

Exercise 4

Application of the Derivative: Related Rates, Linearization & Differentials

1 Linearization & Differentials

1. Determine the linear approximation for $\sin(\theta)$ at $\theta = 0$.
2. Find a linearization of the given function at the indicated number.
 - (a) $f(x) = \ln(x)$ at $x = 1$.
 - (b) $f(x) = x + e^{x-5}$ at $x = 5$
3. Find an approximation of each given quantity by using a local linear approximation.

(a) $(8.1)^{2/3}$

(b) $\frac{(-0.9)^3}{1+(-0.9)^2}$

4. Compute the differential dy for $y = x^2 \sin(2x)$.
5. Compute dy and Δy for $y = \frac{1}{\sqrt{x}}$ as x changes 4 from to 4.02 .
6. Consider the function

$$f(x) = \frac{1}{(1+2x)^3}.$$

- (i) Find a local linear approximation of f at $x = 0$.
 - (ii) Use (i) to approximate $(1.1)^{-3}$.
7. The area of a circle with radius r is $A = \pi r^2$.
 - (i) If the radius of a circle changes from 4 cm to 5 cm, find the exact change in the area.
 - (ii) By using the *approximation from differentials*, what is the approximate change in the area? Determine the *relative error* of this approximation.

2 Related Rates

1. Let $s(t) = 2\sqrt{t} + \cos(\pi t)$ be the position function of an object that moves on a horizontal line. Find the velocity and acceleration functions.
2. Let $s(t) = 2t^3 - 12t^2 + 48$ be the position function of an object that moves on a horizontal line.
 - (a) Find the velocity function $v(t)$ and the acceleration function $a(t)$.
 - (b) Find the position of the object when $a(t) = 0$.
3. A tank in the shape of a right circular cylinder of radius 8 m is being filled with water at a constant rate of $10 \text{ m}^3/\text{min}$. How fast is the level of the water rising? (Or at what rate is the height of the water increasing?)

4. A plate in the shape of an equilateral triangle expands with time. The length of a side increases at a constant rate of 2 cm/hr. At what rate is the area increasing when the side is 8 cm?
5. Ship S_1 sails north and passes a dock with a constant rate of 10 miles per hour at noon. Ship S_2 sails west and passes the same dock at a constant rate of 15 miles per hour at 1:00pm. At what rate is the distance between the two ships changing at 2:00pm?

Additional Problems (Related Rates)

1. A cube is expanding with time. How is the rate at which the volume increases related to the rate at which the length of a side increases?
2. The volume of a rectangular box is $V = xyz$. Given that each side expands at a constant rate of 10 cm/min, find the rate at which the volume is expanding when $x = 1$ cm, $y = 2$ cm, and $z = 3$ cm.
3. A plane at an altitude of 4 km passes directly over a tracking telescope on the ground. When the angle of the elevation is 60° , it is observed that this angle is decreasing at a rate of 30 deg/min. How fast is the plane travelling?
4. A rocket is traveling at a constant rate of 1000 mi/h with a constant angle of 60° to the horizontal ground.
 - (a) At what rate is its altitude increasing?
 - (b) What is the ground speed of the rocket ?
5. An oil tank in the shape of a right cylinder of radius 8 is being filled at a constant rate of 10 m³/min. How fast is the level of the oil rising?
6. Water leaks out the bottom of the conical tank at a constant rate of 1 ft³/min.
 - (a) At what rate is the level of the water changing when the water is 6 ft deep?
 - (b) At what rate is the radius of the water changing when the water is 6 ft deep?
 - (c) Assume the tank was full at $t = 0$. At what rate is the radius of the water changing at $t = 6$ min?
7. Assume that a cube of ice melts in such a manner that it always retains its cubical shape. If the volume of the cube decreases at a rate of 0.25 in³/min, how fast is the surface area of the cube changing when the surface area is 54 in²?
8. A 15-ft ladder is leaning against a wall of a house. The bottom of the ladder is pulled away from the base of the wall at a constant rate of 2 ft/min. At what rate is the top of the ladder sliding down the wall at the instant when the bottom of the ladder is 5 ft from the wall?
9. Sand flows from the top half of the conical hourglass at a constant rate of 4 cm³/s. Express the rate at which the height of the bottom pile increases in terms of the height of the sand.
10. A pulley is secured to the edge of a dock that is 15 ft above the surface of the water. A small boat is being pulled toward the dock by means of a rope on the pulley. The rope is attached to the bow of the boat 3 ft above the water line. If the rope is pulled in at a constant rate of 1 ft/s, how fast does the boat approach the dock when it is 16 ft from the dock?

11. At 8 A.M. ship S_1 is 20 km due north of ship S_2 . Ship S_1 sails south at a rate of 9 km/h and ship S_2 sails west at a rate of 12 km/h. At 9:20 A.M., at what rate is the distance between the two ships changing ?
12. A stone dropped into a still pond causes a circular wave. Assume that the radius of the wave expands at a constant rate of 2 ft/s.
 - (a) How fast does the diameter of the circular wave increase?
 - (b) How fast does the circumference of the circular wave increase?
 - (c) How fast does the area of the circular wave expand when the radius is 3 ft?
 - (d) How fast does the area of the circular wave expand when the area is 8π ft² ?