HW

เล่นพร้อมกัน (ต่างฝ่ายต่างไม่รู้ว่า คนอื่นจะทำอะไร ?)

1. Find Cournot equilibrium when there are 3 firms in the market

$P = a - bq \; ; \; q = q_1 + q_2 + q_3 \longrightarrow P = a - bq_1 - bq_2 - bq_3$

$C_1 = C_2 = C_3 = C \longrightarrow C = C_1 = C_2 = C_3 = \text{constant.}$

What is equilibrium price? : P^{*}

What are firms' profit? $\pi_1 = \pi_2 = \pi_3 = ?$

$$\pi_1 = TR_1 - TC_1 = (\underbrace{a - bq_1 - bq_2 - bq_3}_{P})q_1 - c_1$$

$$\frac{\partial \pi_1}{\partial q_1} = a - 2bq_1 - bq_2 - bq_3 = 0$$

$$a - bq_2 - bq_3 = 2bq_1$$

$$\frac{a}{2b} - 0.5q_2 - 0.5q_3 = q_1$$

. Best Response fn (1)

$$\pi_2 = TR_2 - TC_2 = (a - bq_1 - bq_2 - bq_3)q_2 - c_2$$

$$\frac{\partial \pi_2}{\partial q_2} = a - bq_1 - 2bq_2 - bq_3 = 0$$

$$a - bq_1 - bq_3 = 2bq_2$$

$$\frac{a}{2b} - 0.5q_1 - 0.5q_3 = q_2$$

. Best Response fn (2)

$$\pi_3 = TR_3 - TC_3 = (a - bq_1 - bq_2 - bq_3)q_3 - c_3$$

$$\frac{\partial \pi_3}{\partial q_3} = a - bq_1 - bq_2 - 2bq_3 = 0$$

$$a - bq_1 - bq_2 = 2bq_3$$

$$\frac{a}{2b} - 0.5q_1 - 0.5q_2 = q_3$$

: Best Response fn (3)

แทน q_3 ใน q_1

$$-0.5\frac{a}{2b} + 0.25q_1 + 0.25q_2$$

$$q_1 = \frac{a}{2b} - 0.5q_2 - 0.5\left(\frac{a}{2b} - 0.5q_1 - 0.5q_2\right)$$

$$q_1 = \frac{a}{4b} + 0.25q_1 - 0.25q_2$$

$$0.75q_1 = \frac{a}{4b} - 0.25q_2 = 0.25\frac{a}{b} - 0.25q_2$$

$$q_1 = \frac{1}{3}\left(\frac{a}{b} - q_2\right)$$

แทน q_3 ใน q_2

$$-0.5\frac{a}{2b} + 0.25q_1 + 0.25q_2$$

$$q_2 = \frac{a}{2b} - 0.5q_1 - 0.5\left(\frac{a}{2b} - 0.5q_1 - 0.5q_2\right)$$

$$q_2 = 0.25\frac{a}{b} - 0.25q_1 + 0.25q_2$$

$$0.75q_2 = 0.25\left(\frac{a}{b} - q_1\right)$$

$$q_2 = \frac{1}{3}\left(\frac{a}{b} - q_1\right)$$

แทน q_2 ใน q_1 : $q_1 = \frac{1}{3}\left(\frac{a}{b} - \left(\frac{1}{3}\left(\frac{a}{b} - q_1\right)\right)\right) = \frac{1}{3}\left(\frac{a}{b} - \frac{a}{3b} + \frac{q_1}{3}\right)$

$$q_1 = \frac{a}{3b} - \frac{a}{9b} + \frac{q_1}{9}$$

$$9q_1 = \frac{3a}{b} - \frac{a}{b} + q_1$$

$$8q_1 = \frac{2a}{b}$$

$$q_1 = \frac{a}{4b} \; \#$$

$$q_2 = \frac{a}{3b} - \frac{a}{12b}$$

$$12q_2 = \frac{4a}{b} - \frac{a}{b}$$

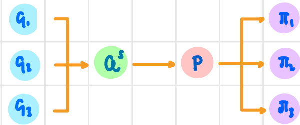
$$q_2 = \frac{a}{4b} \; \#$$

$$q_3 = \frac{a}{2b} - 0.5\frac{a}{4b} - 0.5\frac{a}{4b}$$

$$q_3 = \frac{a}{4b} \; \#$$

$$\therefore \quad P = a - b\left(\frac{a}{4b} + \frac{a}{4b} + \frac{a}{4b}\right)$$

$$P = a - \frac{3}{4}a$$



$$P = 0.25a \quad \#$$

$$\pi_1 = P \cdot q_1 - c_1 = 0.25a \cdot \frac{a}{4b} - c_1 = \frac{a^2}{16b} - c_1 \quad \#$$

$$\pi_2 = P \cdot q_2 - c_2 = 0.25a \cdot \frac{a}{4b} - c_2 = \frac{a^2}{16b} - c_2 \quad \#$$

$$\pi_3 = P \cdot q_2 - c_3 = 0.25a \cdot \frac{a}{4b} - c_3 = \frac{a^2}{16b} - c_3 \quad \#$$

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2. If there are N firm

$q_i^* = f(N)$, $P = (N)$, $\pi_i = f(N)$

$P = a - bQ$; $Q = q_N$

$c_N = c_u = c_\mu = c$

$\pi_i = P(Q)q_i - c_i q_i$

$= (a - b(q_N))q_i - c_i q_i$

Assume that : $q_1 + q_2 + \dots + q_n = A$

$$P = a - b(q_1 + q_2 + \dots + q_n)$$

$$P = a - bq_1 - bq_2 - \dots - bq_n$$

$$\pi_1 = (a - bq_1 - bq_2 - \dots - bq_n) \cdot q_1 - c_1$$

$$\vdots \qquad \vdots \qquad \vdots$$

$$\pi_n = (a - bq_1 - bq_2 - \dots - bq_n) q_n - c_n$$

$$\frac{\partial \pi_1}{\partial q_1} = a - 2bq_1 - bq_2 - \dots - bq_n = 0$$

$$\frac{a}{2b} - 0.5(q_2 + q_3 + \dots + q_n) = q_1$$

$$\frac{a}{2b} - 0.5(q_1 + q_2 + \dots + q_{n-1}) = q_n$$

จัดรูป : $q_1 - 0.5q_1 = \frac{a}{2b} - 0.5(q_1 + q_2 + \dots + q_n)$

$$0.5q_1 = \frac{a}{2b} - 0.5A$$

$$q_1 = \frac{a}{b} - A$$

$$q_2 = \frac{a}{b} - A$$

$$\vdots$$

∴ จัดรูปเหมือนกัน ; $q_n = \frac{a}{b} - A$

q บวกกัน n ครั้ง ได้ nq

จับ $q_1 + q_2 + \dots + q_n \Rightarrow A = n\left(\frac{a}{b} - A\right)$

$$A = n\frac{a}{b} - nA$$

$$(n+1)A = n\frac{a}{b}$$

✱ $A = nq$ $\longrightarrow$ $A = \frac{na}{(n+1)b}$

$$q_i = \frac{a}{(n+1)b} \quad \#$$

$$P = a - b(A) = a - \cancel{b}\left(\frac{na}{(n+1)\cancel{b}}\right)$$

$$P = a - \frac{n}{n+1}a = \frac{a(n+1) - na}{n+1} = \frac{\cancel{na} + a - \cancel{na}}{n+1}$$

$$\boxed{P = \frac{a}{n+1}} \quad \#$$

$$\pi_i = P \cdot q_i - c_i = \frac{a}{n+1} \cdot \frac{a}{(n+1)b} - c_i$$

$$\boxed{\pi_i = \frac{a^2}{(n+1)^2 b} - c_i} \quad \#$$

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3. from Q_2 ; What happen if $N \to \infty$

$N = 1$

if $n \to \infty$: $q_i = \frac{a}{(n+1)b} \longrightarrow 0$ # แต่ละ firm จะขายเข้าใกล้ 0 unit

: $A = nq_i \longrightarrow \infty$ # ทุก firms รวมกันที่ Q เยอะมาก เข้าใกล้ ∞ unit

: $P = \frac{a}{n+1} \longrightarrow 0$ # Supply ↑ ⇒ P ↓ จนเข้าใกล้ 0 #

: $\pi_i = \frac{a^2}{(n+1)^2 b} - c_i \longrightarrow -c_i$ # กำไรแต่ละ firm จะขาดทุน fixed cost = C #

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if $n \to 1$: $q_i = \frac{a}{(n+1)b} = \frac{a}{2b}$ # Monopoly จะขายน้อยลง $Q = \frac{a}{2b} < Q = \frac{na}{(n+1)b}$

: $A = nq_i = Q$ # $n = 1 \therefore$ firm เป็น Monopoly

: $P = \frac{a}{n+1} = \frac{a}{2}$ # $P_M = \frac{a}{2} > P = \frac{a}{n+1}$

: $\pi_i = \frac{a^2}{(n+1)^2 b} - c_i = \frac{a^2}{4b} - c_i$: $\pi_M > \pi_i = \frac{a^2}{(n+1)^2 b} - c_i$