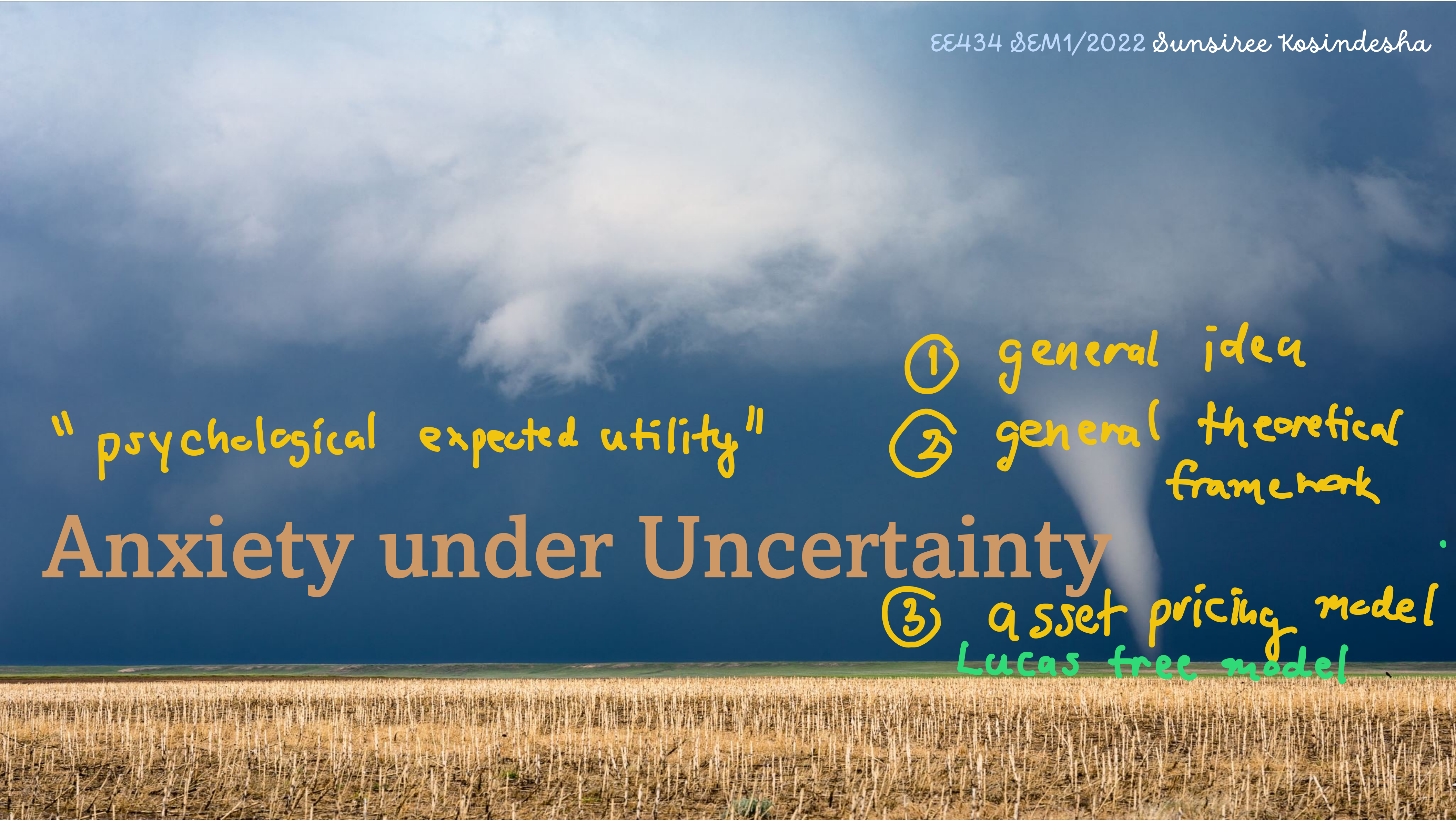
EE434 SEM1/2022 Sunsiree Kosindesha



"psychological expected utility"

① general idea
② general theoretical framework
③ asset pricing model
Lucas free model

Anxiety under Uncertainty

How do you feel about uncertainty?

- We all experience feelings related to our uncertainty about the future, such as hopefulness, anxiety, and suspense.
- These are "anticipatory emotions".
- Anticipatory emotions are believed to arise in reaction to mental images of the outcome of a decision.

Andrew Caplin, John Leahy, Psychological Expected Utility Theory and Anticipatory Feelings, *The Quarterly Journal of Economics*, Volume 116, Issue 1, February 2001, Pages 55–79, https://doi.org/10.1162/003355301556347

How do you feel about uncertainty?

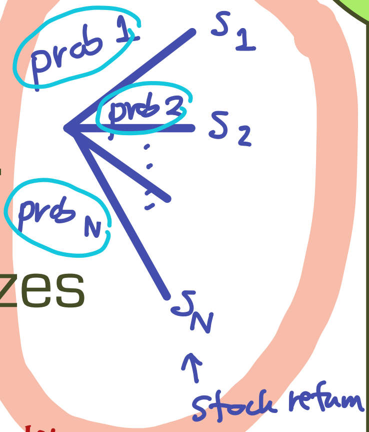
- **Anxiety** is anticipatory and the desire to reduce anxiety motivates many decisions.

- The term anxiety denotes "**apprehension, tension, or uneasiness that stems from the anticipation of danger**".

Andrew Caplin, John Leahy, Psychological Expected Utility Theory and Anticipatory Feelings, *The Quarterly Journal of Economics*, Volume 116, Issue 1, February 2001, Pages 55–79, https://doi.org/10.1162/003355301556347

Caplin&Leahy(2001)'s framework

Caplin and Leahy (2001) extends expected utility theory to situations in which agents experience feelings of anticipation prior to the resolution of uncertainty.



They model how lotteries over future physical prizes influence anticipatory emotions.

argument of utility f^n will be emotions, instead of utility as a function of final wealth

The model is called psychological expected utility.

→ No ref. point, No two-part value f^n
No loss aversion; No PWF

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Caplin&Leahy(2001)'s framework

- Caplin and Leahy (2001) also study the portfolio decisions of an anxious saver.
- The incorporation of anxiety into asset pricing models may help explain the equity premium puzzle and the risk-free rate puzzle.

Andrew Caplin, John Leahy, Psychological Expected Utility Theory and Anticipatory Feelings, *The Quarterly Journal of Economics*, Volume 116, Issue 1, February 2001, Pages 55–79, https://doi.org/10.1162/003355301556347

Caplin&Leahy(2001)'s framework

- Safe assets, by providing secure returns, may reduce anxiety even before final consumption takes place.

- Safe assets therefore provide an extra benefit in addition to the smoothing of final consumption across states. Thus, the risk-free rate can be reduced.

Safe asset, ex. getting 2% return no matter what
→ consumption smoothing across states of nature
→ anxiety - reducing (extra benefit when we consider emotion)
→ less required rate of return
answer to
→ risk-free rate puzzle

Caplin&Leahy(2001)'s framework

- Stocks and other risky assets, however, by increasing the variance of the portfolio, tend to increase anxiety in the period before final consumption takes place.
- Hence owning stocks involves an extra cost in addition to increasing the variance of final consumption, which increases their required return.

Risky asset ex. get 15% return or -10% return
→ Variance of the return of the portfolio is high
→ anxiety - enhancing (extra cost in terms of emotion)
→ more require rate of return
answer → equity premium puzzle

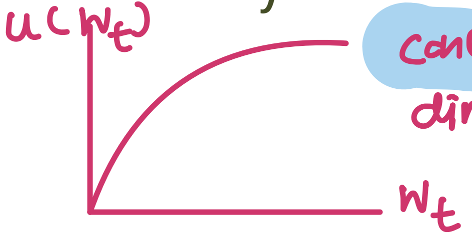
Anxiety vs. Standard Risk Aversion

- Anxious agent can appear to be risk averse, but the two concepts are different.

Behavior that people avoid risk

- Anxiety is emotion, related to the feeling of living with uncertainty.

- In contrast, "standard risk aversion" is a static concept pertaining to the curvature of the utility function within a period.



concavity of utility fn
diminishing mu

Anxiety vs. Risk aversion

- Anxiety complements risk aversion (and other explanations) in the understanding of the equity premium puzzle.

- In addition, anxious agents may appear to "overreact" to small probability events.

anxiety $\xrightarrow{\text{may be}}$ overweighting of small prob. of negative events.

The General Framework

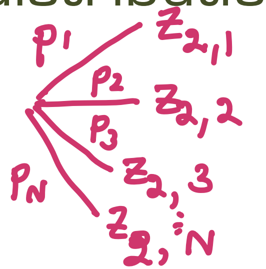
Caplin&Leahy(2001)'s framework

- Assume there are two periods: period $t = 1{,}2$.
- The feelings of anticipation concern the second-period uncertainty that remains unresolved after the outcome of the first-period lottery has been realized.

Caplin&Leahy(2001)'s framework

- Let X_1 represent the possible psychological states in period 1.
  e.g. {hopefulness, anxiety, suspense}
- Let X_2 represent the possible psychological states in period 2.
  e.g. {regret, rejoicing, elation, disappointment}
- Let Z_1 be the space of physical prizes in period 1.

  lotteries
- Let L_2 be the space of probability distributions over period 2 prizes.



- Define $Y_1 = Z_1 \times L_2$

  what we care in terms of prizes
- Define the function $\phi: Y_1 \to X_1$, giving the psychological state resulting from an agent facing the outcome $y_1 \in Y_1$.

  physical mental

Utility function

- Given $y_1 = (z_1, l_2) \in Y_1$,

Psychological expected utility:

$x_1 = \phi(y_1)$

expected utility

$$V_1(y_1) = u_1(\overbrace{\phi(y_1)}^{x_1 = \phi(y_1)}) + \underbrace{E_{l_2}[u_2(x_2)]}_{\text{expected utility}}$$

just like the traditional utility fn

- V_1 looks like a standard time-separable expected utility function except for having the psychological states as argument.

Asset Prices and Anxiety

A model of portfolio choice

- Consider a two-period Lucas tree model of consumption and saving.

- A representative agent is born with an endowment of a consumption good equal to w_1. The consumption good is nonstorable.

Endowment : Consumption good = w_1 (non storable)

can't keep to consume in period 2

For Lucas tree model, see Stokey, Nancy, and Robert Lucas, Recursive Methods in Economic Dynamics (Cambridge, MA: Harvard University Press, 1989), p.300.

A model of portfolio choice

- The agent is also endowed with N productive assets, each in fixed supply (normalized to unity), that yield random quantities of a consumption good in the second period.

- That is, a representative consumer is endowed with 1 unit each of N productive assets.

- Asset $n \in N$ yields s_n units of the consumption good in period 2.

endowment : N productive assets 1 unit of each

Asset 1 , 1 unit at price p_1 → give S_1 units of cons good in $t=2$ (θ_1)

2 , 1 unit at price p_2 → " — " S_2 " ——— " (θ_2)

3 , 1 unit at price p_3 → " — " S_3 " ——— " (θ_3)

$\vdots$

N , 1 unit at price p_N → " — " S_N " ——— " (θ_N)

price of asset in consumption good units

are random.

A model of portfolio choice

- In addition to valuing consumption, the utility of the representative agent depends on the anxiety associated with holding risky assets.

- Let $Z_1 = Z_2 = R$ be the space of physical prizes in period 1 & 2.

- The consumption in each period is $c_t \in Z_t$.

- The space of temporal lotteries is the set of pairs

$$y_1 = (c_1, l_2) \in Y_1$$

c_1

⊙ how much we could eat today

⊙ The probability dist. of consumption tomorrow

, where l_2 represents a lottery over second-period consumption levels.

A model of portfolio choice

- The mapping $\phi: Y_1 \to X_1$ describes the relationship between temporal lotteries and the mental state in period 1.

- Assume that:

$$\boxed{\phi(c_1, l_2) = (c_1, a(l_2))}$$

, where $a: L_2 \to R$ is a differentiable function that measures the anxiety associated with the lottery $l_2 \in L_2$.

anxiety

A model of portfolio choice

anxiety-averse
: more anxiety less utility

- The induced expected utility function takes the form

$$V_1(y_1) = u_1(\phi(y_1)) + E_{l_2}[u_2(x_2)]$$

the heart of this model

$$V_1(c_1, l_2) = \underbrace{u_1(c_1, \boxed{a(l_2)})}_{\text{utility today}} + \underbrace{\beta E_{l_2}[u_2(c_2)]}_{\text{expected utility from tomorrow consumpt}}.$$

Lifetime expected utility of the representative agent

- l_2, a lottery over second-period consumption levels, depends on the portfolio that the agent holds.

- Let θ denote the vector of portfolio shares held by the agent, and θ_n the share of asset n.

- Define $l_2(\theta)$ to be the distribution of the random variable $\sum_n s_n \theta_n$.

$\tilde{a}(\theta) = a(l_2(\theta))$: how the anxiety level depends on the share of productive asset that we are holding.

- Let $\tilde{a}(\theta)$ be the differentiable composite mapping $a(l_2(\theta))$, associating each portfolio to the corresponding level of anxiety.

$a(l_2)$

anxiety depends on lottery l_2

tomorrow's consumption

l_2 —prob 1— $c_{2,1}$

l_2 —prob 2— $c_{2,2}$ if there are N possible states of nature.

⋮

l_2 —prob N— $c_{2,N}$

Risk on tomorrow's consumption depends on / comes from risky returns of the productive assets

The productive asset total return (from N assets) in terms of consumptⁿ good

$$= s_1\theta_1 + s_2\theta_2 + \dots + s_N\theta_N$$

$$= \underbrace{\sum_{n=1}^{N} s_n\theta_n}_{\text{a random variable}} \quad ; \quad n = 1, \dots, N$$

$l_2(\theta)$ — $\sum_{n=1}^{N} s_n\theta_{n,1}$ in state of the world 1

$l_2(\theta)$ — $\sum_{n=1}^{N} s_n\theta_{n,2}$ " ——— " 2

⋮ ⋮

$l_2(\theta)$ — $\sum_{n=1}^{N} s_n\theta_{n,N}$ " ——— " N

A model of portfolio choice

- The agent chooses the level of first-period consumption and the asset portfolio to maximize utility subject to the budget constraint,

total endowment in period 1

consumption + saving =

period 1 budget const. : $$c_1 + \underbrace{\sum_n p_n \theta_n}_{\text{total investment in N productive assets}} = \boxed{w_1 + \underbrace{\sum_n p_n}_{\text{total value of N productive assets}}}$$

period 2 budget const. : $$c_2 = \sum_n s_n \theta_n$$

tomorrow's consumption = total stock return of N assets

,where p_n is the price of asset n in terms of the consumption good in period 1.

lifetime expected utility

$$\max_{c_1,\,\theta_n} V_1(c_1, \ell_2) = u_1(c_1, a(\ell(\theta))) + \beta E_{\ell_2}[u_2(c_2)]$$

note that $\tilde{a}(\theta) = a(\ell(\theta))$

$$\text{s.t.} \quad c_1 + \sum_n p_n \theta_n = \overbrace{\boxed{w_1 + \sum_n p_n}}^{e_1}$$

$$c_2 = \sum_n s_n \theta_n$$

$$\theta = (\overbrace{\theta_1, \theta_2, \ldots, \theta_N}^{\theta_n})$$

$$\max_{\theta_n} u_1(\overbrace{e_1 - \sum_n p_n \theta_n}^{c_1}, \underbrace{a(\ell(\theta))}_{a(\ell(\theta)) = \tilde{a}(\theta)}) + \beta E_{\ell_2}[u_2(\underbrace{\sum_n s_n \theta_n}_{c_2})]$$

$$\text{FOC}: \quad \frac{\partial u_1}{\partial c_1} \overbrace{\boxed{\frac{\partial c_1}{\partial \theta_n}}}^{-p_n \,=} + \frac{\partial u_1}{\partial a} \frac{\partial \tilde{a}}{\partial \theta_n} + \beta E_{\ell_2}\left[\frac{\partial u_2}{\partial c_2} \cdot s_n\right] = 0$$

$$\boxed{\frac{\partial u_1}{\partial c_1} \cdot p_n = \frac{\partial u_1}{\partial a} \cdot \frac{\partial \tilde{a}}{\partial \theta_n} + \beta E_{\ell_2}\left[\frac{\partial u_2}{\partial c_2} \cdot s_n\right]}$$

$$c_1 = e_1 - \sum_n p_n \theta_n$$

$$\frac{\partial c_1}{\partial \theta_n} = \cancelto{0}{\frac{\partial e_1}{\partial \theta_n}} - \overbrace{\frac{\partial}{\partial \theta_n}\left(\sum p_n \theta_n\right)}$$

e.g. $\frac{\partial}{\partial \theta_2}(p_1\theta_1 + p_2\theta_2 + \ldots + p_N\theta_N) = p_2$

$$= 0 - p_n$$

A model of portfolio choice

- The first-order condition for asset n is:

$$\frac{\partial u_1}{\partial c_1} p_n = \frac{\partial u_1}{\partial a} \frac{\partial \tilde{a}}{\partial \theta_n} + \beta E_{l_2}\left[\frac{\partial u_2}{\partial c_2} s_n\right].$$

The first-order condition for asset n

Marginal utility lost when invest in asset n 1 unit

Expected marginal utility from 1 unit of asset n

$$\boxed{\frac{\partial u_1}{\partial c_1} p_n} = \frac{\partial u_1}{\partial a}\frac{\partial \tilde{a}}{\partial \theta_n} + \boxed{\beta E_{l_2}\left[\frac{\partial u_2}{\partial c_2} s_n\right]}.$$

additional term in psychological expected utility

- By reducing consumption by p_n units in the first period, the agent can purchase one unit of asset n, thereby raising consumption by s_n units in the second period.

- The portfolio adjustment, however, also has an effect on the level of anxiety in the first period reflected in the first term on the right-hand side of the FOC.

The competitive equilibrium

- First-period consumption is equal to the endowment:

Demand today = Supply today

$$c_1 = w_1$$

- Second-period consumption is equal to the random output of the assets:

Demand tomorrow = Supply tomorrow

$$c_2 = \sum s_n$$

Asset price

- Rearranging the FOC and plugging in the equilibrium conditions pins down the price of the asset:

standard model without emotion

$\mathbf{1} = (1, \dots, 1) = (\theta_1 = \theta_2 = \dots = \theta_N = 1)$

Price of asset n

$$\boxed{p_n = \frac{\frac{\partial u_1(w_1, \tilde{a}(\mathbf{1}))}{\partial a} \frac{\partial \tilde{a}(\mathbf{1})}{\partial \theta_n} + \beta E_{l_2}\left[\frac{\partial u_2(\sum_n s_n)}{\partial c_2} s_n\right]}{\frac{\partial u_1(w_1, \tilde{a}(\mathbf{1}))}{\partial c_1}}.}$$

c_1

$c_1 = w_1$ & $c_2 = \sum s_2$

$$\frac{\partial u_1}{\partial c_1} \cdot p_n = \frac{\partial u_1}{\partial a} \cdot \frac{\partial \tilde{a}}{\partial \theta_n} + \beta E_{l_2}\left[\frac{\partial u_2}{\partial c_2} \cdot s_n\right]$$

Asset price

positive for asset that causes anxiety

$$p_n = \frac{\overbrace{\frac{\partial u_1(w_1, \tilde{a}(\mathbf{1}))}{\partial a}}^{<0} \frac{\partial \tilde{a}(\mathbf{1})}{\partial \theta_n} + \beta E_{l_2}\left[\frac{\partial u_2(\sum_n s_n)}{\partial c_2} s_n\right]}{\frac{\partial u_1(w_1, \tilde{a}(\mathbf{1}))}{\partial c_1}}.$$

anxiety-averse : $a \uparrow \Rightarrow u_1 \downarrow \therefore \frac{\partial u_1}{\partial a} < 0$

- Since anxiety is aversive, $\frac{\partial u_1}{\partial a}$ is negative. It is immediate that an asset that causes anxiety has a lower price and a higher required rate of return.

Anxiety, Risk, and the Equity Premium

- Suppose that anxiety is decreasing in the mean and increasing in the riskiness of second-period consumption. Suppose that:

$$a(l_2) = -\alpha E_{l_2}(c_2) + \gamma var_{l_2}(c_2), \alpha, \gamma > 0$$

$E_{l_2}(c_2) \uparrow \rightarrow a(l_2) \downarrow$

$var_{l_2}(c_2) \uparrow \rightarrow a(l_2) \uparrow$

- Given that $c_2 = \sum s_n \theta_n$,

$$\frac{\partial\, \tilde{a}(\mathbf{1})}{\partial \theta_n} = -\alpha E_{l_2}(s_n) + 2\gamma cov_{l_2}(c_2, s_2)$$

marginal anxiety for 1 more unit of asset n

Risk-free rate puzzle

➢ For a riskless asset, in which s_n is constant,

$$\frac{\partial\, \tilde{a}(\mathbf{1})}{\partial \theta_n} = -\alpha s_n < 0$$

hold 1 more unit of safe asset → anxiety ↑

→ ~~anxiety ↓~~

Risk-free rate puzzle

- For a riskless asset, in which s_n is constant,

$$\frac{\partial\,\tilde{a}(\mathbf{1})}{\partial\theta_n} = -\alpha s_n < 0$$

$\theta_n \uparrow \to a \downarrow$

< 0

$a \uparrow \to u_1 \downarrow$

< 0

standard model

$$p_n = \frac{\frac{\partial u_1(w_1,\tilde{a}(\mathbf{1}))}{\partial a}\frac{\partial\tilde{a}(\mathbf{1})}{\partial\theta_n} + \beta E_{l_2}\left[\frac{\partial u_2(\sum_n s_n)}{\partial c_2} s_n\right]}{\frac{\partial u_1(w_1,\tilde{a}(\mathbf{1}))}{\partial c_1}} > \frac{\beta E_{l_2}\left[\frac{\partial u_2(\sum_n s_n)}{\partial c_2} s_n\right]}{\frac{\partial u_1(w_1,\tilde{a}(\mathbf{1}))}{\partial c_1}}$$

$p_n >$ price in standard model

- The price of the riskless asset is greater than the price it would take in the standard model.

Risk-free rate puzzle

- The agent is purchasing "peace of mind" along with the asset, and this justifies the low risk-free rate.

Equity premium puzzle

- Since stocks are risky, their purchase will tend to increase both the mean and the variance of second-period consumption.

- The sign of $\frac{\partial \, \tilde{a}(\mathbf{1})}{\partial \theta_n}$ will depend on how these two effects balance out. If γ is sufficiently large relative to α, the effect through the variance will dominate, and $\frac{\partial \, \tilde{a}(\mathbf{1})}{\partial \theta_n}$ will be positive.

Equity premium puzzle

➢ If $\frac{\partial\,\tilde{a}(\mathbf{1})}{\partial\theta_n} > 0$, then

$\theta_n \uparrow \rightarrow \tilde{a} \uparrow$

$$p_n = \frac{\underbrace{\overbrace{\frac{\partial u_1(w_1,\tilde{a}(\mathbf{1}))}{\partial a}}^{<0}\overbrace{\frac{\partial\tilde{a}(\mathbf{1})}{\partial\theta_n}}^{>0}}_{<0} + \beta E_{l_2}\left[\frac{\partial u_2(\sum_n s_n)}{\partial c_2}s_n\right]}{\frac{\partial u_1(w_1,\tilde{a}(\mathbf{1}))}{\partial c_1}} < \frac{\beta E_{l_2}\left[\frac{\partial u_2(\sum_n s_n)}{\partial c_2}s_n\right]}{\frac{\partial u_1(w_1,\tilde{a}(\mathbf{1}))}{\partial c_1}}$$

anxiety-aversive

for risky asset : $p_n \downarrow$ → required return for the asset ↑

→ "equity premium puzzle"

➢ Anxiety will reduce the price of stocks and increase their return relative to the standard model.

Equity premium puzzle

- Stock ownership entails psychic costs. The agent has to live with the anxiety that accompanies the holding of a risky portfolio.

THANKS!

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