = \left(\frac{z^{3/2}K}{(3w)^{3/2}} \right). \text{ This is labor demand function. } w \uparrow \Rightarrow N^d \dots\dots (\uparrow \text{ or } \downarrow).$$

(d) Find the competitive equilibrium values given $h = 16$, $z = 1$, $K = 8$, $G = 0$.

$$Y = zK^{2/3}N^{1/3}$$

$$= (\dots)(\dots)^{2/3}N^{1/3}$$

$$8^{2/3} = (\sqrt[3]{8})^2 = 2^2.$$

$$Y = 4N^{1/3}. \tag{1}$$

Marginal Product of Capital.

$$\begin{aligned}
 MP_N &= \frac{\partial Y}{\partial N} \\
 &= \left(\frac{\dots\dots\dots}{\dots\dots\dots}\right) 4N^{(1/3-\dots\dots)} \\
 &= \left(\frac{\dots\dots\dots}{\dots\dots\dots}\right) N^{\dots\dots\dots}
 \end{aligned}$$

$$MP_N = \frac{4}{3}N^{-2/3} = w. \tag{2}$$

From consumer's optimization solution, $C = w\ell$.

$$C = w\ell$$

Substitute $MP_N = \frac{4}{3}N^{-2/3} = w$

$$C = \dots\dots\dots$$

$$C = \frac{4}{3}N^{-2/3}\ell \tag{3}$$

As $G=0$, $Y = C + G$. Then,

$$C = Y$$

From (1),

$$C = 4N^{1/3} \tag{4}$$

Equation (3) = equation (4).

Substitute $\ell = h - N$ into $\ell = 3N$. Let $h = 16$. Solve for N .

$$N^* = \dots\dots\dots$$

Substitute $N = 4$ into $\ell = 16 - N$ and get

$$\ell^* = \dots\dots\dots$$

From equation (4), substitute $N = 4$ and get C^* .

$$C^* = \dots\dots\dots$$

From equation (2), substitute $N = 4$ and get w^* .

$$w^* = \dots\dots\dots$$

From equation (1), substitute $N = 4$ and get Y^* .

$$Y^* = \dots\dots\dots$$